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**Pringle : equitangential bending active frame and minimal surface robe-net**

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Abstract

Membrane surfaces usually follow the principles of traditional formfinding methods. In analogy to soapfilm experiments, anticlastic curved, tension-only surfaces are formfound by predefining closed boundary conditions. In the realization, these boundary conditions represent essential structural elements, but mostly they are neglected in terms of architecturally designed, space defining elements. In 2009, the unrealized, competition winning entry (Filz) for a roof cover for the Guatemala relief map raises the architectural demand for an integrated solution, where the bordering frame-like structure and its enclosed membrane surface form an architectural and structural unity.

This paper presents the follow-up research, which led to the experimental structure of the “Pringle” in 2017, a minimal surface within an actively bent, annular stripe (Filz and students at the University of Innsbruck). Such a closed elastic stripe with overlenght is frustrated by geometry, forcing it to buckle. Its non-planarity would increase both the curvature and torsion depending on the magnitude of its overlenght but the resulting shapes are determined by minimizing the total elastic energy. The tension forces from the filling, minimal surface could be absorbed by using the strong axis of the annular stripe while its weak axis allowed for elastic bending into the selforganized shape. In retrospective, the analysis and the comparison of results of different setups and configurations of the structure have been analyzed and in a collaborative accomplishment by Filz, Shahzad, and Niiranen at Aalto University, Finland.

Keywords: active bending, structural membrane, minimal surface, experimental structure, geometric non-linearity, FEM, large deformations.

1. Introduction, Context, Overview

Basically this paper gives an overview, respectively provides a review on the research, experimental tests and structural analysis, which aim to overcome the fact that the boundary conditions, which represent essential structural elements in the generation of membrane structures mostly are neglected in terms of architecturally designed, space defining elements in the realization. Predefining closed boundary conditions usually is the first step in the formfinding process for structural membranes. Once these structural and form-generating elements are defined, they are in many cases structurally analyzed and designed but not architecturally questioned any more. On the other hand considering these structural members as integral part of the global structure bears the potential of creating architectural, formal and structural entities with possibly spatially and structurally added value.

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Before going into detailed explanation of the “Pringle”, the concept and general idea, which is rooted in the unrealized, competition winning entry (Filz, 2009) for the permanent tensile cover for the relief map of Guatemala (Fig.01) shall be explained.
Between 2010 and 2013 some variations of the original concept have been explored, which will not be presented in this paper in depth. Basically these variation have been guided by parameters like economical optimization and/or geometrical optimization for fabrication, and issues like comparing temporary and permanent versions.

This paper presents the follow-up research, which led to the experimental structure of the “Pringle”, which was finally realized at the University of Innsbruck, Austria in 2017. The experimental structure has been composed by a minimal surface within an actively bent, annular stripe (Filz and students at the University of Innsbruck). The project seeks to right-proof the hypothesis of the coexistence of an equitangential, bending active, annular frame and minimal surface, and showcasing the use of their geometrical match in 3D in mainly two regards: firstly, the utilization of the strong axis of the annular stripe perfectly absorbs all axial forces from the minimal surface and secondly, to use the annular stripe’s weak axis for elastic bending into its selforganized shape.

Lienhard et. al. describe that active-bending is applied intentional for the shaping process to achieve a predefined geometry. It is one advantage of actively-bent elements that the same straight/flat elements can be used for different curvatures. Actively-bent elements are defined as elements that are transformed from the stress-free start-geometry to their end-geometry by elastic bending. Possible elements are beams and plates with a usually straight respectively flat start-geometry [1]. The authors of this paper agree with above mentioned definition in the sense, that the shape/geometry of actively-bent elements, which basically follow elastica curves, can be predicted. At the same time the geometrical result of active bending is the result of a selforganizing process, where its shape cannot be directly designed but influenced by predefining the setup of boundary conditions. This is of fundamental importance for above mentioned hypothesis.

In retrospective, the structure has been analyzed in a collaborative accomplishment by Filz, Shahzad, and Niiranen at Aalto University, Finland. This analysis looks into mainly two aspects: firstly, into the simulation of the lifting process of the plywood frame forming the spatial configuration of the frame and secondly, into a stress analysis for the final configuration of the frame with tensional ropes. The comparison stress states of the frame with and without initial stresses resulting from the (bending) deformations of the lifting process aims at distinguishing and demonstrating the roles of geometry and initial stresses of the actively bent frame structure.

Figure 01: The relief map of Guatemala (Francisco Vela, 1904) in Minerva Park, Guatemala City and the competition winning entry (Filz, 2009) for its permanent tensile cover.

1.1. Permanent tensile cover for the relief map of Guatemala in Minerva Park, Guatemala City
The relief map of Guatemala in Minerva Park, Guatemala City is a scaled model in an open-air public monument that represents the geography of Guatemala (Fig.01). It was designed in 1904 by Engineer Francisco Vela and covers 1,800 square meters. This relief map shows all natural and non-natural geographical features like rivers, lakes, volcanoes, ports, railways, roads, provinces, districts and important locations and is viewed from platforms located on each side. This work is unique in the world, as no other country has a relief map of such dimensions. Due to its historical and artistic value, it is considered national heritage. Having been exposed to environmental conditions like sun, wind and rain for more than 100 years it has been decided to protect the relief map by a freely spanning, tensile cover.
1.2. Iterative Design Steps

The cover´s outlines in plan have been designed in a way to not only roof but to enclose the relief map of Guatemala and to generate divers paths and spaces for the flow of visitors around the relief map. These outlines have been brought into 3d by different parameters like axis, views and covering the path, making space for viewing platforms and so on. So, the closed splines were tilting and moving in space as a response to infrastructural needs and functional necessities like views or shading.

The designed, closed boundary frame and its effect on the resulting shape of the filling minimal surface have been checked on functional and structural performance. Feedback loops have been generated in order to analyze the structural efficiency of the overall membrane surface as well as its tangential direction and tensile forces at the boundary. The received results has been compared with the initial geometry of the boundary frame, and changed accordingly. First rough iterative steps have been made with the help of the Rhinoceros plugins Grasshopper and Rhino Membrane. Structural analysis and fine-tuning has been done with help of Forten 3000 (Fig.02 a-c). The final progression of the boundary curve was found in iterative steps and is directly linked to the shape in plan and cross-section as well as the dimensioning of the created steel-frame. The result represents an architecturally and structurally optimized 3d-frame with basically elliptical cross-section from iterative response of minimal surface and boundary spline.

2. Equitangential bending active, annular stripe and its minimal surface filling

The shape/geometry of actively-bent elements, which basically follow elastica curves, can be predicted. At the same time the geometrical result of active bending is the result of a selforganizing process, where its shape cannot be directly designed. It can only be influenced by predefining the setup of boundary conditions. The same is valid for physical minimal surfaces. Its surface geometry, surface tension and curvature can only be indirectly designed by choosing and adapting the constellation of boundary conditions. This is of fundamental importance for the following hypothesis: Both “systems” geometrical results are the equilibrium shape of selforganized processes. Therefore there should be at least one predefined, geometrical setup for the bending active annular stripe, respectively its spatially undulating configuration and its selforganized inclination to perfectly match the tangential direction of its filling by a physical minimal surface. So, the aim of the research and project was to generate bidirectional interaction of minimal surface and its boundary condition, a bending active wooden stripe, until the tangential direction of the minimal surface matches the inclination of the stripe. Both structural members inform each other in feedback-loops about their geometry and find a common equilibrium shape.

2.1. Theoretical Background

As described earlier active-bending is applied intentional for the shaping process to achieve a predefined geometry. An annular, circular and elastic (later actively bent) stripe of any width is divided at one or a couple of locations. At these divisions further segments with the same geometry of the annular circular stripe are added, causing self-overlapping over-length. When assembling the elastic stripe to a closed “ring”, the resulting shape depends on the length of the inserted segments and basically forms a
“Pringle”-like shape. So, a closed elastic stripe with over-length is frustrated by geometry, forcing it to buckle. Its non-planarity would increase both the curvature and torsion depending on the magnitude of its over-length but the resulting shapes are determined by minimizing the total elastic energy.

In “Geometric Mechanics of Curved Crease Origami” in the Physical Review Letters published in September 2012 Marcelo A. Dias, Levi H. Dudte, L. Mahadevan, and Christian D. Santangelo [2] describe this phenomenon as follows. “...An annular circular strip that is folded along a central circular curve to form a three-dimensional buckled structure driven by geometrical frustration. We quantify this shape in terms of the radius of the circle, the dihedral angle of the fold, and the mechanical properties of the sheet of paper and the fold itself. When the sheet is isometrically deformed everywhere except along the fold itself, stiff folds result in creases with constant curvature and oscillatory torsion. However, relatively softer folds inherit the broken symmetry of the buckled shape with oscillatory curvature and torsion.....”

A more specified investigation on a numerical solution was done by Dragos I Naicu and Chris J K Williams from the Department of Architecture & Civil Engineering, University of Bath, when this problem that was introduced to them by the first author at the FORM-RULE|RULE-FORM Workshop and Symposium 2014 [3]. In “The bending of very thin, narrow circular strips “ they describe “... the theory and computation of the bending of a very thin, narrow circular strip. The “very thin” means that we assume that the bending stiffness of the strip is small in comparison to the membrane stiffness, which in turn means that the surface of the strip has zero Gaussian curvature and remains developable. The theory applies to both wide and narrow strips, but the solution of the equations is easier if the strip is “narrow” in comparison to the distance between the centerline of the strip and the edge of regression of the tangent developable surface.” The resulting shape is maintained and stabilized by the distribution of internal forces, which tend to be harmonized. If this ring is cut at any point it releases its evenly distributed stresses, develops into a planar geometry and forms an overlapping annular circular stripe. Naturally the spatially undulating ring would form two “low-points” and two “high-points”. For self-standing purposes (tripod) the experimental structure of the “Pringle” was realized as a three-three version.

![Image of Pringle-like shape](image)

**Figure 03:** a) rise of high points and increase of inclination-angle at the high- and low-points at increasing over-length, b) equitangential match of the undulating, annular circular stripe and its filling by soapfilm, c) equitangential matching stripe and minimal surface

2.2. Finding the appropriate Over-length

Based on above mentioned theoretical background a feedback-loop between minimal surface, and the geometry of the actively bent annular circular stripe, which is dependent from the added over length has been generated. It has been assumed that the three-three version would behave symmetrically – three times 120°. With increasing over-length the high points rise and the inclination-angle at the high- and low-points increases (Fig.03a). If the inserted segments are too large the stripe starts to flip over and to touch itself. Basically the rise and the inverse inclination-angle at high- and low-points have been terminated and evaluated at different over-length. The bidirectional interaction of minimal surface and its boundary - “Pringle” iteratively approximated the tangential direction of the minimal surface at the boundary and the transversal inclination, respectively torsion, of the stripe (Fig.03b). The shape matching with the minimal surface results from a stripe with an over-length of about 1/6 (Fig.03c). At the same time we can recognize increased stability of the 3-dimensional form as an equilibrium of curvature and elastic material properties.
2.3. Physical Modelling, Soapfilm and Simulation

In physical model the stripe has been subdivided into six equal circle segments: three third of a circle with three added over-length in the range of 1/4 to 1/8 have been tested. To be able to have an accurate reading and understanding of the most important dimensions and numbers the real dimensions were chosen as follows: diameter 40 cm, width: of annular circular stripe 2.5 cm. Copolyester has been used as material for physical modelling. Its material properties, the resulting bending stresses at chosen dimensions showed matching results with digital simulation. The self-weight has been neglectable and did not influence the results. Smaller physical models have been used for soapfilm experiments in order to proof the equitangential match of the undulating, annular circular stripe and its filling by soapfilm (Fig.03b).

Basically the natural (two high- and two low-points) shape of the undulating stripe is forming a stable equilibrium but a configuration of three high-points and three low-points shows a certain “rolling – effect”. This means that the boundary stripe tends to relax into its initial two-two configuration. For this reason a triangle, which is connecting and determining the distance of the low-points was necessary to stabilize the global shape of the stripe. For the later installation process this triangular connection has been helpful, although the “rolling-effect” has been totally blocked by the prestressed, anticlastically curved membrane.

3. Pringle: Proof of Hypothesis and Concept

Although, for the realization it was decided to aim for a stripe with three instead of two undulations, which would have been the “natural” shape, the geometrical constraints basically stay the same. The dimensions and choice of material for the full scale experimental structure were as follows: The annular stripe was made from three layers of 600mm x 6mm plywood and its pieces were screwed together in three double rows. No glue was added between the layers and the holes for the screws did not allow for longitudinal sliding. The orthotropic material itself consists of five layers of beech wood, with three layers in longitudinal and with two layers in cross direction. Its volumetric mass density is around 600 kg/m3. In planar state the self-overlapping wooden annular stripe had a diameter of 9m. As a kind of foundation and in order to lift the whole structure for functional reasons from the ground an inverted spherical triangle replaced the triangular connection of low-points. The spherical triangle consisted of three times two layers of beech plywood. Other plywood plates were snapped into the spherical triangle, connecting the layers, preventing them from torsional buckling, and providing seating at the same time

(Fig.04 center). When assembling the wooden stripe to a closed “ring”, the resulting shape had a total height of 2,8m, and it was cantilevering 2,5m from the three low-points. Compared to earlier test installations, the wooden ring has been supported by leaf-spring-like elements, in order to avoid buckling and potential snap-through due to the rings total self-weight of about 400kg, although the later inserted robe-net would have counteracted. The above mentioned “rolling –effect” has been totally blocked by the prestressed, anticlastically curved prefabricated robe-net from Liros prestretch high performance yachting robe (Fig.04 right). A closed membrane surface would have been even more beneficial for the stability because it would create much more shear stiffness compared to the robe net, which allows for an angular change of the mesh.
The whole installation process has been managed by physical strength only (Fig.04 left). The annular stripe’s weak axis allowed for connecting the ends of the overlapping as well as for elastic bending of the stripe into its natural shape. The strong axis of the annular stripe perfectly absorbed all axial forces from the minimal surface robe-net and allowed even for climbing on it (Fig.04 center).

An act of vandalism caused a complete snap-through of the whole structure, but was reversible without any visual damage of the structure.

4. Structural analysis

Structural analysis for the structure has been accomplished in two parts. First, the realized final shell configuration with the minimal surface rope net is analyzed for the self-weight of the plywood strip. In this analysis, the initial stresses of the structure are not taken into account, i.e., the geometry of the structure is considered as a given configuration not as a results of a bending process for an originally planar structure. Second, a part of the deformation process for forming the “pringle”-like annular geometry of the strip is analyzed as a quasi-static geometrically nonlinear deformation. This process gives a view on the initial stresses of the bending-active annular strip structure.

4.1. Geometrically linear static structural analysis for the final geometry

The final geometry with and without the rope net is exported from a commercial, computer-aided design software Rhino3D through the STEP file format (.stp) and then imported into a commercial finite element software Comsol Multiphysics for structural analysis. Orthotropic material parameters are assigned to the annular stripe of three layers of plywood (modelled by shell elements), with volumetric mass density $\rho = 600 \text{ kg/m}^3$, Young’s modulus vector $(E_1, E_2, E_3) = (9019, 8481, 676) \text{ MPa}$, shear modulus vector $(G_1, G_2, G_3) = (620, 205, 190) \text{ MPa}$, Poisson’s ratio $\nu = 0.4$ assumed have the same value in all the three directions. For Liros pre-stretch high-performance yachting ropes used in the net (modelled by beam elements) the blank material option of the software is used with the following material constants: $\nu = 0.45$, $\rho = 1380 \text{ kg/m}^3$ and $E = 500 \text{ MPa}$. For the self-weight of the shell structure, gravity load from the “face and volume load option” is selected, and pinned boundary conditions are given to the three points touching the ground. The shell-beam connections are given with the corresponding free rotation manner as well. In total, after a convergence study for the geometry and field approximation, the chosen final mesh consists of 2443 (default MITC) shell elements and 3374 (Euler–Bernoulli) beam elements.

The vertical displacement fields of the annular strip without and with the rope net (top and bottom, respectively) are compared in Figure 05. As expected, the fields are qualitatively identical but the maximum displacement with the net (41.5 mm) is about one fourth of the maximum displacement of the net-augmented one (155 mm). Bending von Mises stress distribution of the shell strip is presented in Figure 06. As can be seen, the stress level with the net is very low (maximum level about $10^{-2} \text{ MPa}$) and the distribution is fairly uniform across the width and perimeter of the strip. Without the net, the stresses increase near the ground points (maximum level about $10^1 \text{ MPa}$).

![Figure 05: Vertical displacement field of the strip without (top) and with (bottom) the rope net.](image-url)
Figure 06: Top surface bending von Mises stress field without (top) and with (bottom) the rope net.

Figure 07 presents the stress distributions in the beam elements of the net. It is worth mentioning that shear, bending and torsional stresses of the beam elements are negligible with respect to the axial ones, as expected. In general, axial forces are quite equally distributed in the net. However, the horizontal middle rope, in particular, has clearly higher tension than any other part of the net. The reason behind this is most probably a dimension anomaly resulting either from the iterative design process within the Rhinoceros software or from the file exchange between the design and analysis software.

Figure 07: The top view of von Mises stresses (left) and local axial stresses (right) in the beam elements.

4.2. Geometrically nonlinear quasi-static structural analysis of the bending process

The bending process of a planar annular strip (see Figure 8 (left)), or better, a helix strip with overlap of angle $\pi/2$ (with pitch 0.005 m), for forming the “pringle” structure is next analysed via geometrically nonlinear finite element (FE) analysis by a commercial FE software Abaqus. The strip geometry is created in Comsol, exported as a Parasolid text file (.x_t) and imported into Abaqus where the geometry is discretized by shell elements (bilinear S4RS). One end of the strip is fixed, whereas a rigid connector is attached to the other end which is moved via quasi-static incremental-iterative steps (the Riks method relying on Newton–Raphson iteration) towards the fixed end. In fact, due to convergence problems or the instability of the structure about 97 percent of the desired forced displacement was finally achieved (as visible in Figure 8). Anyway, the process results in a curved “pringle” geometry presented in Figure 8 (right) with a von Mises bending stress distribution. This geometry touches the ground at two points, unlike the final geometry analysed above. Accordingly, as a future step, our aim is to bend the “pringle” structure in order to form the realized “pringle”-like structure touching the ground at three points, and furthermore, to add the ropes for accomplishing the final structural analysis. With this analysis, we target at demonstrating the roles of geometry and initial stresses of the actively bent frame structure.
Figure 5: The undeformed shape and shell element mesh (left) and the deformed shape with von Mises stress distribution (center and right) of the strip.

5. Conclusion

In conclusion, this research right-proofed the hypothesis of the coexistence of an equitangential, bending active, annular frame and minimal surface. At the same time the experimental structure showcased the potential of an integrated thinking of formgenerating boundary conditions as architectural and structural elements.

The realization of the Pringle took advantage of the strong axis of the annular stripe, which perfectly absorbs all axial forces from the minimal surface and simultaneously used the annular stripe’s weak axis for elastic bending into its selforganized shape as well as for the benefit of the installation process.

Altogether, the structural analysis supports the following conclusions. First, the rope net serves as a tensional membrane-like structural system supporting the shell frame against its self-weight and other prospective vertical loadings. On the other hand, due to the equitangential connection between the frame and the net, combined with the minimal surface property of the net, prospective loadings will result in membrane stresses in the frame, primarily. The role of the initial bending stresses stemming from the geometrically nonlinear bending process for forming the frame geometry, now simulated only partially, remains as an open question.

References

