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Spatial game in cellular automaton evacuation model

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For numerical simulations of crowd dynamics in an evacuation we need a computationally light environment, such as the cellular automaton model (CA). By choosing the right model parameters, different types of crowd behavior and collective effects can be produced. But the CA does not answer why, when, and how these different behaviors and collective effects occur. In this article, we present a model, where we couple a spatial evacuation game to the CA. In the game, an agent chooses its strategy by observing its neighbors' strategies. The game matrix changes with the distance to the exit as the evacuation conditions develop. In the resulting model, an agent's strategy choice alters the parameters that govern its behavior in the CA. Thus, with our model, we are able to simulate how evacuation conditions affect the behavior of the crowd. Also, we show that some of the collective effects observed in evacuations are a result of the simple game the agents play.

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I. INTRODUCTION

When a large crowd evacuates from a public place or building, it gives birth to many collective effects. These collective effects include clogging at bottlenecks and the faster-is-slower effect. In the faster-is-slower effect, the attempt of individuals to move faster results in a slower evacuation for the whole crowd.

There are experiments studying microscopic pedestrian behavior in bottlenecks. In [1] it was shown that a clogging can occur without the flow exceeding the capacity of the bottleneck. It has also been shown that humans form lanes in bottlenecks. These lanes overlap, which is why the phenomenon is called the zipper effect. Because of the zipper effect, the flow in a bottleneck is a stepwise function of the bottleneck width [2,3].

In [4] experimental proof for the faster-is-slower effect is given. In the experiment, a group of people was set to evacuate a room through an exit. Time lapses between consecutive individuals were monitored. An interesting result was that the higher the desired velocity of the evacuees was, the more likely longer time lapses were.

The time lapses are a result of the formation and break down of archlike blocks of humans at exits. These arches are able to hinder the flow at exit. An irregular succession of archlike blocks forming and breaking also causes the underlying phenomena for clogging at bottlenecks [5]. Note that a higher probability for longer time lapses, not only means a slower evacuation, but also a higher probability for people to get suffocated and squeezed.

In contrast, there is also experimental evidence of phenomena, which can improve the evacuation efficiency. In [6] it was shown that placing an obstacle in front of an exit makes the evacuation more efficient. Also, maybe we could learn something from how ants behave when facing danger. In [7] it was shown that the faster-is-slower effect is not witnessed in ants, because ants are always evenly distributed in the space, unlike humans which form dense areas in front of exits.

What is evident, is that for a safe evacuation, it is crucial to understand the mechanism behind dangerous collective effects. The agent-based social force model describes the actual physical forces arising in a rushing crowd [5]. It has been implemented to the software FDS+EVAC, which allows simultaneous simulation of both fire spreading and evacuation process [8]. Collective effects in evacuations can be realistically simulated with FDS+EVAC by setting specific model parameters [8–11]. However, it does not answer why, when, and how these effects occur.

There lies a consensus among social theorists that people behave rationally in evacuation situations [12–14]. In his famous experiment, Mintz showed that escape panic is due to the reward structure innate to the evacuation situation [15]. Moving orderly towards the exit is rewarding, as long as everybody else moves orderly. When one agent starts to rush, rushing behavior becomes rewarding also for the other agents.

Brown attempted to model Mintz' results with game theory. He suggested the agents play a one-shot Prisoner's Dilemma (PD) game [16]. In the game, the agents choose simultaneously, not knowing the other agents' choice, whether to rush or evacuate orderly. The outcome of the game is for all agents to rush. However, this is not exactly in line with Mintz' results, and with how people behave in evacuation situations. In reality, people can observe the behavior of other agents. Thus, it has been suggested that the agents rather play a repeated version of PD, where the agents make their decisions based on what the other agents have played in the previous rounds of the game [17].

Heliövaara *et al.* [18] have pointed out that an evacuation situation is spatial in nature, and, since time is a limited resource, the reward structure should change according to how far from the exit the agent is. Based on these remarks, Heliövaara *et al.* proposed a *spatial game*, where the agents play against their nearest neighbors. The agents can choose between two strategies: *Patient* and *Impatient*. The game will be either Hawk-Dove (HD) or PD, depending on how far they are from the exit. The strategy Patient corresponds to the strategy of evacuating orderly, and Impatient to rushing in Mintz' experiment. The agents are *myopic* in the sense that they choose their strategies based on the previous period play

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of their opponents, not considering the play further back in history, or of future periods.

The spatial game has been implemented to FDS+EVAC in [18]. There, the physical behavior of the agents is obtained by altering the parameters of the social force model in the following way: impatient agents do not avoid contacts with other agents as much; they accelerate faster to their target velocity, and move more nervously. On the other hand, patient agents avoid contact with other agents.

Simulations with this version of FDS+EVAC gives an explanation to the faster-is-slower effect [18]. Due to the game agents play, the proportion of impatient agents grows farther back in the crowd. Impatient agents push the crowd in front of them. As a result of the driving force of the agents back in the crowd and frictional interaction between agents, a clogging is formed [5]. The clogging slows down the evacuation, and becomes more severe as the proportion of impatient agents grows.

However, FDS+EVAC is computationally rather heavy. This is because the social force model is continuous and interaction forces between all agents are modeled. In this paper, our goal is to couple the spatial game [18] to a computationally lighter evacuation model. Such a model could be used by the incident commander, e.g., through the Internet, to give timely and accurate instructions in evacuation situations.

For our purposes, the computationally very light cellular automaton evacuation model (CA) is appropriate. With it, it will be possible to produce many collective effects observed in evacuation situations, for example, lane formation, clogging, and the faster-is-slower effect [19–22]. In CA the agents move in a discrete square grid according to some transition probabilities. These transition probabilities depend on the so-called *coupling parameters*, which describe the degree to which the agents move straight towards the exit, or move to regions of higher crowd flow.

There is some literature on agents' decision-making in CA evacuation models [23–27]. In these articles, game theory has mainly been used to solve conflict situations, i.e., situations where several agents try to occupy the same space simultaneously. The agents play a game when in a conflict, to determine who gets to move to the cell. Thus, the decision-making is limited to these conflicts only. However, it is reasonable to assume that the agents interact with their immediate neighborhood, and make decisions, also when not in a conflict. Furthermore, in large-scale simulations, the games the agents play at different distances from the exit should differ.

In this paper, we propose a model, where the spatial evacuation game presented in [18] is coupled with the CA presented in [21]. The agents observe their neighbors' strategies in the discrete square grid, and choose their strategy accordingly. The game the agents play depends on the distance to the exit and the available time to safely evacuate. The choice of strategy alters the agents' behavior in the CA: If the agent plays Impatient, it will rush straight towards the exit, and, if it chooses Patient, it will move towards the exit using regions of higher crowd flow. These behaviors are obtained by choosing the coupling parameters for the agents accordingly. As far as we know, there are no earlier studies where agents in a CA can have different values for the coupling parameters, or

the agents' coupling parameters would change systematically during the simulation.

Our model takes into account that there are differently behaving individuals in the crowd and that their behavior can change during the evacuation. Simulation results with our model show that some of the collective effects observed in evacuations are a result of the simple game the agents play.

The paper is structured as follows: In Sec. II we present how crowd movement is modeled with a CA; in Sec. III we present the spatial evacuation game; in Sec. IV we couple the CA evacuation model with the game; in Sec. V we present simulation results with our model; Sec. VI is for conclusions.

II. CELLULAR AUTOMATON MODEL

A two-dimensional cellular automaton (CA) is a grid of cells, where each cell has a finite number of states. The state of every cell can be updated in discrete time steps, so that it simulates the movement of a crowd in a two-dimensional space. The update rules can be defined in many ways. Next, we present a well-known CA evacuation model [21] by Schadschneider *et al.* In Sec. IV, the spatial evacuation game will be coupled to this model.

A. Movement in the CA

In the CA, the agents are located in a discrete square grid, so that a single agent occupies a single cell. The agents move towards the exit according to some transition probabilities. A cell is assumed to be 40×40 cm. The maximal possible moving velocity for an agent is one cell per time step, i.e., 40 cm per time step. Empirically the average velocity of a pedestrian is about 1.3 m/s [28]. Thus, a time step in the model corresponds to 0.3 s.

At each time step of the simulation, the agent can move to one of the unoccupied cells in his *von Neumann neighborhood*, i.e., the nearest cells in the orthogonal directions. Diagonal movement is not allowed, since moving diagonally in a square grid would account for a higher velocity than moving orthogonally.

The transition probabilities depend on the values of the static and dynamic floor field in the cells. The static floor field (SF) is based on the geometry of the room. The values associated with the cells of SF increase as we move closer to the exit, and decrease as we move closer to the walls. The dynamic floor field (DF), in turn, represents a *virtual trace* left by the agents. An agent leaving a cell, causes the value of DF in that cell to increase by one unit. Over time, the virtual trace decays and diffuses to surrounding cells. The values of the fields DF and SF are weighted with two *coupling parameters* $k_{DF} \in [0, \infty)$ and $k_{SF} \in [0, \infty)$.

Now, for each agent, the transition probabilities p_{mn} , for a move to a neighbor cell (m, n) are calculated as follows:

$$p_{mn} = \frac{1}{Z} e^{k_{DF} DF_{mn}} e^{k_{SF} SF_{mn}} (1 - \xi_{mn}), \quad (1)$$

where

$$\xi_{mn} = \begin{cases} 1 & \text{for forbidden cells (walls and occupied cells)} \\ 0 & \text{else} \end{cases},$$

FIG. 1. The impact of friction parameter on the agents' movement. With probability μ neither of the agents gets to move, and with probability $1 - \mu$ one of the agents moves.

and the normalization,

$$Z = \sum_{(m,n)} e^{k_{DF} DF_{mn}} e^{k_{SF} SF_{mn}} (1 - \xi_{mn}).$$

The agents' desired movement directions are updated with a *parallel update scheme*, i.e., the directions are updated simultaneously for all agents. The parallel update of movement directions can result in *conflict situations*, i.e., situations where several agents try to occupy the same cell simultaneously. In these situations, all the agents are assigned equal probabilities to move, and with probability $1 - \mu$ one of the agents is allowed to move to the desired cell. Here, $\mu \in [0,1]$ is a so called *friction parameter*. In [21], a conflict situation was interpreted to represent the effect of pressure in the crowd. The impact of the friction parameter is depicted in Fig. 1.

B. Different crowd behaviors

It has been shown that by altering the coupling parameters k_{SF} and k_{DF} different crowd behaviors can be observed [20,21]. The different crowd behaviors were called ordered, disordered, and cooperative in [20,21]. In Fig. 2, the coupling parameter combinations responsible for different behavioral regimes are plotted in a schematic phase diagram.

Before analyzing the behavior of agents in the different regimes, let us consider the dynamic floor field DF. Because the value of DF in a cell at a specific time is proportional to how often agents have been able to leave that cell recently, it can be thought to describe temporal crowd flow in different parts of the grid. For cells where agents get stuck due to conflict situations, the value of the DF is small.

Now we can give an interpretation to the different behavioral regimes. In the *ordered regime*, an agent's transition probabilities mainly depend on SF, i.e., the agent moves straight towards the exit. In the *disordered regime*, an agent's transition probabilities mainly depend on DF, i.e., the agent moves to cells of locally higher crowd flow. Between the ordered and disordered regime is the *cooperative regime*

FIG. 2. Altering the coupling parameters k_{SF} and k_{DF} , in the CA model, produces different crowd behaviors.

around the values $k_{DF} = k_{SF} = 1$. In the cooperative regime, an agent's transition probabilities depend as much on SF as DF. That is, the agent moves towards the exit, using cells of locally higher crowd flow. Typical movement patterns of a single agent of different behavioral regimes are illustrated in Fig. 3. In the figure, the agent is part of the crowd, i.e., it is not moving independently in the space. For the sake of clarity, the movement patterns of the rest of the crowd is not depicted.

In Fig. 3, we can see that if the agent is in the ordered regime, it moves straight towards the exit. In the cooperative regime, the agent moves towards the exit, but using a longer route with a higher average crowd flow. In the disordered regime, the agent just blindly moves in the grid to regions of locally higher crowd flow.

FIG. 3. Typical movement patterns of an agent in the different behavioral regimes.

C. Collective effects in evacuations

With the right parameter choices the CA model is able to generate many of the collective effects in evacuations, e.g., the faster-is-slower effect and clogging at bottlenecks [21].

The faster-is-slower effect, where the attempt of the individuals to evacuate faster results in a slower evacuation for the whole crowd, is a typical phenomenon observed in evacuation situations. In the CA, if an agent would be able to move towards the exit without getting into conflicts, ordered behavior would make the agent evacuate fastest. However, in [21], it was shown that if μ is sufficiently large, a crowd of ordered agents evacuate slowest, and a crowd of cooperative agents fastest.

The reason is that ordered agents often try to move simultaneously to the same cells, which causes conflicts that slow down the evacuation. In the cooperative regime, all the agents move to exit using cells of higher crowd flow. Still, there will not be as many conflicts as in the ordered regime. If too many agents get into conflicts in a cell of higher crowd flow, it ceases to be a cell of higher crowd flow and is a less desirable movement direction. Thus, conflict situations are not an unwanted result of the parallel update scheme, but an important feature to be able to describe the dynamics of an evacuation properly.

A dense crowd trying to move through a bottleneck can cause a clogging. A typical situation where a clogging is formed is when many people try to leave a room at the same time. The crowd will self-organize in a half-circle-like formation in front of the bottleneck; this slows down the movement of the crowd or completely hinders it [5].

In CA, a half-circle-like formation in front of a bottleneck builds up quite fast, without it actually having to slow down the evacuation. Thus, we can't speak of a clogging until the density of the crowd is such that more than about every third cell in the half-circle-like formation is occupied. Then, the number of conflicts drastically increases. An example of a clogging at an exit in the CA model is shown in Fig. 4. In the figure, the gray squares represent agents moving towards the exit.

FIG. 4. A clogging formed at an exit. The gray squares represent agents moving towards the exit.

III. SPATIAL EVACUATION GAME

Next, we present the spatial evacuation game defined in [18]. In the game, n_a agents, indexed by i , $i \in I = \{1, \dots, n_a\}$, are in an evacuation situation. Each agent has an *estimated evacuation time* T_i , which depends on the number λ_i of agents closer to the exit than him, and on the capacity of exit β . T_i is defined as

$$T_i = \frac{\lambda_i}{\beta}. \quad (2)$$

Each agent has a *cost function* that describes the risk of not being able to evacuate before the conditions become intolerable. The cost function depends on T_i .

The game is played for the remaining time to evacuate. The agents interact with their nearest neighbors. Each agent has two strategies to choose from: *Patient* and *Impatient*. It is assumed that in an actual play of the game, these strategies correspond to patient and impatient behaviors of the agents, respectively. Let us denote the average of the evacuation times of agents i and j , $T_{ij} = (T_i + T_j)/2$. When agents i and j interact with each other, we assume that the agents expect the following outcomes:

(1) In an impatient vs patient agent contest, an impatient agent i can overtake his patient neighbor j . This reduces agent i 's evacuation time by ΔT and increases j 's evacuation time by the same amount. The cost of i is reduced by $\Delta u(T_{ij})$ and increased for j by the same amount. Here

$$\Delta u(T_{ij}) = u(T_{ij}) - u(T_{ij} - \Delta T) \simeq u'(T_{ij})\Delta T. \quad (3)$$

(2) In a patient vs patient agent contest, the patient agents do not compete with each other, they keep their positions and their costs do not change.

(3) In an impatient vs impatient agent contest, neither agent can overtake the other, but they will face a conflict and have an equal chance of getting injured. The risk of injury is described by a cost $C > 0$, which affects both agents. The constant C is called the *cost of conflict*.

The above-mentioned conflict, should not be confused with a conflict situation in the CA, which is a mere consequence of the parallel update scheme [21].

From the aforementioned assumptions, a 2×2 game matrix is constructed in Table I.

In the game matrix in Table I, agent i is a row player and agent j a column player. When a particular pair of strategies is chosen, the costs for the two agents are given in the corresponding cell of the matrix. The cost to agent i is the first number in a cell, followed by the cost to agent j . Because this is a cost matrix, the agents want to minimize their outcome in the game. Now, each number in a game matrix can be multiplied by a number > 0 without affecting the equilibria of the game. So, divide every number in our matrix by $\Delta u(T_{ij})$.

TABLE I. The game matrix for the spatial evacuation game.

	Impatient	Patient
Impatient	C, C	$-\Delta u(T_{ij}), \Delta u(T_{ij})$
Patient	$\Delta u(T_{ij}), -\Delta u(T_{ij})$	$0, 0$

Then the game matrix only depends on the parameter $C/\Delta u(T_{ij})$. When $0 < C/\Delta u(T_{ij}) \leq 1$, the game played is PD, and the only Nash equilibrium is (Impatient, Impatient). If $C/\Delta u(T_{ij}) > 1$, the game played is HD, and there are two pure strategy Nash equilibria (Impatient, Patient) and (Patient, Impatient). There is also a mixed strategy equilibrium, where the strategy Impatient is played with probability $\Delta u(T_{ij})/C$, and the strategy Patient with probability $1 - \Delta u(T_{ij})/C$. For general comments on 2×2 games of this form see the appendix in [18].

It should be emphasized that the outcomes of the above game are what the agents think will happen in different encounters. The agents' decision-making is based on these expectations. In Sec. IV, we couple the simple decision-making model with the CA model presented in Sec. II. As the crowd is a large complex system, we are not able to capture the full complexity of the system with the simplified assumptions behind the game and CA model. Hence, when coupling the game strategies with the CA model, the actual outcome of an interaction is not necessarily exactly what the agents expected when selecting their strategies.

A. The cost function

The cost function depends on the parameter T_{ASET} , *available safe egress time*, which is a term extensively used in the literature [29]. It describes the time, in which the conditions in a building on fire become intolerable. Additionally, the cost function depends on a parameter T_0 . It describes how short the time difference between T_{ASET} and T_{ij} has to be before agents i and j start playing the game with each other. If $T_{\text{ASET}} - T_{ij} > T_0$, or, equivalently $T_{ij} < T_{\text{ASET}} - T_0$, agents i and j do not play the game with each other. In order to make calculations with the model, some additional assumptions about the cost function are made.

(1) For $T_{ij} < T_{\text{ASET}} - T_0$ the game is not played. Then $u(T_{ij}) \equiv 0$.

(2) For $T_{ij} \geq T_{\text{ASET}} - T_0$ the game is played. The cost function starts to increase quadratically.

(3) $u'(T_{\text{ASET}}) = C$.

A quadratic cost function that fulfills these conditions is

$$u(T_{ij}) = \begin{cases} 0, & \text{if } T_{ij} < T_{\text{ASET}} - T_0, \\ \frac{C}{2T_0}(T_{ij} - T_{\text{ASET}} + T_0)^2, & \text{if } T_{ij} \geq T_{\text{ASET}} - T_0. \end{cases} \quad (4)$$

The derivative of the cost function is

$$u'(T_{ij}) = \begin{cases} 0, & \text{if } T_{ij} < T_{\text{ASET}} - T_0, \\ \frac{C}{T_0}(T_{ij} - T_{\text{ASET}} + T_0), & \text{if } T_{ij} \geq T_{\text{ASET}} - T_0. \end{cases} \quad (5)$$

Two different cost functions meeting these assumptions are depicted in Fig. 5. The function illustrated by the gray dashed curve starts to grow immediately at $T_{ij} = 0$, because for it $T_{\text{ASET}} - T_0 = 0$. The function illustrated by the black curve, on the other hand, starts to grow when $T_{ij} = 40$, because for it $T_{\text{ASET}} - T_0 = 40$.

FIG. 5. Two different cost functions. The gray dashed curve has the following parameter values: $T_{\text{ASET}} = 60$, $T_0 = 60$, $C = 3$. The black curve has the following parameter values: $T_{\text{ASET}} = 60$, $T_0 = 20$, $C = 3$.

The cost of overtaking can be expressed as

$$\Delta u(T_{ij}) \simeq u'(T_{ij})\Delta T = \frac{C}{T_0}(T_{ij} - T_{\text{ASET}} + T_0)\Delta T. \quad (6)$$

We will suppose that $\Delta T = 1$ s. Then the parameter $C/\Delta(T_{ij})$ appearing in the game matrix is

$$\frac{C}{\Delta u(T_{ij})} \simeq \frac{T_0}{T_{ij} - T_{\text{ASET}} + T_0}. \quad (7)$$

Note that whether the game played is PD or HD, depends only on the value of $T_0/(T_{ij} - T_{\text{ASET}} + T_0)$. Thus the game only depends on the estimated evacuation time T_{ij} , since T_0 and T_{ASET} are constants. When T_{ij} increases, the game turns from HD to PD.

B. Spatial setting and update of strategies

We assume that the agents are located in a discrete square grid. At this point, we do not assume the agents to move. Let N_i be the set of agents in the *Moore neighborhood* of agent $i \in I$, i.e., the agents surrounding agent i immediately in the orthogonal and diagonal directions. Note that when we couple the game model to the CA, the agents in the Moore neighborhood of agent i will change as agent i moves in the square grid.

We assume that in period $t, t \geq 1$, agent i observes the strategies of its neighbors, and updates its strategy based on its best-response function BR_i . The best-response strategy $s_i^{(t)}$ of agent i in period t is defined by

$$s_i^{(t)} = \text{BR}_i(s_{-i}^{(t-1)}; T_i, T_{-i}) = \arg \min_{s'_i \in S} \sum_{j \in N_i} v_i(s'_i, s_j^{(t-1)}; T_{ij}). \quad (8)$$

Here, the function $v_i(s'_i, s_j^{(t-1)}; T_{ij})$ gives the cost defined by the game matrix to agent i , when he plays strategy s'_i , and agent j has played strategy $s_j^{(t-1)}$ on period $(t-1)$. The strategy set $S = \{\text{Patient}, \text{Impatient}\}$. The notation $s_{-i}^{(t-1)}$ is used to denote the strategies of all other agents than agent i at period $t-1$, and T_{-i} includes the estimated evacuation times of these agents. The initial configuration of strategies is $s_i^{(0)} = s_i^0, \forall i \in I$.

The *best-response strategy* (8) is a strategy that minimizes the total cost of an agent against the strategies played by the neighboring agents in the previous period. Thus, the agents are *myopic* in the sense that they choose their strategies based on the previous period play of their opponents, not considering the play further back in history, or of future periods.

For numerical simulations it is convenient to assume that in each period only one agent updates its strategy. To obtain convergence we further assume that each agent in the crowd I updates its strategy several times. Technically, a suitable iteration scheme is the *shuffle update scheme*, where after every n_a period each agent has updated its strategy exactly once, in random order [30].

C. Equilibrium configuration in a square grid

Numerical simulations show that regardless of the number of agents, the spatial equilibrium is found, when all the agents have updated their strategies less than 10 times, or so, with the shuffle update scheme.

As stated earlier, the game turns from a HD to PD as we move farther away from the exit. It can be shown, that in the region, where the agents play HD, we observe subregions with different proportions of impatient agents.

Let the set of impatient agents in the Moore neighborhood of agent i be N_i^{Imp} . Moreover, let the number of agents in the Moore neighborhood of agent i be $|N_i|$ and the number of impatient agents $|N_i^{\text{Imp}}|$, then the number of patient agents is $|N_i| - |N_i^{\text{Imp}}|$.

Recall the costs associated with interactions of different strategists. Now, agent i should play Impatient, if the cost of playing Impatient against his neighbors is less than or equal to the cost of playing Patient:

$$\sum_{j \in N_i^{\text{Imp}}} \frac{T_0}{T_{ij} - T_{\text{ASET}} + T_0} + (|N_i| - |N_i^{\text{Imp}}|)(-1) \leq |N_i^{\text{Imp}}|.$$

To make the analysis simpler, let us approximate $T_i = T_{j,j \in N_i^{\text{Imp}}}$. Then the above inequality can be written,

$$\frac{|N_i^{\text{Imp}}|}{|N_i|} \leq \frac{T_i - T_{\text{ASET}} + T_0}{T_0}. \quad (9)$$

Here, $|N_i^{\text{Imp}}|/|N_i|$ is the proportion of impatient agents in the Moore neighborhood of agent i . Thus, Eq. (9) gives an upper limit for the proportion of impatient agents in the Moore neighborhood of an impatient agent i at a given distance from the exit. If this limit is exceeded, agent i should switch strategy to Patient. We can see that this limit increases as we move farther away from the exit.

In an equilibrium configuration, all agents choose their strategy based on Eq. (9). Thus, the proportion of impatient agents grows as we move farther away from the exit. We will actually notice eight subregions, with different proportions of impatient agents. This is due to the game being played in the Moore neighborhood, which implies exactly eight possible proportions. This can be seen in Fig. 6, which illustrates an equilibrium configuration of the game. Along

FIG. 6. An equilibrium configuration for 3180 agents with parameter values $T_{\text{ASET}} = 2200$ and $T_0 = 2200$. Black squares represent impatient agents and gray squares patient agents.

the semicircles A, B, C , the number of impatient agents in the Moore neighborhood of agents are at most 1, 2, 4, respectively.

More such simulations, with different patient and impatient agent proportions, can be found in [18]. Similar results were obtained in [31] for a square lattice of periodic boundary conditions. There, contrary to our model, the parameters in the HD game did not depend on the agents' locations.

IV. CELLULAR AUTOMATON EVACUATION MODEL COUPLED WITH A SPATIAL GAME

Recall Sec. II; if k_{SF} is high, and k_{DF} low, an individual agent is able to evacuate fast, but the evacuation time of the whole crowd using these coupling parameters is slow. On the other hand, if $k_{\text{DF}} \approx k_{\text{SF}} \approx 1$, an individual agent evacuates slowly, but the evacuation time for the whole crowd is fast [21].

Next, we are going to couple the CA with the spatial game from Sec. III. In the resulting model, an agent's coupling parameters k_{SF} and k_{DF} are a realization of the agent's strategy choice in the spatial game. As far as we know, there are no earlier studies where agents can have different values for k_{SF} and k_{DF} , or that they would change during the simulation.

As discussed in Sec. II, in the CA, the agents move in a discrete grid in discrete time steps. A time step in the CA is equal to 0.3 s. At the beginning of every time step, in the CA, before the agents move, the averaged estimated evacuation times T_{ij} for the neighboring agents $i, j \in I$ are calculated. After that, before moving in the grid, the agents observe the strategies of their neighboring agents and react by updating to their best-response strategy. The agents' best-response strategies are updated until an equilibrium is reached. Thus, the crowd is in an equilibrium configuration always when moving. Note that the equilibrium configuration may slowly change in form over time, since the game parameter T_{ij} changes for the agents.

We let the agent's strategy choice alter its behavior in the CA: playing Impatient results in faster movement for the agent, than playing Patient. Thus, for an agent playing Impatient, the coupling parameters are set to $k_{\text{SF}} = 10$, $k_{\text{DF}} = 1$, and for an agent playing Patient $k_{\text{SF}} = 1$, $k_{\text{DF}} = 1$.

So, in our model, k_{SF} and k_{DF} change for agent i based on the game parameter T_{ij} and neighboring agents' strategies. Again, it should be emphasized that T_{ij} describes what agent i

expects its averaged estimated evacuation time with neighbor j to be, and has nothing to do with the elapsed time in the CA.

After k_{SF} and k_{DF} have been updated for the agents, the agents' positions are updated in parallel according to the transition probabilities determined by Eq. (1). In a case of a conflict, one of the agents is allowed to move with probability $1 - \mu$,

For technical reasons, the agents' positions are updated with parallel update scheme. If the positions were not updated in parallel, no conflicts would occur. These conflicts are a crucial feature to achieve clogging and the faster-is-slower effect with our model. On the other hand, the shuffle update used for updating agents' strategies is not only technically suitable, but is a fair description of how agents really update their strategies, since, in many real social systems, agents do not update their strategies simultaneously. Alternatively, the agents could be set to update their strategies according to a Poisson process.

A. Model description

Next, a recap of the model is given in a form of a step-by-step description. In the beginning of the simulation, the agents are located randomly in the room, and are all considered patient.

Before the simulation, we set a specific value for the available safe egress time T_{ASET} , however, T_{ASET} can change with time. The linear change in the value of T_{ASET} in 1s of the simulation is described by the parameter ΔT_{ASET} . So, ΔT_{ASET} describes the rate at which the evacuation conditions get worse or improve.

Step 1. At the beginning of each time step, the game parameters $T_{ij} = (T_i + T_j)/2, i \neq j, \forall i, j \in I$ are calculated according to Eq. (2). If $T_{ij} < T_{ASET} - T_0, \forall j \in N_i$, agent i does not play the game and behaves patiently.

Step 2. The agents' strategies are updated with the shuffle update scheme until an equilibrium is reached. The agents observe the strategies of the other agents in their Moore neighborhood, and choose a best-response strategy according to Eq. (8).

Step 3. The agents' behavior in the CA model is set to correspond their strategy choice. This is done by altering the agents' coupling parameters as follows:

- (a) For an agent playing Impatient, the coupling parameters are set to $k_{DF} = 1.0$ and $k_{SF} = 10.0$.
- (b) For an agent playing Patient, the coupling parameters are set to $k_{DF} = 1.0$ and $k_{SF} = 1.0$.

Step 4. The agents' positions are updated in parallel according to Eq. (1). In case of a conflict, one of the agents is allowed to move with probability $1 - \mu$.

Step 5. The available safe egress time is updated: $T_{ASET} = T_{ASET} + \Delta T_{ASET} \cdot 0.3$.

Step 6. Go to Step 1. The procedure is repeated until all agents have evacuated the room.

V. EVACUATION SIMULATIONS

We have presented an evacuation model, where the agents' behavior in the CA is a result of the spatial game they play. Preliminary results with the model were obtained in [32]. In the

following, we will extend these simulations for a few typical evacuation situations.

Additionally, the performance of the strategies in an evacuation is investigated. We show that playing the strategy Impatient results in fastest evacuation for an individual, but if the whole crowd adopts it, it will result in slowest possible evacuation for the crowd. This is the well-known faster-is-slower effect, which was also found in the original formulation of the spatial game [18]. In the CA model discussed in Sec. II, the phenomenon is achieved by pre-simulation setting specific model parameters. It is remarkable that here we are able to obtain the phenomenon as a result of the simple game the agents play.

A sufficiently large friction parameter $\mu = 0.6$ is used in all the simulations. Later in this section, we analyze the influence of μ and k_{SF} and k_{DF} on the faster-is-slower effect.

A. Simulation of typical situations

When the remaining time to evacuate is scarce, people start to behave impatiently. This especially holds for people located farther away from the exit. It is striking that our model describes this feature of human beings. It is clearly seen in Fig. 7; recall also the explanation in Sec. III C.

In Fig. 7, there are three snapshots from different stages of an evacuation from a large room. The black squares represent impatient agents and gray squares patient. As can be seen, the agents organize in a half-circle-like formation in front of

FIG. 7. Different stages of an evacuation, with 1000 agents, from a large room. Here $T_0 = T_{ASET} = 100$ and $\Delta T_{ASET} = 0$. Black squares represent impatient agents and gray squares patient agents. (a) Early stages. (b) Middle stages. (c) Late stages.

FIG. 8. Different stages of an evacuation, with 1000 agents, from a large room. Here $T_0 = T_{\text{ASET}} = 100$ and $\Delta T_{\text{ASET}} = -2$. Black squares represent impatient agents and gray squares patient agents. (a) Early stages. (b) Middle stages. (c) Late stages.

the exit already in the early stages of the evacuation. Even though the agents are moving, the whole crowd is in a spatial equilibrium in every snapshot. This is a consequence of fast updating of agents' best-response strategies compared to the time step of their move. This can also be seen in Figs. 8 and 9.

In Fig. 8 the agents evacuate from the same room as in Fig. 7. In comparison to Fig. 7, here $\Delta T_{\text{ASET}} = -2$, which means that the situation gets worse over time. This is typical for a fire evacuation. Due to the worsening conditions, more agents feel pressured to evacuate, resulting in a higher proportion of impatient agents in the crowd.

Recall that impatient agents move straight towards the exit, and thus try to occupy same cells simultaneously, which causes conflicts in the CA. These conflicts slow down the evacuation. Patient agents, on the other hand, move to regions of higher local crowd flow, and thus have a tendency to avoid conflicts. Thus, the evacuation is slower in Fig. 8 than in Fig. 7. This can be seen by comparing the amount of agents in the late stages of the evacuation in both figures. Later in this section, a more in-depth analysis of the subject is given.

In Fig. 9, there is a crowd moving from a room to another through a narrow hallway. The exit is located in the second room. In the beginning, the agents are located in the first room. Already in the early stages of the evacuation, some of the agents have reached the exit, and the hallway is filled with agents. Because, the hallway is narrow conflicts are likely to happen. After the early stages, the amount of conflicts in the

FIG. 9. Different stages of an evacuation, with 800 agents, through a hallway. Here $T_0 = T_{\text{ASET}} = 150$ and $\Delta T_{\text{ASET}} = 0$. Black squares represent impatient agents and gray squares patient agents. (a) Beginning of evacuation. (b) Early stages. (c) Middle stages.

hallway starts to increase. Thus, the flow in the hallway will start to decrease. This is because the proportion of impatient agents in the hallway is increasing.

B. Simulations with fixed strategies

To gain knowledge about the performance of different strategists, we will simulate an evacuation, where the strategies of the agents are held fixed, i.e., the agents do not update their strategies.

In the simulation, 50 impatient and 50 patient agents are randomly distributed in a $7.2 \text{ m} \times 7.2 \text{ m}$ room, with a 0.8 m -wide exit. The agents have 60 s to evacuate, and the simulation is run 40 times with different random initial locations. The results can be seen in Fig. 10. It shows the average number of patient (gray curve) and impatient (black curve) agents inside the room over time, and the difference in these numbers (dashed curve).

Point B and C in Fig. 10 represent the points in time when on the average all the impatient and patient agents, respectively, have evacuated the room. Because B occurs earlier than C, impatient agents are on the average able to evacuate faster than patient agents. Thus, the assumption behind the spatial game that impatient agents can overtake patient agents seems to approximately hold also for our model.

FIG. 10. The average number of patient and impatient agents inside the room over time, and the difference in these numbers.

Impatient agents are able to evacuate faster, because the majority of them are able to rush to the front of the exit before a clogging is formed. Naturally, the agents in front of the exit are also the first ones to evacuate. After they have evacuated, we are left with a crowd in front of the exit, where the proportion of impatient agents is small. The remaining few impatient agents are able to overtake patient agents, but they now have a smaller advantage than before, because the crowd is so dense that there is not much room to move. In Fig. 10, this can be seen by looking at the difference in the average numbers of patient and impatient agents in the room. The difference increases until point A, after which it starts to decrease. Point A represents the time when the group of impatient agents that has first arrived in front of the exit has managed to evacuate.

So, on average an individual impatient agent is able to evacuate faster than a patient agent. Then what effect has the proportion of impatient agents on the evacuation efficiency of the whole crowd? Next, we study how the proportion of impatient agents affects the average egress flow of the crowd.

In the simulation, 200 agents are set in a half-circle in front of the exit, and update their strategies until equilibrium is reached. Immediately afterwards, their strategies are fixed, the exit is opened, and the agents start to evacuate. The simulation is performed 100 times for 11 different values of T_{ASET} , to get results for 11 different proportions of impatient agents. The results of the simulations can be seen in Fig. 11.

In Fig. 11 it is clearly seen that the more agents behave impatiently, the smaller the egress flow is. This is because the more there are impatient agents, the more agents move straight towards to the exit, resulting in an increased amount of conflict situations in the CA. These conflicts decrease the egress flow.

We have shown that even though an impatient agent evacuates faster than a patient agent, the whole crowd evacuates slower the larger the proportion of impatient agents in the crowd is. In other words, the attempt of an individual agent to evacuate faster results in a slower evacuation for the whole crowd. This astounding result shows that in our model the faster-is-slower effect is a result of the two-strategy game the agents play in their nearest surrounding.

FIG. 11. Average egress flow of a crowd, with 200 agents, for different proportions of impatient agents. In the simulations, 11 different values of T_{ASET} were used. $\Delta T_{\text{ASET}} = 0$ in each simulation.

C. Simulations with adaptive strategies

Next, we let the agents update their strategies throughout the simulation. We study how the number of evacuated agents and proportion of impatient agents vary in different scenarios. The scenarios are listed in Table II.

Each scenario in Table II is simulated 50 times. During the simulations, 200 agents are set to evacuate from a room. In Fig. 12 the proportion of impatient agents over time for Scenarios 1–5 are shown.

In Scenarios 1–3 of Table II, the value of T_{ASET} decreases over time. It can be seen that this results in more agents behaving impatiently over time. On the other hand, in Scenarios 4–5 of Table II, the value of T_{ASET} increases over time, which improves the evacuation conditions for the agents. Thus, over time fewer agents behave impatiently. At a point in time, they even stop playing the game, and behave all patiently, because their estimated evacuation times are larger than $T_{\text{ASET}} - T_0$.

Also, the cumulative exit counts were monitored in the simulations; the results are shown in Fig. 13.

It can be seen that the better the conditions are, the more agents are able to evacuate. The cumulative exit count curve grows almost linearly over time in all scenarios. Actually, for Scenarios 1–3, there is a slight decrease in the slope of the curve over time because as the proportion of impatient agents grows, the egress flow decreases. And, in Scenarios 4–5, there is a slight increase in the slope of the curve over time. This is because, as the proportion of patient agents grows, the egress flow increases.

TABLE II. The initial values of T_{ASET} , T_0 , and the linear change of T_{ASET} in 1 s of the simulation, i.e., ΔT_{ASET} , for Scenarios 1–5.

Scenario	T_{ASET}	T_0	ΔT_{ASET}
Scenario 1	200	100	−2
Scenario 2	100	100	−2
Scenario 3	100	100	−0.5
Scenario 4	10	100	+4
Scenario 5	10	100	+1

FIG. 12. Proportion of impatient agents over time for Scenarios 1–5.

The same simulations have been run with FDS+EVAC in [18]. There, the cumulative exit count curves differ quantitatively. In [18] the slopes of the curves increase or decrease, depending on ΔT_{ASET} , more over time.

As has been mentioned before, in [18], impatient agents actually push the crowd in front of them. The driving force of the impatient agents in the back of the crowd, and the frictional interactions between the agents, result in archlike blocks in front of the exit, which form and break down [5].

In our model, an increase in the proportion of impatient agents increases the amount of conflicts. These conflicts slow down the evacuation, but not as drastically as the archlike blocks in FDS+EVAC. The reason is that an increase in impatient

agents only increases conflicts in the locations where the impatient agents are, not necessarily in front of the exit. Whereas in [18], an increase in impatient agents adds up to the driving force of the back of the crowd. Thus, an increase in impatient agents in [18] has a more severe effect on the egress flow in FDS+EVAC, and because of that the slope of the cumulative exit count curve will decrease more.

Additionally, in FDS+EVAC, the agents can make a use of their body ellipse, and thus two persons can move through an exit simultaneously [8]. This makes an evacuation for a crowd with a high proportion of patient agents more efficient, and thus the slope of the cumulative exit count curve will increase more in FDS+EVAC.

D. Influence of μ and different strategists' k_{SF} and k_{DF} on the faster-is-slower effect

So far, we have only run simulations with the parameter values: $\mu = 0.6$, and $k_{\text{SF}} = 10, k_{\text{DF}} = 1$ for impatient agents and $k_{\text{SF}} = 1, k_{\text{DF}} = 1$ for patient agents. In Sec. VB, it was seen that using these parameter values the faster-is-slower effect is a result of the game the agents play. Next, we investigate whether there are other suitable parameter values.

First, we investigate the influence of μ on the faster-is-slower effect. We have run the same simulations as in Fig. 10 with different values of μ . For each value of μ , we have calculated the average evacuation time for a patient and an impatient agent, $\bar{t}_{\text{Pat}}$ and $\bar{t}_{\text{Imp}}$, respectively. The difference $\bar{t}_{\text{Pat}} - \bar{t}_{\text{Imp}}$ for given μ is shown in Table III.

For all values of μ in Table III, $\bar{t}_{\text{Pat}} - \bar{t}_{\text{Imp}}$ is positive, i.e., impatient agents are on average able to overtake patient agents and evacuate faster.

It seems that the advantage impatient agents have over patient agents grows as μ increases, because $\bar{t}_{\text{Pat}} - \bar{t}_{\text{Imp}}$ is larger for larger values of μ . However, in Table III, it is not shown that also $\bar{t}_{\text{Imp}}$ is larger for larger values of μ . So, the advantage impatient agents have over patient agents, in relation to their own evacuation time, in fact, decreases as μ increases.

In addition to impatient agents being able to overtake patient agents, for the faster-is-slower effect to take place, the egress flow needs to decrease, as the proportion of impatient agents increases. To study how the egress flow depends on the proportion of impatient agents, we have run the same simulations as in Fig. 11 for different values of μ . The results are shown in Fig. 14. Notice that the vertical axis scale is larger in Fig. 14 than in Fig. 11.

For $\mu = 0.6$ and $\mu = 0.9$, the faster-is-slower effect takes place, because the average egress flow decreases as the proportion of impatient agents increases. Whereas, for $\mu = 0$

FIG. 13. Cumulative exit count over time for Scenarios 1–5.

TABLE III. Difference in average evacuation time for a patient and an impatient agent, i.e., $\bar{t}_{\text{Pat}} - \bar{t}_{\text{Imp}}$ for given μ .

μ	Impatient ($k_{\text{SF}}/k_{\text{DF}}$)	Patient ($k_{\text{SF}}/k_{\text{DF}}$)	$\bar{t}_{\text{Pat}} - \bar{t}_{\text{Imp}}$
0	10/1	1/1	11.8918
0.3	10/1	1/1	12.1972
0.6	10/1	1/1	12.5511
0.9	10/1	1/1	13.7004

FIG. 14. Average egress flow of a crowd for different proportions of impatient agents. The parameter μ is altered. In all cases $k_{SF} = 10, k_{DF} = 1$ for impatient agents and $k_{SF} = 1, k_{DF} = 1$ for patient agents.

and $\mu = 0.3$, the attempt of agents to move faster results in a faster evacuation for the whole crowd.

Recall Fig. 2; $k_{SF} = 10, k_{DF} = 1$, used here for impatient agents, is in the ordered regime, and $k_{SF} = k_{DF} = 1$, used here for patient agents, in the cooperative regime. If μ is sufficiently large, a crowd of ordered agents evacuates slower than a crowd of cooperative agents [21]. Inversely, if μ is low enough, a crowd of ordered agents will evacuate faster than a crowd of cooperative agents.

Thus, there should exist a limit value for μ , at which the faster-is-faster effect turns into faster-is-slower effect. Based on Fig. 14, it is between 0.3 and 0.6. This value should vary with crowd density, and k_{SF} and k_{DF} for impatient and patient agents.

In Fig. 14, it can also be seen that for larger values of μ , the faster-is-slower effect becomes more pronounced. However, if $\mu \rightarrow 1$, the egress flow should approach 0, for all proportions of impatient agents, because the probability for an agent to move in a conflict would approach 0.

Next, we investigate the effect impatient agents' k_{SF} and k_{DF} have on the faster-is-slower effect. We have run the same simulations as in Fig. 10 with different values of impatient agents' k_{SF} and k_{DF} . The difference $\bar{t}_{Pat} - \bar{t}_{Imp}$ for given parameter values is shown in Table IV.

Again, because $\bar{t}_{Pat} - \bar{t}_{Imp}$ is positive in all the cases, impatient agents are able on average to overtake patient agents and evacuate faster.

Next, we investigate how the egress flow depends on the proportion of impatient agents for different values of impatient agents' k_{SF} and k_{DF} . Same simulations were run as in Fig. 11 for

TABLE IV. Difference in average evacuation time for a patient and an impatient agent, i.e., $\bar{t}_{Pat} - \bar{t}_{Imp}$, for given parameter values.

μ	Impatient (k_{SF}/k_{DF})	Patient (k_{SF}/k_{DF})	$\bar{t}_{Pat} - \bar{t}_{Imp}$
0.6	10/4	1/1	11.8356
0.6	10/0	1/1	13.5004
0.6	3/1	1/1	11.0358

FIG. 15. Average egress flow of a crowd for different proportions of impatient agents. k_{SF} and k_{DF} are altered for impatient agents. In all cases, $\mu = 0.6$, and $k_{SF} = 1, k_{DF} = 1$ for patient agents.

different values of impatient agents' k_{SF} and k_{DF} . The results are shown in Fig. 15.

For all the parameter combinations in Fig. 15, an increase in the proportion of impatient agents decreases the egress flow, i.e., the faster-is-slower effect takes place. From Fig. 15, we see that k_{SF} can be set as low as 3 for impatient agents. On the other hand, there is no upper limit for k_{SF} for impatient agents, because as $k_{SF} \rightarrow \infty$, the agent walks as straight forward to the exit as possible, which makes the agent evacuate as fast as possible and cause conflicts as much as possible [20].

If k_{SF} is fixed for an agent, increasing k_{DF} makes the agent evacuate slower, and cause less conflicts [20,21]. Thus, there should be an upper limit for the impatient agents' k_{DF} for a fixed k_{SF} . In Fig. 15, it can be seen that if $k_{SF} = 10$, k_{DF} can be set at least up to 4.

If k_{SF} is fixed and large enough for an agent, decreasing k_{DF} makes the agent evacuate faster, and cause more conflicts [20]. Thus, the lower limit for k_{DF} for an impatient agent is 0. In Fig. 15, it can be seen that $k_{SF} = 10, k_{DF} = 0$ is suitable for an impatient agent.

Finally, we investigate the influence of patient agents' k_{SF} and k_{DF} on the faster-is-slower effect. We have run the same simulations as in Fig. 10 with different values of patient agents' k_{SF} and k_{DF} . The difference $\bar{t}_{Pat} - \bar{t}_{Imp}$ for given parameter values is shown in Table V.

Again, because $\bar{t}_{Pat} - \bar{t}_{Imp}$ is positive for all parameter combinations, impatient agents are on average able to overtake patient agents and evacuate faster. We still need to investigate if the egress flow decreases, as the proportion of impatient agents increases, for different values of patient agents' k_{SF} and k_{DF} .

TABLE V. Difference in average evacuation time for a patient and an impatient agent, i.e., $\bar{t}_{Pat} - \bar{t}_{Imp}$, for given parameter values.

μ	Impatient (k_{SF}/k_{DF})	Patient (k_{SF}/k_{DF})	$\bar{t}_{Pat} - \bar{t}_{Imp}$
0.6	10/1	1/0.5	11.1327
0.6	10/1	2/2	9.3429
0.6	10/1	10/10	6.9855

FIG. 16. Average egress flow of a crowd for different proportions of impatient agents. k_{SF} and k_{DF} are altered for patient agents. In all cases, $\mu = 0.6$ and $k_{SF} = 10, k_{DF} = 1$ for impatient agents.

In Fig. 16, there are shown the results of the same simulations as in Fig. 11 for the parameter combinations in Table V.

For all the parameter combinations, an increase in the proportion of impatient agents decreases the egress flow, i.e., the faster-is-slower effect takes place. Naturally, $k_{SF} = 1, k_{DF} = 0.5$ is suitable for patient agents, since it is in the cooperative regime of Fig. 2. However, it is surprising to see that $k_{SF} = k_{DF} = 2$ and $k_{SF} = k_{DF} = 10$ are suitable for patient agents.

Also, we have run simulations with $k_{DF} > k_{SF}$ for patient agents, in which results are not presented here, since $k_{DF} > k_{SF}$ resulted in patient agents performing a random walk in the room. Thus, for patient agents, k_{DF} should not be so high that the dynamic floor field DF dominates the agents' movement. It seems that there is an upper limit for patient agent's k_{DF} which is $k_{DF} \approx k_{SF}$, at least when $k_{SF} \leq 10$.

We omitted the investigation of large and really small values of k_{SF} and k_{DF} for both impatient and patient agents. The larger the coupling parameters are, the more amplified the differences in the values of the static SF and dynamic floor field DF become. Thus, the agent will either follow DF or SF, which means that the behavior either resembles a random walk or is suitable for impatient agents. On the other hand, if $k_{SF}, k_{DF} \rightarrow 0$ the agent's movement resembles a random walk [20].

We can conclude that our model is not defined merely for a narrow range of parameters, but there is room to alter both μ , and k_{SF} and k_{DF} for patient and impatient agents. For a patient agent, k_{SF} and k_{DF} needs to be chosen in such a way that a patient agent evacuates slower than an impatient agent, but causes less conflicts than an impatient agent. Thus, the coupling parameters for patient and impatient agents cannot be chosen independently. It seems to hold approximately, that

the coupling to DF in relation to the coupling to SF, i.e., k_{DF}/k_{SF} , needs to be larger for patient than impatient agents. Coincidentally, in [20] the ratio k_{DF}/k_{SF} was compared to the panic parameter introduced by Helbing *et al.* in [5].

VI. CONCLUSIONS

The aim in this paper was to develop a computationally light evacuation simulation model, which quantitatively answers the questions posed in [18], and in earlier studies, namely, why, when, and how collective effects of crowd dynamics occur.

For simulation of the movement of the crowd, we choose the computationally very light CA [21]. With the CA, different crowd behaviors can be produced by altering the coupling parameters k_{SF} and k_{DF} for the agents. In [21], the coupling parameters are adjusted before simulations. Thus, there all the agents behave in the same way, at all times and everywhere.

To equip the agents with decision-making abilities, we coupled the spatial evacuation game [18] to the CA. In the game, an agent observes the strategies of its neighboring agents and reacts by choosing its best-response strategy. The agents have two strategies to choose from: Patient and Impatient. An agent's best-response strategy depends, in addition to the neighboring agents' strategies, on the agent's estimated evacuation time T_{ij} and available safe egress time T_{ASET} .

The choice of strategy determines the agent's coupling parameters k_{SF} and k_{DF} . To our knowledge, there are no previous studies where agents in the CA have different values for the coupling parameters, or the coupling parameters would change for the agents during the simulation.

The agents update their best-response strategies so that they are in a one-shot spatial equilibrium configuration always when moving. Because the game parameter T_{ij} changes for the agents when they move, the equilibrium also changes as the crowd moves. In equilibrium, the proportion of impatient agents grows farther away from the exit. This reflects the fact that human beings farther away from the exit feel more pressured to evacuate and tend to behave more impatiently. Since the assumptions behind the model are fairly simple, and the best-response dynamics was found to converge quite fast, it is plausible that similar equilibrium patterns could exist in real-life situations.

In Sec. V, we showed that impatient agents are able to reach the exit faster than patient agents. However, the more agents were impatient, the slower the evacuation was for the whole crowd. This is because, the more there are impatient agents, the more agents move straight towards the exit, resulting in an increased amount of conflict situations in the CA. These conflicts slow down the evacuation. This remarkable result shows that when we couple the spatial game to the CA evacuation model, the faster-is-slower effect is found as a result of the Hawk-Dove game the agents play in their nearest surrounding.

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