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# Optical characteristics of the metal-wire dielectric periodic structure: hyperbolic eigenwaves

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## ABSTRACT

We are carefully analyzing the properties of the eigenwaves of 2D-structure with regularly ordered metal nanorods in lossless dielectric. Plane-wave-decomposition was used in numerical calculations. New “hyperbolic” solutions were found.

**Keywords:** hyperbolic medium, nanowires, eigenwaves

## 1. INTRODUCTION

The class of artificial media which has emerged as one of the most important at optical frequencies are hyperbolic metamaterials. These non-magnetic media have a dielectric tensor which is extremely anisotropic and cannot be found in nature at optical frequencies. The name is derived from the isofrequency curve of the medium, which is hyperbolic as opposed to circular as in conventional media. As known, engineering of optical properties of media using artificial nanostructures known as metamaterials has led to breakthrough devices with capabilities from super-resolution imaging to invisibility. The interaction of light with these materials can be very different [1, 2], depending on the size of the smallest elements. One of the perspective classes of the metamaterials is wire media, i.e. media composed of lattices of aligned metal rods embedded in a dielectric matrix [3]. One of the examples of its unusual optical properties is those exhibited by so-called “indefinite media” [4]. The resulting physical effects were successfully confirmed in recent experiments [5].

The two most straightforward implementations of hyperbolic metamaterials are sub-wavelength metal-dielectric multilayers [4],[6] and metallic nanowires in a dielectric host [2, 3]. Such designs can be adopted for all wavelength ranges of interest from the UV, visible, near-IR to mid-IR using the appropriate choice of metal and metallic filling fraction. We note that the key design aspects are the choice of the metal and its plasmonic response. In the visible range, silver forms the best choice because of its low optical losses [2].

One of the ways for investigations of the optical properties of the hyperbolic medium is the approximation of effective medium. In contrast to discrete structures such as photonic crystals, wire media can be homogenized [4, 6, 7]. Homogenization is only valid when the wavelength of light is much larger than the unit cell of the metamaterial. In hyperbolic metamaterials, the interesting properties arise due to propagating waves that have an effective wavelength inside the medium which is much smaller than the free space wavelength (high-k states).

We have investigated the periodic structure with metal wires embedded into dielectric, which as expected exhibits hyperbolic properties. It was shown recently that these structures can produce super-Planck thermal radiation and may enhance the gain properties of the dopants [8, 9]. Holey fibers containing elements sizes down to 30 nm have been reported recently, and thus optical metamaterials can be produced by drawing [10-12]. In spite of numerous publications in this field further investigations of the details of wave propagation in these materials should be carried out aiming manufacturing and utilizing of these materials. First important question is whether the optical phases and hyperbolic

isofrequency surfaces predicted by effective medium theory are achievable in the practical structure. Second question is estimation of the wavelength region where real metal dielectric wire structure can be treated like hyperbolic medium. In this paper we investigate this problem by numerical calculations eigenwaves of this structure.

## 2. EQUATION AND NUMERICAL MODEL

We have investigated the periodic structure with silver wires embedded into glass (see Fig.1), which as expected exhibits hyperbolic properties. Such structures could potentially be fabricated via fiber drawing, either via direct co-drawing, in which a macroscopically sized metal-dielectric preform is heated and reduced in size by several orders of magnitude [10, 11], or via pumping of liquid metal into existing micro- and nanoholes [12]. The real example of metal-dielectric preform with rods diameter is about 18  $\mu\text{m}$ , period is 125 $\mu\text{m}$  on Fig.1 (b). The structure under investigation (Fig.1 (a)) has parameters as follow: radius of silver wires is 20nm, period is 375nm,  $n_g=1.58$ ,  $\epsilon_m=-9.4+i 0.309$ .

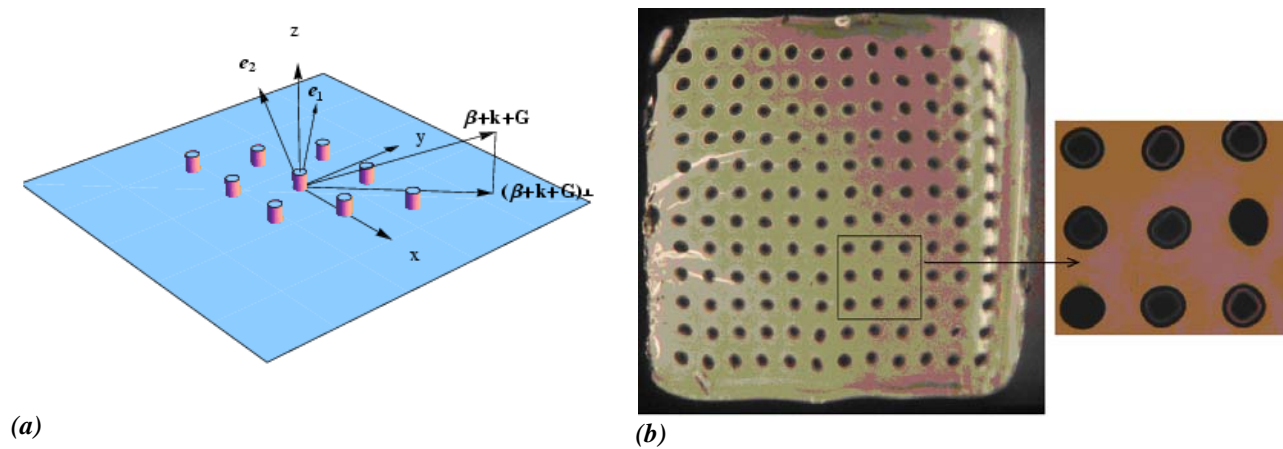


Fig. 1: Medium structure and coordinate system (a). Metal-glass preform (b): rods diameter is about 18  $\mu\text{m}$ , period is 125 $\mu\text{m}$ .

As we say above, the hyperbolic media characterized by extremely anisotropic dielectric tensor. The components of dielectric permittivity tensor have opposite signs:

$$\epsilon = \begin{pmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix}, \quad \epsilon_{xx} = \epsilon_{yy} \neq \epsilon_{zz}, \quad \epsilon_{xx} = \epsilon_{yy} > 0, \quad \epsilon_{zz} < 0. \quad (1)$$

As opposed to discrete structures such as photonic crystals, wire media can be homogenized. As known, homogenization is only valid when the wavelength of light is much larger than the unit cell of the metamaterial. The effective permittivity of the silver nanowire composite can be found after homogenization using the Maxwell-Garnett rules, which gives

$$\epsilon_{eff} = \epsilon_d(1 - f) + \epsilon_m f, \quad f = \frac{\pi r^2}{P^2 \frac{\sqrt{3}}{2}}, \quad (2)$$

where  $f$  - is filling factor,  $P$  - period of the structure. This relation was used for estimation of the geometrical parameters of the structure which can provide negative effective index to make wired medium hyperbolic.

For calculation of the dispersion properties of the wired medium we have used plane wave method. We should start with wave equation for monochromatic wave. For metal with constant permittivity (not dependent on frequency) we can use the equation for the magnetic field  $\vec{H}(x, y, z)$ :

$$\left[ \nabla \times \frac{1}{\varepsilon(x,y)} \left[ \nabla \times \vec{H} \right] \right] = K^2 \vec{H}. \quad (3)$$

Using plane wave expansion of  $\frac{1}{\varepsilon(x,y)}$  and Floquet solutions for  $H(x,y,z) = \exp[i(\vec{k} + \vec{\beta})\vec{r}] \sum_{\vec{G}} \vec{H}_{\vec{G}} \exp(i\vec{G}\vec{r})$ ,

where  $\vec{G}$  are the vectors of reciprocal lattice one can get well-known equations [15, 16] for field amplitudes  $H_{\vec{G}}$  :

$$\sum_{\vec{G}'} \left[ \vec{k} + \vec{\beta} + \vec{G} \right] \left[ \vec{k} + \vec{\beta} + \vec{G}' \right] \left( \frac{1}{\varepsilon} \right)_{\vec{G}-\vec{G}'} \begin{bmatrix} \vec{e}_{2\vec{G}'} \cdot \vec{e}_{2\vec{G}} & -\vec{e}_{1\vec{G}'} \cdot \vec{e}_{2\vec{G}} \\ -\vec{e}_{2\vec{G}'} \cdot \vec{e}_{1\vec{G}} & \vec{e}_{1\vec{G}'} \cdot \vec{e}_{1\vec{G}} \end{bmatrix} \begin{pmatrix} H_{1\vec{G}'} \\ H_{2\vec{G}'} \end{pmatrix} = -K^2 \begin{pmatrix} H_{1\vec{G}} \\ H_{2\vec{G}} \end{pmatrix}, \quad (4)$$

where layout of unit vectors  $\vec{e}_{1\vec{G}}$  and  $\vec{e}_{2\vec{G}}$  is shown on Fig.1 (a).

For metal with frequency-dependent permittivity it is better to use another approach based on the equations for  $H_x(x,y)$  and  $H_y(x,y)$  only. Corresponding equations are as follows [15]:

$$\nabla_{\perp}^2 H_x + K^2 \varepsilon H_x - \frac{\partial \ln \varepsilon}{\partial y} \left( \frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial x} \right) = \beta^2 H_x, \quad (5)$$

$$\nabla_{\perp}^2 H_y + K^2 \varepsilon H_y - \frac{\partial \ln \varepsilon}{\partial x} \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) = \beta^2 H_y. \quad (6)$$

For the numerical solution of these equations we also used plane wave expansion of  $\varepsilon$  and Floquet solutions for  $H_{x,y}$ . Isofrequencies for given  $K$  is  $\beta = \beta(K, k_x, k_y)$ .

### 3. NUMERICAL RESULT

Equations for plane wave amplitudes  $H_{x\vec{G}}$ ,  $H_{y\vec{G}}$  include harmonics  $\varepsilon_{\vec{G}}$ :  $-(\vec{k} + \vec{G})^2 H_{x\vec{G}} + K^2 \varepsilon_{\vec{G}} H_{x\vec{G}} + \dots = \beta^2 H_{x\vec{G}}$ , where dots denote other terms including interaction of waves with different  $\vec{G}$  due to  $\varepsilon_{\vec{G}}$ . Neglecting the wave interaction it is easy to show that the isofrequencies are the intersections of spheres located at points parameterized by  $\vec{G}$  in reciprocal space (Fig.2). Interaction of waves leads to intersection of these spheres with anti-crossing.

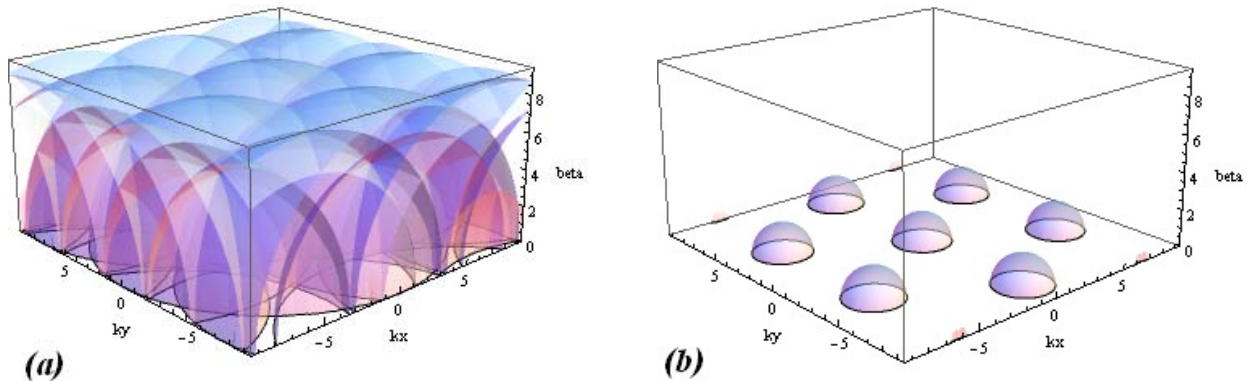


Fig.2. The intersections of spheres located at points parameterized by  $\vec{G}$  in reciprocal space, slightly deformed by plane-waves interaction. (a)  $K=3\pi$ , (b)  $K=0.5\pi$ .

In Fig.3 (a) isofrequencies for  $K = 2.5, 5, 7.5, 10 \mu\text{m}^{-1}$ , are shown by blue curves corresponds to real part of  $\beta$ , red curves corresponds to imaginary part of  $\beta$ . Data for  $\epsilon_m$  have been taken from [14]. Fig.3 (b) shows details of the region shown on the Fig.3 (a) by the rectangle.

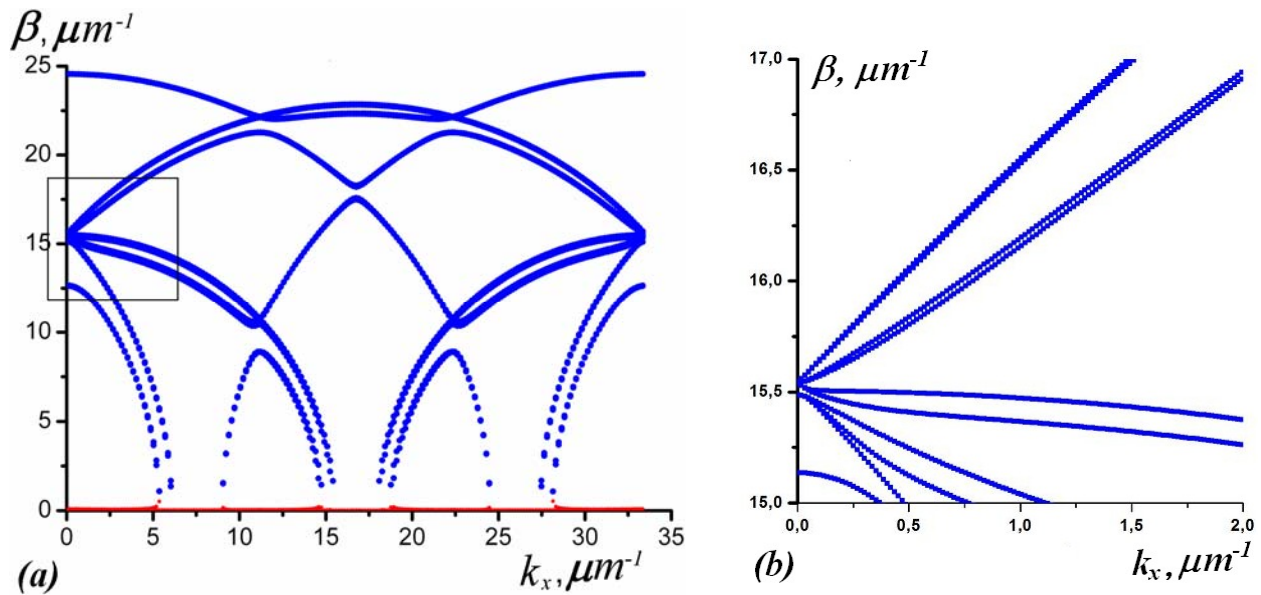


Fig.3. (a) Isofrequencies for  $K = 2.5, 5, 7.5, 10 \mu\text{m}^{-1}$ , data for  $\epsilon_m$  have been taken from [14] blue curves correspond to real part of  $\beta$ , the red curves correspond to imaginary part of  $\beta$ ; (b) details of the region shown on the Fig.3(a) by the rectangle.

It is easy to see the existence of hyperbolic dispersion at the curves which corresponds to first order space harmonics. At large values of transverse wavenumbers the dispersion becomes normal, not hyperbolic [3]. However, there are other regimes (denoted by blue dots) in which two additional zeroes are appeared via tangent bifurcation. In this case more complicated picture of waves exists with additional “hyperbolic” waves.

#### 4. CONCLUSION

Concluding, we have confirmed that wired medium can be treated as hyperbolic medium in definite range of direction of plane wave propagation and frequencies. We have shown that there exist also other “hyperbolic” waves having large transverse wave numbers.

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