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Dynamic Element Matching in Digital-to-Analog Converters with Code-Dependent Output Resistance

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Abstract—This paper evaluates the performance of dynamic element matching (DEM) in digital-to-analog-converters, when the unit conversion cells of the converter have finite output resistance. DEM is already known to be effective against the static amplitude, timing and pulse shaped mismatches. However, the effect of output resistance and its mismatches has not been studied. A comprehensive code-dependent output resistance model for the current-steering DAC is presented. System level simulations show that the non-linearity caused by the output resistance, in the absence of mismatches, is not shaped by the DEM encoder since the output resistance is same for all the conversion cells. In this paper we demonstrate that, in the presence of mismatches, the DEM encoder is able to shape the non-linearity they cause since the output resistance now varies among different conversion cells.

I. INTRODUCTION

Continuous evolution of modern wireless communication systems has placed an increased stress on the design of digital-to-analog converters. These converters now have to meet tighter constraints placed on their performance in order to fully utilize the potentials of modern digital RF circuits.

Current steering DACs use CMOS transistors for implementing current sources and switches and are a common choice for communication systems. One of the main sources of non-linearity in these current steering DACs are the static amplitude, timing, pulse shape and output resistance mismatches among the conversion cells. These mismatches occur due to the stochastic nature of the fabrication process and have been analyzed in detail in [1]–[3].

There are a number of methods available to deal with these mismatches. One common approach is to use dynamic element matching (DEM) techniques. Among these encoding techniques, the tree-structured DEM [4], presented in Fig. 1, will be considered in this paper. Although the analysis, modeling and simulations are carried out assuming the tree-structured DEM architecture, there is no indication that results obtained will not hold for other DEM encoding architectures.

There is a lot of literature analyzing the ability of DEM encoders to shape mismatches in the DACs [4]–[9]. There is also literature dealing with output resistance and its mismatches and the non-linearity they cause [2], [3]. However, these two topics are always analyzed separately. There is a general lack of literature discussing the impacts of output resistance and its mismatches on the performance of DEM encoders.

This paper starts with a brief overview of general DEM encoding process, presented in Section II. Section III presents

Fig. 1. Segmented tree-structured DEM encoder with 4-bit MSB and 6-bit LSB segmentation.

a comprehensive code-dependent output resistance model for current-steering DACs and derives and analyzes the output current expressions. Section IV presents the system simulation results, which show that the DEM encoder is ineffective against the non-linearity caused by a finite output resistance, in the absence of mismatches. In the presence of mismatches, however, the DEM encoder is able to deal with the non-linearity caused by these mismatches. Finally, Section V provides the conclusions.

II. DYNAMIC ELEMENT MATCHING

The DEM encoders work on the principle of scrambling the order of conversion cells of the DAC, on a sample-by-sample basis [4]–[9]. This scrambling is controlled by a pseudo random switching sequence. Assuming a fully differential implementation, in the presence of static mismatches among the conversion cells, a fast and random enough scrambling operation would force the mismatch errors to be zero on average. This removes the input dependence and results in non-linearity being converted into spectrally shaped noise.

The DEM encoders can be implemented using several different architectures. Some examples are the data-weighted averaging (DWA) [6], individual level averaging (ILA) [7], random swapping [8], and tree-structured encoder [4]. This paper uses the tree-structured encoder among the DEM algorithms since it can be implemented with an arbitrary number of 1-bit DACs, is able to spectrally shape the mismatches in these 1-bit DACs and can easily be pipelined for high-speed designs. Fig. 1 presents the internal structure of DEM encoder, used in [5] and also in the rest of paper.

Fig. 2. Current-steering DAC (a) circuit-level schematic and (b) code-dependent output resistance model.

The detailed mathematical treatment of the tree-structured DEM operation, internal structure of the switching blocks and the switching sequence generator have been extensively analyzed [4]. The important thing to note here is the scrambling nature of DEM encoder's operation.

III. CODE-DEPENDENT OUTPUT RESISTANCE MODEL

Fig. 2(a) presents the circuit-level schematic of a current-steering DAC. Differential load resistance and current source resistances of p and n sub-cells are also shown. The conversion cell switches are controlled by the digital encoder (not shown in the figure). Fig. 2(b) presents the code-dependent output resistance model derived from this schematic. The model represents all the conversion cells of the DAC, divided into p and n sub-cells. The current mismatches are ignored, simplifying the sub-cell current sources to $x[n] \cdot I_{LSB}$ and $(1-x[n]) \cdot I_{LSB}$, where $x[n]$ is the DEM encoder input, and I_{LSB} the current in the LSB conversion cell. The model uses four conductances to completely define the total output resistance. These are the individual on and off conductances, of p and n sub-cells and are defined as:

$$G_{ONp}[n] = \sum_{i=1}^N b_i[n] \cdot G_{ONp,i} \quad (1)$$

$$G_{ONm}[n] = \sum_{i=1}^N (1 - b_i[n]) \cdot G_{ONm,i} \quad (2)$$

$$G_{OFFp}[n] = \sum_{i=1}^N (1 - b_i[n]) \cdot G_{OFFp,i} \quad (3)$$

$$G_{OFFm}[n] = \sum_{i=1}^N b_i[n] \cdot G_{OFFm,i} \quad (4)$$

Where $b_i[n]$ is the encoder output and can be set to 1 or 0, enabling the p sub-cell or the n sub-cell respectively, in

the i -th conversion cell, N is the total number of conversion cells and $G_{ONp,i}$, $G_{ONm,i}$, $G_{OFFp,i}$ and $G_{OFFm,i}$ represent the two on and two off conductances of the p and n sub-cells respectively, for the i -th conversion cell. Each of these conductances represents normally distributed random numbers with a mean and a standard deviation.

As an example let us consider the on conductance of p-cell in the i -th conversion cell. This conductance is given as:

$$G_{ONp,i} = G_{ON} + G_{ONpmism,i} \quad (5)$$

Here G_{ON} is the mean value of conductance and $G_{ONpmism,i}$ is the random mismatch. The mismatches are generated with a standard deviation, σ , given as:

$$\sigma = \frac{\sigma_p}{100} \cdot \sqrt{W_{c,i}} \cdot G_{ON} \quad (6)$$

Where σ_p is the percentage mismatch in the conductance value and $W_{c,i}$ represents the weight of the i -th conversion cell. Since the total conductance is a sum of random independent variables (individual cell conductances), the standard deviation, σ , is directly proportional to the square root of the individual conversion cell weights.

All other conductances can be found by similar reasoning.

A. Output Currents

The total output current is the sum of sub-cell currents:

$$I_{out} = I^+ - I^- \quad (7)$$

Using the current divider formula, the sub-cell currents are:

$$I^+ = x[n] \cdot I_{LSB} \cdot \left[\frac{1}{g_{OFFp}[n] + g_{ONp}[n] + 1} \right] \quad (8)$$

$$I^- = (1 - x[n]) \cdot I_{LSB} \cdot \left[\frac{1}{g_{OFFm}[n] + g_{ONm}[n] + 1} \right] \quad (9)$$

where the relative conductances are defined as:

$$g_{ONp}[n] = \frac{G_{ONp}[n]}{2 \cdot G_L} \quad (10)$$

$$g_{ONm}[n] = \frac{G_{ONm}[n]}{2 \cdot G_L} \quad (11)$$

$$g_{OFFp}[n] = \frac{G_{OFFp}[n]}{2 \cdot G_L} \quad (12)$$

$$g_{OFFm}[n] = \frac{G_{OFFm}[n]}{2 \cdot G_L} \quad (13)$$

Equations (8) and (9) show the dependence of output current on the the relative output conductances of the current sources. Applying the first order Taylor approximation:

$$\frac{1}{1+y} \approx 1 - y \quad \text{for } y \ll 1 \quad (14)$$

The conductance values are in the orders of 10^{-6} and R_L is set to 50Ω , as will be shown in the next section.

The p sub-cell current hence becomes:

$$I^+ \approx x[n] \cdot I_{LSB} \cdot \left[1 - g_{OFFp}[n] - g_{ONp}[n] \right] \quad (15)$$

Assuming that the off conductance is zero and multiplying $x[n]$ inside the brackets.

$$I^+ \approx I_{LSB} \cdot \left[x[n] - x[n]g_{ONp}[n] \right] \quad (16)$$

From (10), it is evident that (1) also holds for $g_{ONp}[n]$ and $g_{ONp,i}$ can be split into mean and mismatch values using (5). Hence I^+ can be written as:

$$I^+ \approx I_{LSB} \cdot \left[x[n] - x[n] \sum_{i=1}^N b_i[n]g_{ON} - x[n] \sum_{i=1}^N b_i[n]g_{ONpmism,i} \right] \quad (17)$$

Where:

$$g_{ON} = \frac{G_{ON}}{2 \cdot G_L} \quad (18)$$

$$g_{ONpmism,i} = \frac{G_{ONpmism,i}}{2 \cdot G_L} \quad (19)$$

$$g_{ONp,i} = \frac{G_{ONp,i}}{2 \cdot G_L} \quad (20)$$

g_{ON} is same for all conversion cells and independent of i . The number conservation rule [4], given as:

$$x[n] = \sum_{i=1}^N b_i[n] \quad (21)$$

results in (17) being transformed into:

$$I^+ \approx I_{LSB} \cdot \left[x[n] - x^2[n]g_{ON} - x[n] \sum_{i=1}^N b_i[n]g_{ONpmism,i} \right] \quad (22)$$

The three terms inside the square brackets of (22) represent the three different contributions to the p sub-cell current. $x[n]$ represents the ideal output, $x^2[n]g_{ON}$ represents the non-linear contribution to output due to a finite on conductance, which is not corrected by the DEM encoder. Also higher order non-linearities would be visible, if we would use higher order Taylor expansion, however, the first order is sufficient to study the behavior of conductance mismatch between units under mismatch shaping. The $x[n] \sum_{i=1}^N b_i[n]g_{ONpmism,i}$ represents the contribution due to mismatches in the on conductance.

The contribution of off conductance to the p sub-cell current in (22) will be quite similar to the on conductance contribution. Also the expression of n sub-cell current will be similar to (22). Hence, the following analysis will refer to all the conductances as a whole.

The contribution due to the conductance mismatches involves a time domain multiplication of DEM input with the sum of DEM outputs. However, since the DEM implements all the $b_i[n]$ in a similar manner, ensuring that each $b_i[n]$ has a similar spectral density, the sum can simply be replaced with

Fig. 3. Block diagram of the system level implementation [5].

Fig. 4. PSD of (a) $x[n]$, (b) $b_{28}[n]$ and (c) $x[n] \cdot b_{28}[n]$.

any individual DEM output, without the loss of generality. Using $b_{28}[n]$, the MSB output of DEM encoder, as an example, the time domain multiplication ($x[n] \cdot b_{28}[n]$) is simply a frequency domain convolution.

Figs. 4(a), 4(b) and 4(c) present the PSD of $x[n]$, $b_{28}[n]$ and $x[n] \cdot b_{28}[n]$. The simulation was carried out using an LTE transmitter, whose block level diagram is shown in Fig. 3 and is taken from [5], where it was used to create notches in the RX-band for SAW-less operation. An LTE signal with a bandwidth of 15 MHz, a duplex distance of 95 MHz and a sampling frequency of 888 MHz is used as input. The carrier frequency was set to twice the sampling frequency. This LTE input signal will be referred to as LTE 1 throughout the remaining text. Fig. 4(c) shows that the product ($x[n] \cdot b_{28}[n]$) has a shaped PSD, just like $x[n]$ and $b_{28}[n]$, shown in Figs. 4(a) and 4(b) respectively. Although the RX-band notches are not as deep as in [5], the PSD is visibly shaped and the difference can be explained by the nature of convolution. Hence it can be concluded that the non-linearity caused by the mismatches in the four conductances (or simply the output resistance) is shaped by the DEM encoder.

IV. SYSTEM LEVEL SIMULATIONS

System level simulations, using the block diagram of Fig. 3, were carried out to verify the effectiveness of DEM encoder against a finite output resistance and its mismatches, which are the only sources of non-linearity in the system.

Fig. 5 presents the RF-DAC output spectra comparison for changing R_L/R_{ON} ratio when the mismatches are set to zero. Here, R_{ON} and R_{OFF} are the inverse of G_{ON} and G_{OFF} ,

Fig. 5. RF-DAC output spectra comparison for changing R_L/R_{ON} ratio.

Fig. 6. Effect of using DEM encoder on output resistance mismatches. Here, $\sigma_p = 100\%$, $R_{ON} = 1\text{M}\Omega$ and $R_{OFF} = 10\text{M}\Omega$

respectively. LTE 1 was used as an input and the simulations were carried out using a constant R_{OFF} value of $10\text{M}\Omega$ since here we are only interested in seeing the effect of changing R_{ON} . However, by extension, the results obtained also apply for changing R_{OFF} when R_{ON} is set to a constant value.

The simulation results show that for comparable values of R_{ON} and R_L the output has a significant non-linearity, visible from the bandwidth increase. These results are consistent with (8) and (9) where the dependence of output currents, on the relative conductances of the current sources, was shown.

These comparable values are just the test cases. For our practical system, R_{ON} is $1\text{M}\Omega$ and R_L is 50Ω , resulting in an R_L/R_{ON} ratio of 50×10^{-6} . Hence, our system is immune to the non-linearity caused by the output resistance as evident from the simulation results. The R_{ON} of $1\text{M}\Omega$ is in the order of what can be achieved with typical CMOS implementation.

Fig. 6 shows the effect of using DEM encoder in the presence of resistance mismatches. The LTE signal used as input and the carrier frequency are same as the ones used in Fig. 5. The σ_p is 100% , R_{ON} is $1\text{M}\Omega$ and R_{OFF} is again $10\text{M}\Omega$. R_{ON} and R_{OFF} are set to different values because using same values for both would mean an output non-linearity arising purely from mismatches which is not a realistic case. The simulation result shows the ability of the DEM encoder to effectively shape the mismatches in the output resistance.

Fig. 7 shows the RX-band noise values for changing σ_p . The noise values are shown for both DEM on and off cases with each value showing an average of 10 simulation runs, to obtain smoother curves. The LTE input signals used for Fig. 7(a) and (b) have been indicated on the respective figures. R_{ON} and R_{OFF} are set to $1\text{M}\Omega$ and $10\text{M}\Omega$ respectively.

The simulation results are as expected, with DEM on case

Fig. 7. RX-band noise vs σ_p with (a) and (b) using different LTE signals.

showing a lesser RX-band noise as compared to the DEM off case for a given σ_p . Increasing the σ_p increases the RX-band noise but the noise improvement from the DEM off to DEM on case is also increased.

V. CONCLUSION

This paper evaluates the impact of finite output resistance and its mismatches on the performance of DEM encoder. A comprehensive code-dependent output resistance model is presented and the output current expressions are derived and analyzed. System level simulations show the relative immunity of the simulated system to the non-linearity caused by the output resistance and its mismatches, the inability of the DEM encoder to shape the finite output resistance non-linearity and its ability to shape the additional non-linearity caused by the presence of resistance mismatches.

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