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A Convolutionally Encoded OSTBC System with SNR-Adaptive Constellations for Low-Latency and Low-Complexity Communications

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Abstract—The 5G and beyond wireless networks target novel use-cases with stringent requirements on physical layer parameters such as latency, which may necessitate design decisions with trade-offs. Choosing a channel coding scheme which gives the best error performance may not be the most appropriate choice in mission-critical machine-type communications which require very low latency. In such contexts, convolutional codes may be preferable due to their faster decoding and simplicity. Motivated by this observation, we introduce a convolutionally encoded OSTBC system with SNR-adaptive constellations for low-latency and low-complexity use-cases. Numerical results obtained for the versatile Nakagami-m channel demonstrate that the investigated scheme results in considerable gains in terms of error performance and decoding latency.

Keywords—constellation shaping, convolutional coding, low-latency communications, 5G network

I. INTRODUCTION

The use case and application diversity in wireless networks will set new requirements for the radio access technologies [1]. Previously, reducing the error rate was the primary goal of the channel coding techniques for several decades. The invention of the capacity-approaching/achieving codes such as the polar, LDPC, and turbo codes in the last two decades has been among the key developments resulting from this motivation at the expense of complexity in encoder/decoder structures.

However, it may not be possible to always deploy these powerful coding techniques in 5G and beyond networks since other design consideration may become more dominant than before, such that; emerging requirements in terms of decoding latency and implementation complexity, especially for ultra-reliable and low-latency communications [2]. This trend can be observed in machine-centric communications where the superiority of capacity-approaching/achieving codes may disappear in low-latency communications due to iterative decoding process [3]. Specifically, the mentioned powerful coding techniques suffer from tackling decoding latency requirements [4], [5] or having error floor region [3]. The advantages of convolutional encoders over capacity-approaching codes were summarized in [6].

From this point of view, convolutional encoders accompanied with constellation design which minimize the error of a given convolutionally coded scenario can be promising especially when convolutional encoder can be strengthened with a SNR-adaptive constellation design. It was already proven that a single constellation might not be the best one for all possible SNR ranges [7] and interesting use cases of SNR-adaptive optimized constellations have been recently provided by digital video broadcasting standards [8]. The advancements in computing ability enable researchers to design constellations design without any predefined assumption on the symbol locations since most current optimization frameworks have the ability to work with very large search space without any hardware and software limitations.

Different optimization formulations can be found in [9]–[12] regarding various wireless and fiber-optical scenarios. However, it was shown in [13] that working with any choice of constellation may require more generalized error performance analysis for some pair of constellation and encoder in coded scenarios. A detailed description of the performance analysis over convolutionally encoded scenarios with any given constellation can be found in [14] showing that the analysis based on all-zero transmitted sequences is no longer valid for convolutionally encoded scenarios where irregular constellations are employed.

Besides constellation design and channel coding, diversity and space-time coding can be used to decrease outage probability and latency when the delays caused by automatic repeat request techniques are unacceptable. The encoded space-time coding systems along with different channel coding techniques were already proposed in [15]–[17] where trellis coded modulation and convolutional encoders were mostly used as outer coders for different channel models. Meanwhile, the performance analysis of these studies depend on all-zero transmit sequences which may not be valid for some type of constellations as shown in [14].

Motivated by these facts, we study a SNR-adaptive convolutionally encoded OSTBC system assuming Nakagami-m fading channel. The sets of SNR-adaptive optimized irregular constellations particularly designed for a convolutionally encoded OSTBC system are deduced first. These constellations are defined for different average SNR and Nakagami-m shaping parameter values and result in better error performance and lower decoding delay compared to conventional fixed M-ary constellation schemes.
II. CONVOLUTIONALLY ENCODED OSTBC SYSTEM

We consider a convolutionally encoded OSTBC transmission model which consists of K orthogonal transmission stages as shown in Fig. 1. This model may appear when using coordinated multi-point (CoMP) and relaying where the source and relay link includes error-free transmission. An N_s-length information bit block belonging to lth frame, \( h_l \), is first encoded by a rate-R convolutional encoder; then, the encoded bits, \( c_l = [c_{l,1} \cdots c_{l,N_s}] \), are fed into the symbol mapper where a choice of M-ary constellation is applied. Note that a choice of constellation for each transmission stage can differ, while the same encoded bits are considered in each stage. Then, output symbols for \( k \)th orthogonal phase, \( s^{(k)}_l = [s^{(k)}_{l,1} \cdots s^{(k)}_{l,N_s}] \), are transmitted after using OSTBC encoder. For instance, the transmitting code-block of the Alamouti coded scheme [18] for \( k \)th stage, \( S^{(k)}_l \), can be expressed as

\[
S^{(k)}_l = \begin{bmatrix}
   s^{(k)}_{1,1} & -s^{(k)}_{1,2} & \cdots & -s^{(k)}_{1,i-1} & s^{(k)}_{1,i+1} & \cdots & s^{(k)}_{1,N_s} \\
   s^{(k)}_{2,1} & s^{(k)}_{2,2} & \cdots & s^{(k)}_{2,i-1} & -s^{(k)}_{2,i+1} & \cdots & -s^{(k)}_{2,N_s}
\end{bmatrix},
\]

where \((\cdot)^*\) denotes the complex conjugate of the corresponding output symbol \( i \in \{1, N_s/2\} \). Herein, each column corresponds to different time slot and each row denotes different transmitting antenna.

The channel coefficients during lth frame, \( h_l = [h_{l,1}, \ldots, h_{l,N_s}] \), are modeled by quasi-static Nakagami-m fading model with a shaping parameter \( m \) and an average fading power \( \Omega \). By doing so, the channel coefficients for one code-block, \( (h_{l,2i-1}, h_{l,2i}) \), stay the same during the transmission of \( s^{(k)}_{l,2i-1} \) and \( s^{(k)}_{l,2i} \). The end-to-end transmission for lth frame can be formulated as

\[
R_l = \sum_{k=1}^{K} H^{(k)}_i S^{(k)}_l + N^{(k)}_i,
\]

where \( H^{(k)}_i = [h^{(k)}_{1,1} h^{(k)}_{1,2} \cdots h^{(k)}_{1,2i-1} h^{(k)}_{1,2i}] \) and \( N^{(k)}_i \) represents the additive white Gaussian noise (AWGNN) matrix with zero-mean and \( N_0/2 \) variance per dimension. We assume independent fading between the K orthogonal transmission stages, with possibly different shaping parameters \( m_k \) and link powers \( \Omega_k \). The average received SNR at the receiver for kth stage can be explicitly defined as \( \gamma_k = \Omega_k / N_0 \) where \( \Omega_k \) can be interpreted as the path-loss term.

III. SNR-ADAPTIVE TRANSMISSION MODEL

In this section, SNR-adaptive transmission model is described in a detailed way; starting from error performance calculation for the convolutionally encoded OSTBC system along with any given constellation and it is followed by an optimization framework to seek out for optimized symbol point locations without any predefined structure.

A. Error performance bound

In order to establish an optimization framework without any constraint on symbol point locations for any given convolutionally encoded scenario, the product-state matrix method [19] is required in the calculation of the upper bound on the BER, \( P_e \). Since this method considers all possible combinations of the encoded and the decoded states rather than simply assuming all zero bits transmitted from the encoder, there are \( N^4 \) product-states for a \( N \)-state convolutional encoder. The detailed description of the product-state matrix technique for convolutional encoded case can be found in [14].

In case of the convolutionally encoded OSTBC, the probability \( \mathcal{D}_{(u,v),(\bar{u},\bar{v})} \) of the transition from \( u \)th state to \( v \)th state at the encoder while \( \bar{u} \rightarrow \bar{v} \) occurs in the receiver under maximum likelihood decoding can be reformulated as in [14]

\[
\mathcal{D}_{(u,v),(\bar{u},\bar{v})} = \prod_{k=1}^{K} \left( 1 + \Omega_k |s^{(k)} - \hat{s}^{(k)}|^2 / (4N_0m_k) \right)^{-Dm_k},
\]

where \(|s^{(k)} - \hat{s}^{(k)}|^2\) corresponds the squared Euclidean distance between the corresponding output symbols related to the transitions, \( u \rightarrow \bar{u} \) and \( v \rightarrow \bar{v} \) within integer-valued Nakagami-\( m \) fading scenarios and \( D \) denotes the diversity order of the OSTBC encoder. After calculating each entry of the product-state matrix, \( S \), from (3), and the entries of \( S \) are rearranged as \( S = [S_{GB} S_{Gb}; S_{BG} S_{Bb}] \) [14]. Then, \( P_b \) can be found as [19], [20]

\[
P_b \leq \frac{1}{l} \frac{\partial}{\partial l} l^T S_{GB} l + (l^T S_{Gb})^T \left[ I - S_{BB} \right]^{-1} S_{Bb} l \bigg|_{l=1},
\]

where \( l \) is the number of information bits per output symbol, \( I \) and \( I \) denote the unity and identity matrices, respectively.

B. Finding SNR-adaptive optimized irregular constellations

Particle swarm optimization (PSO) algorithm is applied to find SNR-adaptive optimized irregular constellations, \( \chi (m, \gamma) \), which can vary with \( m \) and average SNR, \( \gamma = \Omega / N_0 \). Specifically, it is aimed to minimize (4) for a given convolutional encoder and \( \gamma \) under the average symbol energy constraint:

\[
\sum_{i=0}^{M-1} |s_i|^2 < 1, \quad s_i \in \mathbb{C}, \forall i.
\]

The PSO technique has smaller computational load and fewer tuning parameter than other evolutionary algorithms [21]. The used parameters for constellation search can be found in [22].
The PSO optimizer requires the following parameters before going through the constellation search: the fading parameter \( m \), the average fading power \( \Omega \), the modulation order \( M \), the swarm size \( P \), and optimizer parameters \( (r_1, r_2, w, N_{iter}) \). After initializing the positions of \( P \) particles\(^5\), \( x_i^0 \), and their velocities, \( v_i^0 \), the fitness value of each particle is calculated from (4). Then, the best value of the swarm, \( g_p^0 \), which is the one giving the minimum value of \( P_h \), is calculated. The velocities and positions of the particles are updated as follows:

\[
\begin{align}
  x_i^n &= x_i^{n-1} + v_i^n \\
  v_i^n &= v_i^{n-1} + r_1 c_1 (g_p^{n-1} - x_i^{n-1}) + r_2 c_2 (g_p^{n-1} - x_i^{n-1})
\end{align}
\]  

(6a)

(6b)

Calculations of (4) are carried out by considering these updated values and the ones giving the lowest \( P_h \) values are kept as the updated particles. At the end of \( N_{iter} \) iteration, the \( x_i^{N_{iter}} \) yields to the optimized irregular constellation, \( \chi (m, \bar{\gamma}) \).

C. SNR-adaptive transmission

Once the optimization framework is completed by determining the optimized symbol point coordinates over a range of \( \bar{\gamma} \) values based on the channel characteristics, the set of optimized constellations, \( (\gamma, m) \) are stored in look-up tables (LUTs) along with their corresponding \( \bar{\gamma} \) and \( m \) values as their labels. Then, the optimized irregular constellation, \( \chi (m, \bar{\gamma}) \), can be used during the transmission which corresponds to the same \( m \) and \( \bar{\gamma} \) values.

This SNR-adaptive transmission model is illustrated in Fig. 2 where the LUTs store the information of constellations used in both the transmitter and the receiver. Note that the choices of optimized irregular constellations may also differ once the used puncturing pattern changes at the transmitter so this information is expected to be available in both sides as well.

IV. Simulation Results

In the following, we consider a convolutionally encoded transmission system along with Alamouti scheme in order to validate the error performance improvement due to using the SNR-adaptive optimized irregular constellations. A rate-1/2 convolutional encoder, \([5, 7]_8\) is employed along with Alamouti encoder which consists of two transmit and single receive antenna. We employ Alamouti space-time code, because it is capacity optimal for this set-up. In simulations, different modulation orders and Nakagami shaping parameters, \( m \), are applied to compare the performance improvement in different scenarios. The frame-length, \( N_b \), is set to 120 bits. The optimized irregular constellations are obtained for a range of average SNR values with a 1 dB interval. The soft-decision Viterbi decoding is carried out at the receiver side.

A. Error performance

To begin with, the SNR-adaptive optimized irregular constellations for 4-ary signalling cases are compared with 4-QAM constellation which is used for all considered \( \bar{\gamma} \) values. In Fig. 3, simulated BER results are shown for \( m = 1 \) and \( m = 3 \) where the convolutionally encoded scenario without Alamouti schemes are also included. It is found that a considerable error performance gain can be obtained by using SNR-adaptive transmission. Some snapshots of optimized 4-ary constellations are presented in Fig. 4 showing that pairs of symbol point locations have become closer to each other with the increasing value of \( \bar{\gamma} \).

In addition to the 4-ary signalling cases, SNR-adaptive transmission model is also tested over 16-ary signaling cases along with the different \( m \) values. Optimized 16-ary constellations for Rayleigh fading are illustrated in Fig. 5 where the symbol point locations tend to gather in four different groups when \( \bar{\gamma} \) is increasing. The simulated BER results for SNR-adaptive transmission are compared with Gray-mapped 16-QAM constellations in Fig. 6 and showing the gain from the SNR-adaptive transmission.

B. Lower decoding latency

Let us next investigate how the error performance gain obtained from SNR-adaptive transmission can give some advantage in terms of a lower decoding latency. The decoding latency can be defined as the time interval between the time instants at which the information is received and at which the decoding process is completed [23]. In this paper, for simplicity, the window-length of back-search limit in Viterbi decoder, \( \tau \), is selected as a quantity of the decoding latency following the approach of [6]. To compare the decoding latencies between the SNR-adaptive transmission with optimized irregular constellations and the SNR-independent transmission with conventional \( M \)-QAM, the required average SNR values to obtain the predefined BER thresholds, \( \text{BER}_{th} \), are calculated via Monte Carlo simulations.

---

\(\text{CC} (5, 7) + 4\)-QAM (m=1)

\(\text{CC} (5, 7) + 4\)-QAM (m=3)

\(\text{CC} (5, 7) + \text{Alamouti} (2\text{Tx}, 1\text{Rx}) + 4\)-QAM (m=1)

\(\text{CC} (5, 7) + \text{Alamouti} (2\text{Tx}, 1\text{Rx}) + \text{SNR-adaptive} 4\text{-ary} (m=1)

\(\text{CC} (5, 7) + 4\)-QAM (m=3)

\(\text{CC} (5, 7) + \text{Alamouti} (2\text{Tx}, 1\text{Rx}) + 4\)-QAM (m=3)

\(\text{CC} (5, 7) + \text{Alamouti} (2\text{Tx}, 1\text{Rx}) + \text{SNR-adaptive} 4\text{-ary} (m=3)

---

Fig. 2: The proposed SNR-adaptive transmission model for the convolutionally encoded OSTBC scheme.

Fig. 3: The bit error rate comparison of the convolutionally encoded Alamouti coded SNR-adaptive 4-ary case and the convolutionally encoded Alamouti coded 4-QAM case for Nakagami-\( m \) channels, \( m = 1 \) and \( m = 3 \).
with different decoding iterations is given in Fig. 9. Over the terms of error rates and decoding latency can be obtained from SNR-adaptive convolutionally encoded OSTBC system. From the simulations, it is observed that the notable gain in diversity gain without any expense of decoding complexity. The powerful channel coding techniques, the decoding delay values and convolutionally coded transmissions can enhance with increasing number of the decoder iteration, it comes with the expense of higher decoding performance can be reached. From the simulations, turbo-TCM encoder given in [24] is employed to turbo-TCM scenarios.

V. CONCLUSION
Non-iterative/one shot-decoding characteristic and the simplicity makes convolutional encoders attractive in particular 5G use cases where low-latency and low complexity are important attributes. Encouraged by this, SNR-adaptive convolutionally encoded OSTBC system is proposed where the powerful channel coding techniques, the decoding delay comparison which includes the turbo-TCM encoded scenario with different decoding iterations is given in Fig. 9. Over the simulations, turbo-TCM encoder given in [24] is employed along with the different numbers of decoding iterations, \( Q \), along with 16-QAM constellation. Although the error performance can be enhanced with increasing number of the decoder iteration, it comes with the expense of higher decoding latency values and convolutionally coded transmissions can be a good option to reach lower decoding latencies compared to turbo-TCM scenarios.

REFERENCES
Fig. 5: Optimized SNR-adaptive irregular constellations for convolutionally encoded $[5, 7]_{18}$ Alamouti coded scheme at $\gamma = 10$ dB, $\gamma = 14$ dB and $\gamma = 20$ dB for Rayleigh fading channels ($m = 1$).

Fig. 6: The bit error rate comparison of the convolutionally encoded Alamouti coded SNR-adaptive 16-ary case and the convolutionally encoded Alamouti coded 16-QAM case for Nakagami-\(m\) channels, $m = 1$ and $m = 3$.

Fig. 7: The decoding latency comparison of the convolutionally encoded Alamouti coded SNR-adaptive 4-ary case and the convolutionally encoded Alamouti coded QAM case for Nakagami-\(m\) channels, $m = 1$ and $m = 3$.

Fig. 8: The decoding latency comparison of the convolutionally encoded Alamouti coded SNR-adaptive 16-ary case and the convolutionally encoded Alamouti coded 16-QAM case for Nakagami-\(m\) channels, $m = 1$ and $m = 3$.

Fig. 9: The decoding latency comparison of the convolutionally encoded Alamouti coded SNR-adaptive 16-ary case, the convolutionally encoded Alamouti coded 16-QAM case, and the turbo-TCM encoded Alamouti coded 16-QAM cases with different iterative decoding numbers, $Q = \{1, 2, 3, 4\}$ over Rayleigh channels ($m = 1$).