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Menzel, Christoph; Hebestreit, E.; Alaei, Rasoul; Albooyeh, Mohammad; Mühlig, Stefan; Burger, S.; Rockstuhl, Carsten; Simovski, Constantin R.; Tretyakov, Sergei A.; Lederer, Falk; Pertsch, Thomas

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Published in:
Physical Review B

DOI:
[10.1103/PhysRevB.89.155125](https://doi.org/10.1103/PhysRevB.89.155125)

Published: 01/01/2014

Document Version
Publisher's PDF, also known as Version of record

Please cite the original version:
Menzel, C., Hebestreit, E., Alaei, R., Albooyeh, M., Mühlig, S., Burger, S., Rockstuhl, C., Simovski, C. R., Tretyakov, S. A., Lederer, F., & Pertsch, T. (2014). Extreme coupling: A route towards local magnetic metamaterials. *Physical Review B*, 89(15), 1-8. [155125]. <https://doi.org/10.1103/PhysRevB.89.155125>

Extreme coupling: A route towards local magnetic metamaterials

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(Received 3 December 2013; revised manuscript received 8 April 2014; published 18 April 2014)

Genuinely homogeneous metamaterials, which may be characterized by local effective constitutive relations, are required for many spectacular metamaterial applications. Such metamaterials have to be made of meta-atoms, i.e., subwavelength resonators, which exhibit only electric and or magnetic dipole and negligible higher-order multipolar polarizabilities in the spectral range of interest. Here, we show that these desired meta-atoms can be designed by exploiting the extreme coupling regime. Appropriate meta-atoms are identified by performing a multipole analysis of the field scattered from the respective meta-atom. To design those particular meta-atoms it is important to disclose the frequency and angular-dependent polarizability of both dipole moments. We demonstrate the applicability of a purely analytical model to accurately calculate for a normally incident plane wave reflection and transmission from meta-surfaces made of periodically arranged meta-atoms. With our work we identify a possible route towards the engineering of artificial materials while only considering the response from its constituents. Our approach is generally applicable to all spectral domains and can be used to evaluate and design metamaterials made from different constituting materials, e.g., metals, dielectrics, or semiconductors.

DOI: [10.1103/PhysRevB.89.155125](https://doi.org/10.1103/PhysRevB.89.155125)

PACS number(s): 42.25.-p, 42.70.-a, 78.20.Bh, 78.67.Bf

I. INTRODUCTION

With the promise of man-made materials to overcome the limits imposed by natural materials, the field of metamaterials tremendously evolved in optics and electrical engineering. Once these metamaterials with tailored artificial electric and magnetic properties are at hand, a huge variety of surprising and fascinating devices and applications may come in reach that can rely in their design on various tools, e.g., transformation optics [1]. However, most of these applications, such as shaping the wave beams, polarization transforms, frequency and polarization selective absorption (see, e.g., Refs. [2–7]), and some others require strictly local metamaterials. These are metamaterials where the electric and magnetic response represented by permittivity $\varepsilon_{ij}(\omega)$ and permeability $\mu_{ij}(\omega)$ only depends on the frequency ω . Thus, such materials are free of spatial dispersion [8,9] by means of a missing explicit dependency of $\varepsilon_{ij}(\omega)$ and $\mu_{ij}(\omega)$ on the wave vector $\mathbf{k}$.¹

For metasurfaces, i.e., 2D metamaterials, effective surface susceptibilities [10] $\chi_{ij}^{e,m}(\omega)$ are used instead of the bulk parameters $\varepsilon_{ij}(\omega)$ and $\mu_{ij}(\omega)$, where the same requirements with respect to spatial dispersion have to be met. In any case, the induced polarizations or polarizabilities, being either magnetic or electric, shall only depend on the magnetic and electric field at the specific spatial position. However, it is highly detrimental that the majority of metamaterials proposed thus far either exhibit strong spatial dispersion accompanied by a complicated angular-dependent optical response or, even worse, they are not even tested for spatial dispersion [11,12]. Such complicated angular-dependent response, unfortunately, often prohibits a simplified description of the material response and eventually limits their use for optical devices fundamentally. In simple terms, most of the proposed meta-atoms are simply too large compared to the wavelength to form local materials.

To achieve local magnetic metamaterials, several not necessarily distinct requirements have to be met: The meta-atoms as well as their mutual separation or lattice constant have to be small compared to the wavelength of the exciting field in the spectral domain of interest. Moreover, the response of the individual meta-atom to an external electromagnetic field has to be purely electric and or magnetic dipolar. Restricting to highly symmetric and achiral particles, any moment beyond the electric and magnetic dipolar one would cause a spatially dispersive metamaterial [9]. This is irrespective of the possible spatial arrangement, which might be a source of spatial

¹Of course, any artificial magnetism is a consequence of a spatially dispersive response of the material on the exciting field, which might be described by a permittivity $\tilde{\varepsilon}_{ij}(\omega, \mathbf{k})$ in general. However, in case of weak spatial dispersion the essentially spatially dispersive response might be reexpressed in terms of a distinct local permittivity $\varepsilon_{ij}(\omega)$ and a permeability $\mu_{ij}(\omega)$ where the latter accounts for a specific form of second order, so-called weak spatial dispersion [9].

dispersion as well.² Furthermore, these dipolar polarizabilities must not depend on the propagation direction or angle of incidence. Once the requirement on a pure electric and or magnetic dipolar response of a meta-atom is fulfilled, one can take advantage of a substantial volume of literature to further homogenize metasurfaces [10,13–15] and metamaterials [16–18] and to understand all peculiarities of light propagation through such materials. However, to make use of this theoretical backbone, true and experimentally accessible local meta-atoms need to be identified that obey the above requirements.

In this paper we will focus on the functionality of such individual meta-atoms and discuss their properties in terms of a multipole analysis [19–23], which gained considerable interest recently for single-particle analysis as well as arrays of scatterers [24–28]. At the example of meta-atoms with an artificial magnetic response we will show that the multipole analysis is an appropriate tool to identify the desired meta-atoms, which might eventually be used to build up local metamaterials [29–33]. For the sake of simplicity we will term these meta-atoms local meta-atoms in what follows. Starting with common designs for the cut-plate pair [34] and the split-ring resonator [35] in the optical domain, we will show that the exploitation of the extreme coupling regime as first studied in the context of metamaterials in Ref. [36] constitutes a viable route to eventually achieve local meta-atoms with a magnetic response. Complementary to the important issue of the coupling between neighboring meta-atoms [37–41], the coupling we are exploiting here takes place within the individual meta-atom. Of the suggested ways to design a metasurface, using local artificial magnetism is not the only one. Optically dense arrays of solid transparent particles operating at a magnetic Mie resonance may also possess local magnetic response [29–33]. These metasurfaces, as a rule, are targeted to the infrared range, though in accordance with Ref. [29] for the visible range such magnetic metasurfaces can be also designed. However, metasurfaces of densely arranged semiconductor nanoparticles correspond to specific technologies very different from those used for fabricating our plasmonic nanostructures. We believe that both these directions deserve further development.

Once the desired meta-atoms are achieved, it has to be proven that they can be used as building blocks for a desired metamaterial with given optical properties. Therefore, by taking advantage of a simple analytical model [15] we show that it is possible to reliably predict the optical response of an array of local meta-atoms, i.e., a metasurface, upon normal incidence plane wave illumination. We will show that as long as the contribution of higher multipolar polarizabilities is negligible one can consider such a metasurface as a local metamaterial.

The paper is structured as follows. By employing at first a film waveguide analysis and subsequently the multipole analysis, we show how to reduce the electrical size of the cut-plate pair to fulfill the requirements imposed on local

meta-atoms. We study the influence of geometrical parameters on the overall response of the cut-plate pair in detail and the multipolar composition of the scattered field. A regime is identified where the scattering response is dominated only by an electric and magnetic dipole moment. The developed approach is then applied to split-ring resonators as well to underpin its versatility. Eventually we show at the example of local cut-plate pairs that the scattering of light at periodically arranged magnetic meta-atoms operated in the extreme coupling regime can be described adequately by an analytical model.

II. CUT-PLATE PAIR IN THE EXTREME COUPLING REGIME

The cut-plate pair (CPP), which is also known as the double plate [42] or cut-wire pair [34], became a pivotal example for artificial magnetic meta-atoms during the last decade. For moderate structural sizes the optical response of the CPP may be understood in terms of the hybridization of two electric dipolar plasmonic modes into a symmetric and an antisymmetric mode [43]. The latter one exhibits a magnetic dipolar response and is known as the magnetic resonance. The CPP's superior performance originates from its potential to operate in the microwave [44] and infrared [45] as well as in the visible domain [46]. In particular, the magnetic resonance can be tuned quite easily by adjusting the length or diameter of the metallic plates as well as the thickness of the dielectric spacer between the plates. However, the primary goal of obtaining a homogeneous, local magnetic metamaterial was indicated just recently by bringing the metallic plates into an extreme coupling regime [36]. Whereas for common spacer thicknesses of around 30 nm the splitting between the symmetric and the antisymmetric mode is rather small in the optical domain, it gets considerably larger in the extreme coupling regime where the spacer is much thinner. In previous work the bulk properties of metamaterials in the extreme coupling regime have been discussed in terms of a Bloch modal approach [36,47,48]. Here we focus on the single-particle response and the identification of promising meta-atoms for homogeneous magnetic metamaterials. We identify that a scattering response in the extreme coupling regime for the individual meta-atom, dominated by an electric and magnetic dipole, is the physical reason for the achievement of a local metamaterial.

To identify possible routes of reducing the wavelength-to-particle size ratio, we employ a film waveguide analysis at first [49]. With such approach a simple estimate of the resonance frequencies beyond the mere qualitative hybridization picture is possible. For this purpose the plates or discs of the cut-plate pair are assumed to be infinitely extended in lateral directions forming a three-layer metal-insulator-metal (MIM) film waveguide.

For an insulator thickness smaller than the wavelength, the waveguide supports just a single propagating mode. By calculating the propagation constant $k_{\text{eff}}(\omega)$ of this MIM-waveguide mode and neglecting in a first approximation the phase accumulated upon reflection at the end facet of a finite waveguide with length D , the fundamental resonance frequency can be estimated by evaluating the condition

²The source of spatial dispersion might be either the single-particle response or the spatial arrangement on a periodic lattice or both simultaneously.

FIG. 1. (Color online) (a) Geometry of the cut-plate pair (CPP). The CPP can be understood as a finite metal-insulator-metal waveguide resonator as indicated by the semitransparent film. The diameter of the CPP is fixed to 180 nm. The metal layers are made of gold [51]. The refractive index of the spacer film is assumed to be 1.5. (b), (c) Imaginary parts of the induced electric and magnetic dipole moments p_x and m_z for different gap thicknesses d and fixed metal-layer thickness $d_{Au} = 30$ nm. The legend indicates the thickness d . (d), (e) Imaginary parts of the induced electric and magnetic dipole moments p_x and m_z for different metal-layer thickness d_{Au} and fixed gap thickness $d = 5$ nm. The legend indicates the thickness d_{Au} in nm. Each of the curves is normalized to the results obtained for a CPP with $d = 5$ nm and $d_{Au} = 30$ nm. The CPPs are illuminated with an x -polarized plane wave propagating in the y direction. For the ratio λ/D the free space wavelength is used. Note the different frequency axes in (b), (c) and (d), (e) for better resolution of the resonance shift.

$k_{eff}(\omega)D = \pi$ [see Fig. 1(a)]. Hence, by considering k_{eff} for different waveguides, we can conclude on the resonance shift upon variation of the waveguide parameters, e.g., the thickness of the layers and the diameter or length of the finite resonator. Note that the actual resonance condition is slightly different for circular CPPs. There, the resonance is related to the roots of a Bessel function [50] whereas above the roots of a sinusoidal function have been evaluated. This is in agreement with the assumption of a planar cavity. To obtain meta-atoms that sustain resonances deep in the subwavelength regime, it is required to identify geometrical parameters such that k_{eff} is as large as possible and, hence, the fundamental resonance occurs at the smallest possible resonator. A careful analysis of the dispersive properties of the MIM waveguide shows that the propagation constant for the fundamental TM-polarized mode (magnetic field parallel to the MIM plates) increases considerably by decreasing either the thickness of the insulator or the

metal layers [49]. This suggests that the thinner these layers the larger the effective index of the waveguide and, hence, the smaller the diameter of the CPP can be while preserving the fundamental resonance frequency. However, scaling down the whole particle was previously shown to be detrimental for the strength of the artificial magnetism [12], since the excitation strength of the magnetic resonance ultimately vanishes upon downsizing. Therefore, such simplistic scaling is not a useful solution. Instead, the response of the CPP upon plane wave excitation has to be analyzed directly to identify a meaningful compromise between a resonance frequency that shall be as small as possible and a resonance strength that shall be as large as possible. Those results are discussed in the next paragraph.

To support the quasianalytical findings we computed the scattered near field around a single CPP illuminated with linearly polarized, monochromatic plane waves at an incidence angle normal to the x - z plane [see Fig. 1(a)]. To analyze the single-particle response of the CPP we used a multipole analysis of the scattered field, as detailed in Ref. [19]. Here, the scattered field is expanded into a series of spherical harmonics, where the expansion coefficients are closely related to spherical multipole moments. The respective Cartesian multipole moments can be then straightforwardly calculated. The CPP were chosen to be cylindrical symmetric with varying diameter D and thicknesses d and d_{Au} of the individual plates. The calculations for the scattered field were performed using the full-wave Maxwell solver JCMSUITE [52] based on a finite element approach [53]. This method is highly suitable since it combines high-order convergence of FEM with geometrical flexibility of unstructured, adaptive spatial discretization. The results upon geometric variations are shown in Fig. 1. There, for simplicity, only the imaginary parts of the Lorentzian-shaped electric and magnetic dipolar moments are shown, indicating the electric and magnetic resonance strength as well as their spectral position. Note that all graphs are normalized to the results of a particle with $D = 180$ nm, $d_{Au} = 30$ nm, and $d = 5$ nm.

In Figs. 1(b) and 1(c) we show the imaginary parts of p_x and m_z upon variation of the spacer thickness d . Clearly, the electric resonance is almost unaffected in shape and size. This contradicts the perception of the hybridization picture, where the symmetric resonance is shifted to larger frequencies with increasing coupling strength. However, the plates are quite small with a diameter of $D = 180$ nm, such that the electric resonance already approaches the saturation limit imposed by the material dispersion of gold, i.e., the plasma frequency constitutes an upper limit. Hence, the shift is quite small but appears clearly towards higher frequencies. The electric resonance is basically determined by the overall amount of metal used for the CPP. The magnetic resonance, however, is strongly shifting towards smaller frequencies for a thinner spacer. This is expected from the MIM-film-waveguide analysis. Furthermore, the resonance strength is slightly decreasing with decreasing d , indicating a reduced excitation strength. With regard to the dependency of the response on the variation of the metal plate thickness, as shown in Figs. 1(d) and 1(e), the situation changes considerably. The electric resonance is sharpened and slightly shifted towards smaller frequencies with decreasing metal plate thickness, i.e., thinner metal plates lead to less scattering and narrower resonances where the

FIG. 2. (Color online) (a) Logarithmic plot of the scattering cross section normalized to ω^4 . The solid curves indicate the electric (blue) and magnetic dipole (red) as well as the electric (green) and the magnetic quadrupole (cyan) contributions for the CPP. The dotted curves trace the values of each multipolar moment at the magnetic resonance while changing the gap thickness d from 30 nm to 1 nm. (b), (c) Real and imaginary part of the tangential electric and magnetic dipole moment $p_t = (p_x, p_z)$ and $m_t = (m_x, m_z)$ when changing the angle of incidence (0° , 45° , 90° with respect to the axis of rotational symmetry) or the polarization (TE and TM polarization is used). The CPP used here is characterized by $D = 180$ nm, $d = 5$ nm and $d_{\text{Au}} = 30$ nm. The different lines obtained for different polarizations and angles of incidence coincide very well for small frequencies, where the deviation gets larger with increasing frequency. Note the normalization of the dipole moments to the exciting field.

resonance position is basically determined by the diameter of the particle.

Overall, the influence on the electric dipole is of minor importance. The magnetic resonance is again shifted towards smaller frequencies. However, the shift is quite small compared to the results shown in Fig. 1(c) and more importantly the resonance strength is tremendously reduced, i.e., the excitation strength of the magnetic resonance gets negligible with decreasing metal plate thickness. This explains the observations made in Ref. [12] that the magnetic resonance vanishes upon downsizing the CPP simultaneously in all its dimensions. We can also conclude that a certain amount of metal is necessary to sustain magnetic resonances of a sufficient strength. Therefore, the metal thickness should have a reasonable value.

In conclusion, reducing the spacer thickness allows us to increase at the magnetic resonance the ratio of free-space wavelength to particle size from $\lambda/D = 5.6$ to values larger than 11 for moderate spacer thicknesses of 5 nm. This is sufficient to meet the first requirement of quasistatic magnetic meta-atoms.

To meet the second requirement imposed on potentially local magnetic metamaterials the dominance of a dipolar scattering response has to be ensured. This requires a minimum contribution of higher-order moments since those prohibit a successful homogenization in general [9,16]. Therefore, we plotted the leading electric (blue) and magnetic dipolar (red) as well as the electric (green) and magnetic quadrupolar (cyan) contributions to the scattering cross section in Fig. 2(a).

Different line types signify different quantities associated to the multipole moments as detailed further below. First of all, it has to be noted that the scaled resonance strength $C_{\text{sca}}^l/\omega^4$ is displayed to account for the fact that the scattering cross section of an electric dipole scales with $C_{\text{sca}}^{l=1} = \omega^4 |p|^2$ where l indicates the multipolar order and m the angular momentum, which is responsible for the geometrical orientation. ($C_{\text{sca}}^l = \sum_m C_{\text{sca}}^{lm}$) [19]. The lines associated to each multipole moment are drawn thin in general but are drawn thicker in spectral domains of importance, i.e., where the respective resonance occurs, to draw the attention of the reader to this spectral domain. Furthermore, we have shown as dotted lines the value of each moment at the magnetic resonance while changing the spacer thickness as already shown in Figs. 1(b) and 1(c). Whereas the entire frequency dependence of the magnetic dipolar (red) and the electric quadrupolar (green) contributions are shown for the limiting cases of $d = 1$ nm (left) and $d = 30$ nm (right), the entire frequency dispersion of the electric dipolar (blue) and the magnetic quadrupolar (cyan) contribution are shown just for the case $d = 30$ nm since their change upon d is quite negligible [cf. Fig. 1(b)].

In addition to the obvious shift of the magnetic resonance with decreasing d , the strength of the undesired electric and magnetic quadrupolar contributions decrease considerably; not just with respect to the absolute value, but also with respect to the dominating electric and magnetic dipole moment. The dominance of dipolar contributions is an important requirement on local meta-atoms, since higher-order multipolar moments lead to spatial dispersion in general. Unfortunately, the strength of the magnetic dipole decreases as well. From the point of view of scattering cross sections, a spacer thickness of $d = 15$ nm, where the difference between magnetic dipolar and higher-order contributions is maximal, might be ideal. However, the ratio of λ/D is only ≈ 7 here, which is rather small for a successful homogenization in terms of local effective parameters in general.

Besides the requirement of large aspect ratios and dipolar dominance, the independence of the meta-atoms' response upon change of the angle of incidence has to be guaranteed. Therefore, we calculated the scattered field upon linear polarized (TE, TM) plane wave excitation for 0° , 45° , and 90° degrees with respect to the axis of rotational symmetry exemplarily for a CPP with $D = 180$ nm, $d_{\text{Au}} = 30$ nm, and $d = 5$ nm. We plotted the tangential dipolar moments, which are relevant at normal incidence, in SI units after normalization to the unit electric field in Figs. 2(b) and 2(c). Obviously, the graphs for different angles of incidence coincide very well, where the deviations get larger with increasing frequency. However, at the important magnetic resonance around 160 THz the graphs match almost perfectly, verifying the required angular independence of the dipole polarizabilities.

The observed effects of extreme coupling within individual particles are in line with the fact that increasing coupling leads to more uniform distribution of induced current in the magnetic mode (see Fig. 3).

When the distance between the two plates is comparatively large and the coupling between the plates is weak, the induced current becomes very small at the plate edges. This

FIG. 3. (Color online) The modulus of the total electric field inside the CPP at the magnetic-mode resonance for different distances between the plates: (a) 30 nm, (b) 5 nm, and (c) 1 nm.

happens because for large separations between the plates the capacitance between the plates is small, and the current at the ends of the plates experiences a very high impedance (open ends). In this regime there is considerable accumulation of charge at the edges of the two plates, and, in terms of the multipole theory, this means strong excitation of the electric quadrupole and higher-order multipole moments. When the distance between the plates decreases, strong coupling via large distributed capacitance makes the distribution of induced current pretty uniform. Since there is little accumulation of charge inside the particle, this mode is characterized by strong dominance of the magnetic dipolar moment over all higher-order modes. This is illustrated by Fig. 3, where the modulus of the total electric field inside CPP is plotted for three different separations between the plates. For smaller gaps (stronger coupling), the distributions become more and more uniform. In Fig. 3 the modulus of the field rather than its real part was plotted. Displaying the latter it is hardly possible to identify the resonance characteristics due to the interference of different multipoles as well as the interference with the incident field in this case. In fact, it is often difficult to gain insight into resonator dynamics by plots of the total field. That is why we prefer and promote the multipole analysis for physical interpretation.

With all these results in mind we can safely conclude that the meta-atoms devised from the principle of extreme coupling obey all desired properties. They have a strong scattering response that is dominated by either electric or magnetic dipole contributions and, moreover, the polarizabilities of these meta-atoms are independent on the propagation direction. Operated in the extreme coupling regime, the CPP is hence a promising base for truly local, artificially magnetic metamaterials.

III. SPLIT-RING RESONATORS IN THE EXTREME COUPLING REGIME

After the successful application of the multipole analysis to extremely coupled CPPs for the identification of local meta-atoms, in a second step we apply the concept of extreme coupling to the well-known split-ring resonator (SRR) [56]. This is done with the purpose to demonstrate that the suggested design route is naturally not limited to a specific type of meta-atom. Quite on the contrary, the approach is versatile and can be applied to many other structures and material platforms. For a SRR the coupling is maximized by reducing the split size down to the few nanometer level. Here, the extreme coupling

FIG. 4. (Color online) (a) Logarithmic plot of the scattering cross section normalized to ω^4 . The solid curves indicate the electric (blue) and magnetic dipole (red) as well as the electric (green) and the magnetic quadrupole (cyan) contributions for the SRR. The dotted curves trace the values of each multipolar moment at the magnetic resonance while changing the split size g . (b) Schematic drawing of the split-ring resonator. The thickness is fixed to $d = 30$ nm. The width of the quadratic SRR is fixed to $W = 180$ nm. The split gap size g is varied. (c) Modulus of the magnetic dipole moment for changing split gap sizes as indicated in the legend. For illumination a y -polarized plane wave propagating in the z direction is used.

takes place between the wire ends forming the split gap, leading to a dramatic increase of the gap's capacity [57,58].

Whereas thin dielectric layers as used for the CPP might be realized quite easily by atomic layer deposition [54], a split gap of just a few nanometers is much more difficult to achieve on large-scale samples. By standard electron beam lithography such small gaps are almost impossible due to proximity effects. However, it was shown that such small gaps can be reproducibly achieved by using thin membranes [55,59] or by using advanced nanofabrication techniques, e.g., helium ion lithography [60].

For an operation in the optical domain, we have chosen a split-ring resonator made of gold [51] as shown in Fig. 4(b).

While keeping all the other geometrical parameters fixed the split gap was varied between 5 and 100 nm. The SRR is illuminated by a y -polarized plane wave propagating in the z direction, such that a pronounced magnetic dipolar contribution (m_z) can be expected accompanied by an electric dipole (p_y) parallel to the incident field for the fundamental SRR resonance [19]. For this illumination direction the generally important issue of electromagnetic coupling and bianisotropy of split-ring resonators [61] plays no role. The calculations are performed by using COMSOL [62]. The results for the multipolar resolved scattering cross sections are shown in Fig. 4(a). Again, C_{sca}^l is shown just for the largest (large resonance frequency) and the smallest gap (small resonance frequency). The trace of resonance maxima is again indicated by dotted points. The results for the induced magnetic dipoles are shown in Fig. 4(c). Similar to the CPP a strong, nonlinear shift of the magnetic resonance with decreasing gap size towards smaller frequencies can be observed at a reasonably small decrease in excitation strength. This strong shift results

in an increase of the ratio of resonance wavelength to particle size λ/W from approximately 9.6 for a 100 nm gap to approximately 17 for a 5 nm gap. Contrary to the CPP, the ratio between the magnetic and the electric dipole is approximately constant for all gap sizes, keeping the magnetic contribution reasonably large for all configurations. The ratio of dipolar to quadrupolar moments is even increasing with decreasing gap size, enhancing the dominance of dipolar contribution over higher-order multipolar contributions. Comparable to the CPP, the SRR operated in the extreme coupling regime is fulfilling the requirements on a meta-atom to built up a local magnetic metamaterial. With respect to the strength of the magnetic dipole compared to the electric dipole and the ratio of wavelength to particle size, it even outperforms the CPP.

IV. MODELING A CUT-PLATE PAIR METASURFACE

It is always desirable to decrease numerical efforts in the study of physical phenomena. Analytical models, if applicable, are good tools to achieve this goal [10,13,14,17,18,30,31]. In the present work, we apply a local analytical model based on the dipole approximation which can be found, e.g., in Ref. [15]. This model describes the reflection and transmission of a normally incident plane wave at a periodic optically dense arrangement of small scatterers modeled as pairs of electric and magnetic dipoles. This model implies the absence of bianisotropy, i.e., the electric (magnetic) dipole is excited only by electric (magnetic) local field. A more general model extended towards bianisotropic planar grids at arbitrary oblique illumination can be found in Ref. [63]. More elaborated models can be found in Ref. [10] (for the oblique incidence in absence of bianisotropy) and Ref. [64] (for the bianisotropic case). Furthermore, other sophisticated approaches, taking into account Green's dyadic and leading to the same results for bianisotropic planar arrays, were reported in Refs. [14,65]. However, with the purpose to show the adequacy of the local model, replacing the original metasurface with a sheet of dipole electric and magnetic polarizations, we focus on the simple example of normal illumination here.

The planar array of cut-plate pairs is situated in the x - z plane [Fig. 1(a)]. The array is excited from the $y > 0$ direction with an incident plane wave characterized by an electric field $\mathbf{E}_i = E_x^{\text{inc}} \exp(-ik_y y) \exp(-i\omega t) \hat{x}$, where E_x^{inc} , k_y , ω , and $\hat{x}$ denote the amplitude, propagation constant, angular frequency, and the x -axis unit vector, respectively. In this model, each cut-plate pair represents a polarizable entity with an electric and magnetic polarization of p_x and m_z , respectively

$$\begin{pmatrix} p_x \\ m_z \end{pmatrix} = \begin{pmatrix} \alpha_{xx}^{\text{ee}} & 0 \\ 0 & \alpha_{zz}^{\text{mm}} \end{pmatrix} \begin{pmatrix} E_x^{\text{loc}} \\ H_z^{\text{loc}} \end{pmatrix}. \quad (1)$$

Here, E_x^{loc} and H_z^{loc} are the x and z components of the local electric and magnetic fields at the place of each particle and α_{xx}^{ee} and α_{zz}^{mm} are the tangential components (x and z) of the individual electric and magnetic polarizability of each particle, respectively. The asymmetric geometry forces all cross components ($\alpha_{xy, xz, yz}^{\text{ee, mm}}$) of the polarizabilities to be zero. Furthermore, the effect of bianisotropy is absent ($\alpha^{\text{em}} = \alpha^{\text{me}} = 0$, i.e., all magnetoelectric components are zero) since there is no asymmetry in the geometry with respect to the incident

field. For the specific illumination no normal components of the electric or magnetic dipoles are excited. Therefore, the reduced scalar form of Eqs. (1) is used instead of the complex full-rank matrix equations used in the extended studies (see, e.g., Ref. [63]). The local fields comprise contributions from the incident ($E_x^{\text{inc}}/H_z^{\text{inc}}$) and interaction ($E_x^{\text{int}}/H_z^{\text{int}}$) fields from other particles at the location of the considered particle:

$$E_x^{\text{loc}} = E_x^{\text{inc}} + E_x^{\text{int}}, \quad (2)$$

$$H_z^{\text{loc}} = H_z^{\text{inc}} + H_z^{\text{int}}. \quad (3)$$

Since all particles on the periodic array are identical we have

$$\begin{pmatrix} E_x^{\text{int}} \\ H_z^{\text{int}} \end{pmatrix} = \begin{pmatrix} \beta_{xx}^{\text{ee}} & 0 \\ 0 & \beta_{zz}^{\text{mm}} \end{pmatrix} \begin{pmatrix} p_x \\ m_z \end{pmatrix}. \quad (4)$$

Here, β_{xx}^{ee} and β_{zz}^{mm} are the tangential interaction constants, which take into account local field corrections. The tangential interaction constants between electric and magnetic dipoles ($\beta_{xz, zx}^{\text{em, me}}$) are zero since both electric and magnetic dipoles are located in the same plane. Moreover, no normally oriented dipoles will be excited upon normally incident plane waves such that the off-diagonal tensor elements $\beta_{yz, zy}^{\text{mm, ee, em, me}}$ do not enter the equations to be solved. For a dense array of resonant inclusions, as in the present case, the necessary interaction constants are analytically given by

$$\beta_{xx}^{\text{ee}} = -i \frac{\eta_0 \omega}{4P^2} \left(1 - \frac{1}{ikR_0} \right) e^{-ikR_0} \quad (5)$$

$$\beta_{zz}^{\text{mm}} = \beta_{ee}/\eta_0^2 \quad (6)$$

with $R_0 = P/1.438$ being an effective interparticle distance [67,68]. Here, η_0 is the free-space impedance, ω the frequency, k the vacuum wave number and P the period of the square lattice.

The applicability of this analytical model requires locality of the meta-atoms, which was proven above. From the electric and magnetic dipole polarizabilities α_{xx}^{ee} and α_{zz}^{mm} obtained earlier (see Fig. 1) and analytically derived electric and magnetic interaction constants β_{xx}^{ee} and β_{zz}^{mm} , we obtain the complex reflection r and transmission t coefficients at normal incidence as [66]

$$r = \frac{i\omega}{2P^2} \left(\frac{1}{\eta_0} \frac{\alpha_{zz}^{\text{mm}}}{1 - \alpha_{zz}^{\text{mm}} \beta_{zz}^{\text{mm}}} - \eta_0 \frac{\alpha_{xx}^{\text{ee}}}{1 - \alpha_{xx}^{\text{ee}} \beta_{xx}^{\text{ee}}} \right) \quad (7)$$

$$t = 1 - \frac{i\omega}{2P^2} \left(\frac{1}{\eta_0} \frac{\alpha_{zz}^{\text{mm}}}{1 - \alpha_{zz}^{\text{mm}} \beta_{zz}^{\text{mm}}} + \eta_0 \frac{\alpha_{xx}^{\text{ee}}}{1 - \alpha_{xx}^{\text{ee}} \beta_{xx}^{\text{ee}}} \right). \quad (8)$$

The first term appearing inside the parentheses of Eqs. (7) and (8) represents the contribution of the magnetic mode while the second term represents the electric mode contribution to the scattered field. These formulas involve the response of individual scatterers ($\alpha_{xx/zz}^{\text{ee/mm}}$) as well as the interaction ($\beta_{xx/zz}^{\text{ee/mm}}$) between them. Note that just the period P as well as the dipolar polarizabilities enter these equations. There is, hence, no free parameter that might be used for fitting analytical to rigorous results.

In Fig. 5 we compare the results of the analytical model to rigorous simulations of a periodic arrangement of CPPs [62]. Two different CPP meta-atoms are considered, where only one

FIG. 5. (Color online) Reflectance (black) and transmittance (blue) at a periodic ensemble of cut-plate pairs on a square lattice. The solid lines are the results obtained from rigorous simulations. The dashed lines are for the analytical model based on the dipole polarizabilities. The cut-plate plate has a diameter of $D = 180$ nm, a metal layer thickness of $d_{\text{Au}} = 30$ nm and a spacer thickness d of 5 nm (a) and 30 nm (b) respectively.

of them has been previously identified as a local meta-atom at resonance. The period of the square lattice is $P = 250$ nm, the diameter of the cut-plate pairs is $D = 180$ nm, and the thickness of the metal layer is $d_{\text{Au}} = 30$ nm.

The thickness of the spacer that controls the coupling strength and, hence, the resonance frequency of the magnetic resonance is chosen as $d = 5$ nm in Fig. 5(a) and $d = 30$ nm in Fig. 5(b), i.e., we compare extremely coupled CPPs with standard ones. At first, it can be seen that the agreement between analytical and rigorous results is surprisingly good at lower frequencies. However, the deviation between both results, in particular for the transmission, is nonlinearly increasing with increasing frequency. The relative error, which is not shown here, increases parabolically with the frequency. For frequencies less than 200 THz, however, the deviation is practically negligible. Therefore, the cut-plate pair operated in the extreme coupling regime in Fig. 5(a) with the magnetic resonance at 150 THz can be described very well by the analytical model. The cut-plate pair in standard implementation in Fig. 5(b), however, has its magnetic resonance at 300 THz where the analytical model starts to strongly deviate from the rigorous results. It can be seen that not just the magnitude of the optical coefficients deviates but also differences exist in the resonance frequency.

These results perfectly agree with the conclusion drawn from the multipole analysis (see Fig. 2). Since the analytical model is based on the assumption of local electric and magnetic dipoles only, the deviation between analytical and rigorous results increases with increasing contribution of higher-order multipoles, which is by itself increasing with the frequency. Just in the case of extreme coupling the artificially magnetic meta-atoms are local at resonance and the analytical model can be applied.

V. CONCLUSION

In this paper we have shown at the example of cut-plate pairs and split-ring resonators how to design local magnetic meta-atoms by exploiting the extreme coupling regime. To achieve a purely local response we had to ensure that the optical response of the meta-atoms is purely electric and or magnetic dipolar with wavelength-to-particle size ratios approximately or greater than 10. To identify potentially local meta-atoms a multipole analysis was successfully employed. This type of multipole analysis permits access to complex multipole moments and polarizabilities. This will allow for an understanding and tailoring of optical antennas with, e.g., enhanced directivity as well. Eventually we have employed an analytical model for modeling the scattering of light at periodically arranged cut-plate pairs. A comparison to rigorous simulations shows a perfect agreement for extremely coupled cut-plate pairs at resonance. These results underline the versatility of the concept of extreme coupling as well as of the multipole analysis and prove the applicability of analytical models to meta-surfaces composed of local meta-atom.

ACKNOWLEDGMENTS

This work was supported by the German Federal Ministry of Education and Research (PhoNa), by the Thuringian State Government (MeMa) and Deutsche Forschungsgemeinschaft (DFG Research Center MATHEON). C.M. gratefully acknowledges financial support by the Carl Zeiss Foundation. C.R. would like to acknowledge the German Science Foundation for support within the project RO 3640/3-1.

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