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Published in:
Physical Review B

DOI:
10.1103/PhysRevB.90.224502

Published: 01/01/2014

Document Version
Publisher's PDF, also known as Version of record

Please cite the original version:
Manninen, M. S., Rysti, J., Todoshchenko, I., & Tuoriniemi, J. (2014). Quasiparticle damping of surface waves in superfluid 3He and 4He. Physical Review B, 90(22), 1-4. [224502]. https://doi.org/10.1103/PhysRevB.90.224502

Quasiparticle damping of surface waves in superfluid ^{3}He and ^{4}He

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(Received 16 October 2014; revised manuscript received 20 November 2014; published 1 December 2014)

Oscillations on free surfaces of superfluids at the inviscid limit are damped by quasiparticle scattering. We study this effect in both superfluid ^{3}He and superfluid ^{4}He, deep below the respective critical temperatures. Surface oscillators offer several benefits over immersed mechanical oscillators traditionally used for similar purposes. Damping is modeled as specular scattering of ballistic quasiparticles from the moving free surface. The model is in reasonable agreement with our measurements for superfluid ^{4}He but significant deviation is found for ^{3}He.

DOI: 10.1103/PhysRevB.90.224502 PACS number(s): 67.25.dg, 03.75.Kk, 67.30.hb, 67.90.+z

All sorts of oscillating bodies have been used for decades to study dissipation mechanisms in superfluids [1–8]. The purpose is often to determine the density of thermal quasiparticles in order to deduce the temperature of the superfluid. The influence of the quasiparticles is to damp the motion of the oscillator as the quasiparticles scatter from the surfaces carrying away a fraction of the momentum in the event. Theoretical treatment of the process requires the knowledge of the roughness of the surfaces leading to either specular [3,9] or diffuse [10] scattering or, more generally, some intermediate of the two extremes. Also, any solid object has a much higher density than helium, so that the relative change in momentum per event is very small, thus limiting the sensitivity of practical devices. Moreover, mechanical oscillators suffer from internal damping of the device itself.

The free surface of superfluid helium set into oscillatory motion is probably the most ideal tool for studying the interaction of quasiparticles with impenetrable boundaries. No additional mass besides helium itself is involved in the motion and the quasiparticle scattering is presumably perfectly specular. This has been verified experimentally in the case of superfluid ^{4}He [11]. As the temperature is reduced deep below the superfluid transition temperature T_c, the quasiparticle mean free path increases very rapidly, and at about $T \approx T_c/4$ it can usually be assumed to exceed the typical dimensions of the experiment. This means that the quasiparticles essentially do not interact with each other, just with the boundaries of the fluid volume, and can be treated as ballistic entities. This simplifies the theoretical treatment a great deal.

The crucial difference between the two helium isotopes is their bosonic (^{4}He) or fermionic (^{3}He) character, which largely dictates their behavior at very low temperatures. While bosonic (even number of elementary particles) ^{4}He is superfluid below 2 K, fermionic (odd number of elementary particles) ^{3}He atoms need to first form pairs, leading to rather complex superfluid properties [12]. At saturated vapor pressure (practically zero pressure) ^{3}He becomes superfluid at about 1 mK and exhibits an isotropic B phase, unless a sufficiently large magnetic field is applied.

Different quantum statistics of the two helium isotopes implies that the thermal quasiparticles have entirely different characters in these two superfluids and their dispersion relation and scattering properties differ in fundamental ways. Therefore, it is of interest to study both of these media in the very same experimental cell, with no alterations in the geometry, using oscillations of the free surface as the indicator of the quasiparticle properties.

Surface-wave resonances in helium fluids have been utilized for determining the surface tension, but only in ^{4}He have such studies been extended to the superfluid state [13]. However, very few data are available on the temperature dependence of damping of such resonances. We are aware of just one systematic study of this aspect in superfluid ^{4}He, utilizing electrons trapped on the free surface [14]. In bulk superfluid ^{3}He only one prior report on the observation of surface waves exists [15]. On thin superfluid films another type of surface oscillation occurs, known as third sound, which has been observed on both ^{4}He and ^{3}He [16,17].

In this article we present a simple model for damping of the surface waves on bulk superfluid helium in the ballistic quasiparticle limit and compare the results with our measurements on both ^{4}He and ^{3}He in the same experimental setup.

Let us consider specular scattering of ballistic quasiparticles from a moving object, in this case the free surface. Indeed, surface waves can be modeled as a moving object in fluid from the quasiparticle point of view since, for any surface element with area ΔA moving at velocity $\mathbf{v}_o$, one finds a counterpart, another surface element moving at the same speed but in the opposite direction $-\mathbf{v}_o$ (see Fig. 1).

According to the momentum and energy conservation laws the energy difference between incoming (E_1) and scattered (E_2) excitations is

$$\Delta E = E_2 - E_1 = \mathbf{v}_o \cdot (\mathbf{p}_2 - \mathbf{p}_1) = \mathbf{v}_o \cdot \Delta\mathbf{p}, \qquad (1)$$

where $\mathbf{p}_1$ and $\mathbf{p}_2$ are the momenta of the incoming and scattered excitations, respectively.

Damping of the moving object is due to the elastic momentum transfer $\Delta\mathbf{p}$ between the object and quasiparticles. The damping force $\mathbf{F}$ is

$$\mathbf{F} = \int_\Omega \frac{-\Delta\mathbf{p}|(\mathbf{v}_g - \mathbf{v}_o)\cdot\hat{\mathbf{v}}_o|\Delta A}{h^3\{\exp[E/(k_BT)] \pm 1\}} d\mathbf{p}, \qquad (2)$$

where $\mathbf{v}_g = \nabla_\mathbf{p} E$ is the group velocity of quasiparticles with energy E, $|(\mathbf{v}_g - \mathbf{v}_o)\cdot\hat{\mathbf{v}}_o|\Delta A$ is their volumetric flow rate towards the moving object, and $d\mathbf{p}/(h^3\{\exp[E/(k_BT)] \pm 1\})$ is

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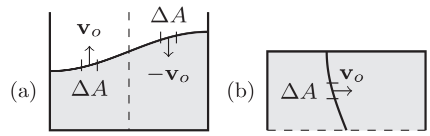

FIG. 1. Alternative formulation of a standing surface wave (a), where the two halves can be redrawn with a common impervious interface (b) moving at velocity $\mathbf{v}_o$.

the number density of thermally excited quasiparticles within $d\mathbf{p}$ for bosons (−) or fermions (+) at temperature T.

In bosonic ^{4}He the low-energy quasiparticles can be considered as phonon-like excitations with energy $E = u|\mathbf{p}|$ and constant group velocity $\mathbf{v}_g = u\hat{\mathbf{p}}$ as sketched in Fig. 2(a) in one dimension. Since $\mathbf{v}_g \uparrow\uparrow \mathbf{p}$, they are always particle-like excitations. According to Eqs. (1) and (2) the damping force in three dimensions is

$$\mathbf{F}_B = -\frac{8\pi^5(k_BT)^4}{15h^3u^4}\Delta A\mathbf{v}_o = -P_B(T)\Delta A\mathbf{v}_o, \tag{3}$$

which defines the temperature-dependent factor $P_B(T)$ for bosonic excitations. The only medium-dependent parameter here is the speed of sound u, which is $u = 239$ m/s in ^{4}He at saturated vapor pressure in the zero-temperature limit [18].

In fermionic ^{3}He the Bogoliubov quasiparticle energy spectrum is more complicated [see Fig. 2(b)]. As for superconductors there is an energy gap E_Δ at Fermi momentum $p_F = 8.28\times10^{-25}$ kg m/s [19]. For superfluid ^{3}He at zero pressure the energy gap is close to the BCS value, $E_\Delta = 1.764k_BT_c$ [20]. Besides the particle-like quasiparticles there are also hole-like quasiparticles where $\mathbf{v}_g \uparrow\downarrow \mathbf{p}$. As depicted in Fig. 2(b) the normal particle-to-particle or hole-to-hole scattering is not always allowed, but a quasiparticle may be Andreev-reflected from hole c_1 to particle c_2 (or vice versa), in the process transferring only a negligible amount of momentum compared to normal scattering [4]. Thus Eq. (2) gives

$$\mathbf{F}_F = -\frac{4\pi p_F^4}{h^3\exp[E_\Delta/(k_BT)]}\Delta A\mathbf{v}_o = -P_F(T)\Delta A\mathbf{v}_o \tag{4}$$

in the limit $|\mathbf{v}_o|p_F \ll k_BT \ll k_BT_c$, while the Andreev-scattered states $(E < E_\Delta + 2\mathbf{p}\cdot\mathbf{v}_o)$ with negligible momentum transfer are excluded from the region of integration Ω, allowing us to use $\Delta\mathbf{p} = 2p_F(\hat{\mathbf{p}}\cdot\hat{\mathbf{v}}_o)\hat{\mathbf{v}}_o$ within Ω.

For an arbitrary standing-surface-wave mode in any geometry oscillating at frequency f, the vertical deviation from the equilibrium can be written as $z = z_0(x,y)\cos(2\pi ft)$. The total energy of the wave is

$$E_{\text{total}} = \frac{1}{2}\rho g\int_A z_0^2\,dA, \tag{5}$$

where ρ and g are the fluid density and gravitational acceleration, respectively. Surface energy has been neglected as a small contribution for wavelengths much longer than the capillary length. In helium fluids this is a safe assumption below frequencies of about 20 Hz.

According to Eqs. (3) and (4) the damping force is proportional to the area and the velocity of the free surface element with temperature-dependent multiplicative factor $-P(T)$, different for bosons and fermions. The total energy dissipated in one cycle is

$$E_{\text{loss}} = \int_A\int_{t=0}^{1/f} P\mathbf{v}_o^2\,dt\,dA = 2\pi^2 fP\int_A z_0^2\,dA. \tag{6}$$

The quality factor Q of the oscillation is then

$$Q = 2\pi\frac{E_{\text{total}}}{E_{\text{loss}}} = \frac{\rho g}{2\pi fP} = \frac{f}{\Delta f} \Rightarrow \frac{\Delta f}{f^2} = \frac{2\pi P(T)}{\rho g}, \tag{7}$$

where Δf is the resonance frequency width. An important observation is that the scaled frequency width $\Delta f/f^2$ does not depend on the geometry of the surface or on the resonance frequency, but only on the temperature and known physical parameters.

In our experiment helium was refrigerated by a nuclear demagnetization cryostat [21] in a cell with a central cuboid volume (length, 10 mm; width, 10 mm; height, 25 mm) that was connected from two opposite corners to a surrounding annular channel (diameter, 25 mm; width, 1 mm; height, 25 mm). The bottom of the central cuboid was 0.4 mm lower than the bottom of the annulus. A photograph of the cell can be found in Ref. [22].

The surface level and its local oscillations were detected capacitively with two independent interdigital capacitors mounted on opposite vertical walls of the cuboid volume. The surface waves could be generated either by ambient vibrational noise or by active drive [22].

Temperature readings are most reliable above 10 mK, where several independent thermometers are available in the cryostat and good thermal contact with the helium sample can be guaranteed. An independent calibration point was provided by the superfluid transition temperature of ^{3}He, while deep in the superfluid state of ^{3}He we had to rely on adiabatic changes of the magnetic field on the copper nuclear refrigerant. At the lowest temperatures, below 0.2 mK, temperature differences developed between the helium sample

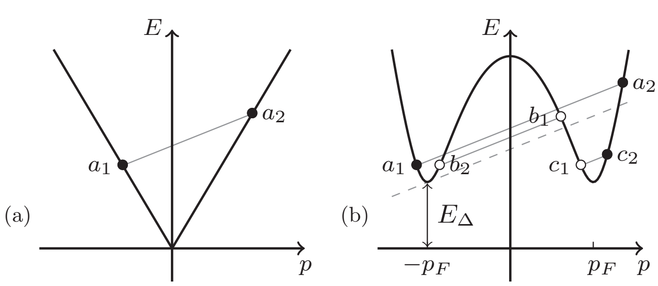

FIG. 2. Energy spectra of collective excitations in one dimension for bosonic ^{4}He (a) and fermionic ^{3}He (b) in the superfluid state. In ^{4}He the spectrum at low energies is linear (phononic branch) and the excitations are particle-like (filled circles) quasiparticles. In ^{3}He the relevant excitations are so-called Bogoliubov quasiparticles with isotropic energy gap E_Δ at Fermi momentum p_F and there are both particle-like (filled circles) and hole-like (open circles) quasiparticles. Representative scattering processes a, b, and c from the moving surface are represented by thin gray lines and explained in the text. The slope of the lines corresponds to the velocity of the moving surface, $\mathbf{v}_o$. Fermi quasiparticles below the dashed line in (b) experience Andreev reflection and contribute very little to the momentum transfer. Characteristic features like E_Δ and $\mathbf{v}_o$ are exaggerated for clarity.

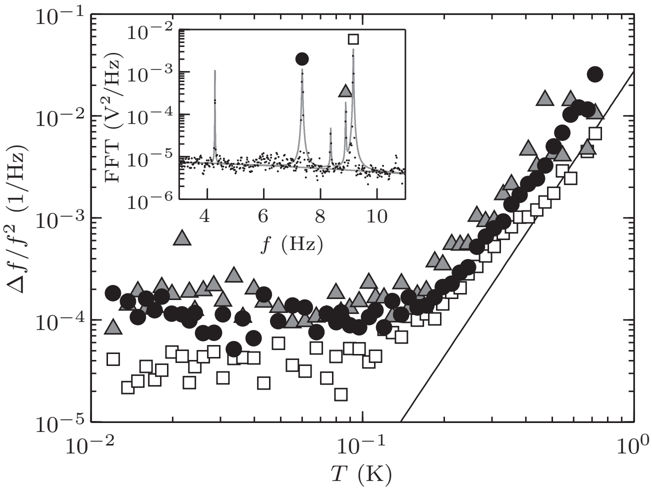

FIG. 3. Resonance frequency width Δf scaled by the resonance frequency squared f^2 in superfluid ^{4}He. The solid line represents the expected behavior according to Eqs. (3) and (7). There are no fitted parameters. The inset shows an example of the frequency spectra as driven by ambient noise at $T = 15$ mK together with a fitted curve. The peaks indicated by the three symbols refer to the data in the main frame. The finite frequency resolution results in leveling off of the data to about $\Delta f \approx 10$ mHz at the lowest temperatures.

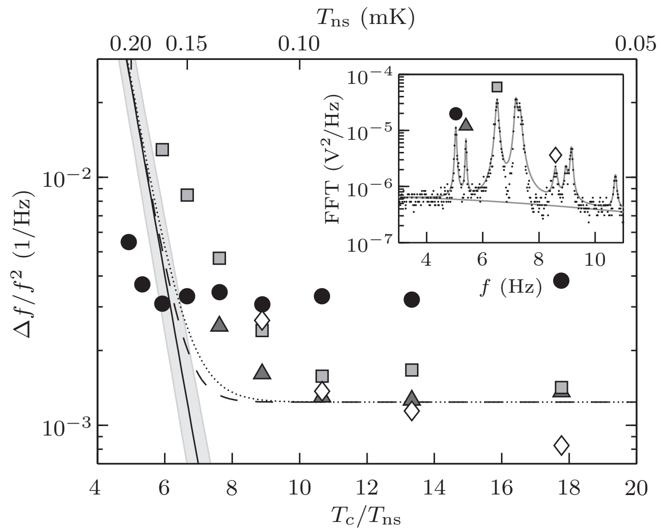

FIG. 4. Scaled resonance frequency width $\Delta f/f^2$ in superfluid ^{3}He as a function of the inverse temperature of the nuclear stage $T_{\rm ns}$, scaled by the superfluid transition temperature $T_c \approx 1$ mK. The solid straight line is the expected behavior according to Eqs. (4) and (7), with E_Δ being the BCS energy gap. Gray shading around the solid black line represents an alteration in E_Δ by $\pm 5\%$. Inset: An example of raw spectra with a fit, at $T_{\rm ns} = 0.11$ mK. The resonance width remains within resolution at any temperature, as ^{3}He cannot be cooled as deep into the superfluid state as ^{4}He in relative terms. Instead, the leveling-off is obviously due to losing thermal contact with the sample at about $T_c/T \approx 7$–10. The then-expected behavior [23] is illustrated by the dashed line. The dotted line assumes a constant-temperature-independent extra damping of $\Delta f/f^2 = 1.2\times10^{-3}$ Hz^{-1}.

and the copper refrigerant despite the extensive sintered heat exchangers on all available surfaces of the sample cell.

Surface-wave resonances in superfluid ^{4}He were observed up to 60 Hz frequency, including several higher order modes. Scaled frequency widths $\Delta f/f^2$ of the clearest resonances are shown in Fig. 3, with an example of raw frequency spectra below 11 Hz in the inset. At higher frequencies the surface tension would become more and more significant, altering the simple frequency scaling used. The helium level in the cuboid volume was $h = 5.2$ mm in this case. Results at other helium levels, down to 1.1 mm, were consistent with these.

The resonance frequencies of the low-frequency modes do not exactly match those expected from the geometry, and in fact, there are more peaks visible in our data than the rectangular and annular volumes should support. As explained above, though, it is not necessary to know the geometry or the wave profile, once we scale the resonance width by the resonance frequency squared. This takes care of complications due to possibly poorly defined geometry. It is convincing, therefore, that our scaled data fall into reasonably unified set and turn closely to the theoretical curve without any fitting parameters. At the lowest temperatures the resonances become so narrow that there is essentially just one spectrum point off the baseline at each resonance, limiting the lowest measurable frequency width to about 10 mHz. As the width is expected to scale as f^2T^4, it follows that the lower the resonance frequency, the higher the temperature where resolution is lost.

In superfluid ^{3}He the surface-wave resonances were observable below about $0.2T_c$. Scaled frequency widths of selected resonances are shown in Fig. 4, with a representative frequency spectrum in the inset. The peaks excluded from the analysis seemed to consist of multiple resonances inseparable from one another. This could be deduced on the basis of driven resonances, an example of which is shown in Ref. [22]. For the data illustrated in Fig. 4, the helium level was $h = 3.9$ mm in the cuboid volume. Two other helium levels were examined, showing similar overall tendencies.

There are remarkable deviations from the expected behavior but the correspondence cannot be improved by treating the energy gap E_Δ as a free parameter. The theoretical line in Fig. 4 would essentially shift in the horizontal direction, whereas the data suggest a less steep temperature dependence. Indeed, the best-resolved resonance (squares in Fig. 4) and two other, sparser sets of data (triangles and diamonds) form more or less consistent sets but the observed temperature dependence is much weaker than expected on the basis of Eqs. (4) and (7). Yet another set (circles) is of a completely different character. That mode has the lowest resonance frequency, so that it most likely has the highest amplitude at the outer annular volume. The resonance remains quite broad at the lowest temperatures, which could indicate just a somewhat higher temperature at the perimeter of the cell, which is weakly coupled to the central volume. However, in contrast to all expectations the resonance becomes *too narrow* at the highest temperatures, while the conditions most definitely should have become better equilibrated in terms of temperature no matter where that particular mode had the highest amplitude. We emphasize that this anomaly is not due to any simple analysis artifact, as this tendency is clearly visible in the raw spectra as well.

The leveling-off of the width at the lowest temperatures is not due to insufficient spectral resolution in this case since even 10 times narrower resonances were observed in supefluid ^{4}He with the same setup. Instead, helium temperature probably saturated due to Kapitza resistance, which has been treated according to an empirical model [23] to produce the dashed saturating curve in Fig. 4. However, a fairly large gap between this curve and the data remains. Alternatively, some other additional dissipation mechanism with a temperature-independent contribution to $P_F(T)$ could be assumed besides ballistic quasiparticles. This results in the bending dotted curve in Fig. 4, which is not much better and could not be distinguished from the Kapitza effect given the statistics of our data. For the time being we are not able to fully explain these results on ^{3}He.

Setting aside the problem of the imperfect fit to the theory, we can still comment on the sensitivity to temperature of the surface-wave resonances deep in the superfluid state. All other types of oscillators utilized so far lose their sensitivity because the damping due to the fluid practically vanishes and the device displays merely its own internal damping at the lowest possible temperatures in superfluid helium. This is not so for the surface-wave resonator in ^{3}He. There was still a margin of about a factor of 10 in the present experiment before the instrumental resolution would have become the limiting factor. If we interpret the leveling-off of the data as being caused by the saturating temperature, we get a lowest helium temperature of about $T_c/T \approx 10$, which is roughly the same as the lowest temperatures ever measured in ^{3}He [24].

In conclusion, the measured damping of surface waves in superfluid ^{4}He between 0.1 and 0.6 K corresponds well with the model of specular scattering of ballistic quasiparticles from the oscillating free surface. The low-temperature limit of sensitivity was set by the instrumental resolution, cutting off the temperature dependence below about 100 and 150 mK. In superfluid ^{3}He, however, there is a remarkable discrepancy between the specular scattering model and the experiment. In this case the resolution was not limited by the measuring scheme but, instead, either by additional damping mechanisms in superfluid helium or by the saturating temperature's not following that of the refrigerator at the very lowest temperatures. Any adjustment of the energy gap value suggested by the BCS theory does not improve the correspondence between our data and the theory.

We thank V. B. Eltsov, P. J. Heikkinen, J.-P. Kaikkonen, A. J. Niskanen, A. Ya. Parshin, V. Peri, A. Salmela, A. Sebedash, and V. Tsepelin for valuable discussions and assistance. This work was supported in part by the European Union FP7/2007–2013, Grant No. 228464 Microkelvin and by the Academy of Finland, CoE 2012–2017, Grant No. 250280 LTQ. This research made use of the Aalto University Low Temperature Laboratory infrastructure. We also acknowledge the grants from Jenny and Antti Wihuri Foundation and the National Doctoral Programme in Materials Physics.

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