Decoupling of first sound from second sound in dilute $^3$He–superfluid $^4$He mixtures

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Bulk superfluid helium supports two sound modes: first sound is an ordinary pressure wave, while second sound is a temperature wave, unique to superfluid systems. These sound modes do not usually exist independently, but rather variations in pressure are accompanied by variations in temperature, and vice versa. We studied the coupling between first and second sound in dilute $^3$He–superfluid $^4$He mixtures, between 1.6 and 2.2 K, at $^3$He concentrations ranging from 0% to 11%, under saturated vapor pressure, using a quartz tuning fork oscillator. Second sound coupled to first sound can create anomalies in the resonance response of the fork, which disappear only at very specific temperatures and concentrations, where two terms governing the coupling cancel each other, and second sound and first sound become decoupled.

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I. INTRODUCTION

There exist two possible sound modes in bulk superfluid helium: first sound is an ordinary pressure (or density) wave, whereas second sound is a temperature (or entropy) wave. Second sound is unique to superfluid systems, where temperature fluctuations can propagate as waves due to the existence of two independent velocity fields of normal fluid and superfluid component. In normal systems, all temperature fluctuations are so strongly damped that such a wave cannot exist. In terms of Tisza’s [1,2] and Landau’s [3,4] two-fluid model for superfluid helium, first sound is the mode where the normal fluid and superfluid component oscillate in phase, while in second sound they oscillate antiphase. Since $^3$He in dilute $^3$He–$^4$He mixtures is in normal state at all accessible temperatures, it contributes to the normal flow component, which gives another interpretation for second sound: $^3$He concentration wave.

First sound and second sound do not usually exist independently from each other, but rather pressure fluctuations of first sound are accompanied by second sound’s fluctuations in temperature, and vice versa. In pure $^4$He, the coupling is due to the thermal expansion of the liquid, even though it is extremely small. The addition of $^3$He modifies not only the superfluid transition temperature of $^4$He, but also the coupling between the two sound modes. In this paper we show that it is possible to find conditions where second sound and first sound become decoupled from each other, when thermal expansion contribution and $^3$He contribution to the coupling cancel each other.

Our studies were conducted using quartz tuning forks, which are commercially mass produced piezoelectric oscillators, whose intended frequency is usually around 32 kHz. They can be used to measure, for example, temperature, pressure, concentration, viscosity, and turbulence in liquid helium [5–8]. Velocity of second sound in superfluid helium is of order 10 m/s, and its characteristic wavelength, at the used frequency, matches the dimensions of common quartz tuning forks. Consequently, at certain temperatures, second sound is able to form standing waves in the fluid surrounding the fork, whereas first sound, with velocity of order 100 m/s, is usually not. When the sound modes are coupled, second sound can drive first sound, and the effect of this driven first sound can be seen as an anomaly in the resonance response of the fork.

These kind of anomalies in the quartz tuning fork response, or second sound resonances, have been observed before [9–11], but the detailed mechanism producing the anomalies has not been thoroughly investigated. Calculations of the coupling factors between first and second sound in helium mixtures have been presented by Brusov et al. [12], but, as first noted by Rysti [13], they made a sign error in their calculations, which washed out the decoupling behavior.

Before presenting the experimental setup and the results of our experiment, we first go briefly over the revised calculation of the coupling factors governing the conversion between first sound and second sound.

II. SOUND CONVERSION AND COUPLING FACTORS

Coupling between first sound and second sound in pure $^4$He is due to the thermal expansion of the liquid, which connects changes in temperature to changes in pressure and the other way around. Since the thermal expansion coefficient of superfluid $^4$He is extremely small, the coupling between the sound modes is very weak. Addition of the lighter isotope $^3$He modifies the coupling so that at low concentrations it becomes even weaker, eventually vanishing at specific temperatures and concentrations. As the concentration is further increased, the $^3$He contribution to the coupling starts to dominate the system and the coupling grows stronger.

We can derive expressions for sound conversions by starting from the linearized two-fluid hydrodynamical equations presented by Khalatnikov [14], from which we reach a set of equations characterizing the sound propagation in $^3$He–$^4$He mixtures:

$$\frac{\partial^2 \rho}{\partial t^2} = \nabla^2 P, \quad (1)$$
$$\frac{\rho_n \partial^2 \sigma}{\rho \partial t^2} = \sigma \nabla^2 T + c \nabla^2 \left( \frac{Z}{\rho} \right). \quad (2)$$

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Here $\rho, t, P, \sigma, T$, and $c$ are density, time, pressure, specific entropy, temperature, and $^3$He mass concentration, respectively, whereas $\rho_s (\rho_0)$ is normal fluid (superfluid) density. Furthermore, $Z \equiv \rho (\mu_3 - \mu_4)$, where $\mu_3$ and $\mu_4$ are the chemical potentials of $^3$He and $^4$He, respectively. Equation (1) is the first sound wave equation, Eq. (2) the second sound wave equation, and Eq. (3) is the result of the conservation of entropy and $^3$He “impurities.”

Next, we can choose $T, P$, and $c$ to be our independent variables, and consider small perturbations around their equilibrium values, so that $T = T_0 + \tilde{T}(r,t)$, where $T_0$ is the equilibrium value, and $\tilde{T}$ is the small deviation, and similarly for the other variables. We further assume that the perturbations are of plane-wave form $\propto \exp [i\omega (z - ct)]$, where $\omega$ is the angular frequency, $u$ is the velocity of the wave, and $z$ is the direction of propagation. When we next eliminate $c$ from Eqs. (1)–(3) we obtain a linear set of equations of form:

$$A_{00}(u_1^2)\tilde{T} + A_{01}(u_2^2)\tilde{P} = 0, \quad (4a)$$

$$A_{10}(u_3^2)\tilde{T} + A_{11}(u_4^2)\tilde{P} = 0, \quad (4b)$$

where Eq. (4a) is the linearized first sound wave equation, and Eq. (4b) is the linearized second sound wave equation. If we assume that the eigenvalues of this system are pure first sound ($u_1$) and pure second sound ($u_2$), we can consider the two equations above independently. In order to see how second sound creates first sound, we insert $u = u_1$ into the first sound wave equation, Eq. (4a), i.e., we use second sound as source for the first sound. This way, with appropriate simplifications, we get

$$\tilde{P} = -\frac{A_{00}(u_2^2)}{A_{01}(u_1^2)}\tilde{T}$$

$$= \left[ \frac{\partial \rho}{\partial T} \right]_{P,c} + c_0 \frac{\partial \rho}{\partial \sigma} \frac{\partial \sigma}{\partial T} \left( \frac{\partial \sigma}{\partial \rho} \right)_{T,P} \frac{u_1^2 u_2^2}{u_1^2 - u_2^2} \tilde{T}$$

$$\equiv \alpha \tilde{T}, \quad (5)$$

where $\tilde{\sigma} \equiv \sigma_0 - c_0 \frac{\partial \sigma}{\partial \rho}$, and $\alpha$ is the coupling factor governing conversion of second sound into first sound. Similarly, if we insert $u = u_2$ into the second sound wave equation, Eq. (4b), we obtain

$$\tilde{T} = -\frac{A_{11}(u_2^2)}{A_{10}(u_1^2)}\tilde{P} = \left[ \frac{\partial \rho}{\partial T} \right]_{P,c} + c_0 \frac{\partial \rho}{\partial \sigma} \frac{\partial \sigma}{\partial T} \left( \frac{\partial \sigma}{\partial \rho} \right)_{T,P} \tilde{P}$$

$$\equiv \beta \tilde{P}, \quad (6)$$

where $U_1 \equiv \frac{u_1}{\rho_0}$, and $U_2 \equiv \frac{u_2}{\rho_0}$, and $\beta$ is the coupling factor characterizing conversion of first sound into second sound. These equations are similar to what Brusov et al. [12] had obtained, except that, as noted by Rysti [13], a sign error in the bracketed term of Eq. (5) prevented them from noticing the possibility of decoupling between the two sound modes. For superfluid helium ($\partial \rho/\partial T$)$_{P,c}$, which is proportional to the thermal expansion coefficient, is positive, and since $c_0/\tilde{\sigma}$, and $(\partial \sigma/\partial T)$$_{T,P}$ are also positive, but $(\partial \rho/\partial \sigma)$$_{T,P}$ is negative, it is possible that the two terms in brackets cancel out each other at certain $^3$He concentrations and temperatures, resulting in first sound decoupling from second sound. The velocity term of $\alpha$ does not change sign since $u_1 > u_2$. On the other hand, the coupling factor $\beta$ is small, always negative, and practically constant in the temperature and concentration region of our experiment. Only very close to the $^4$He superfluid transition temperature $T_\lambda$, its value starts to depart from the constant value. Since $\beta$ is always finite, first sound can always generate second sound in the region considered here, whereas there exists specific temperatures and concentrations where the opposite is not possible; second sound cannot always create first sound.

Figure 1 shows the decoupling conditions, as well as calculated values for the bracketed term of the coupling factor $\alpha$, obtained by using the entropy data of Refs. [15–17], and by assuming ideal mixture between the two helium isotopes. Superfluid and normal fluid densities in mixture were evaluated according to Refs. [13,18–21]. Furthermore, the density of the mixture was evaluated by using molar volume formula given by Dobbs [22], and the pure $^4$He density formula given by Niemela and Donnelly [23] scaled by the $^3$He concentration dependence of $T_\lambda$, to produce correct density behavior near $T_\lambda$. 

FIG. 1. (a) Calculated temperature [relative to $^4$He superfluid transition temperature $T_\lambda = T_\lambda(x_3)$], where first sound decouples from second sound ($\alpha = 0$), plotted as a function of molar $^3$He concentration $x_3$. (b) Values of the bracketed term of $\alpha$ in Eq. (5) as a function of relative temperature at different $^3$He concentrations. To clarify the relation between the plots, a few examples of equivalent points in the figures are indicated by $\triangle$, $\square$, and $\circ$. 

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At 0.75\(T_s\) \(\approx\) 1.6 K, the decoupling occurs already around 0.3% molar \(^3\)He concentration, and at higher temperatures the decoupling condition moves to higher concentrations, up to about 3% near \(T_s\).

### III. EXPERIMENTAL DETAILS

#### A. Cell and cooling system

Our 2 cm\(^3\) experimental cell, shown in Fig. 2, was a simple copper container that had two horizontal tubes soldered at the bottom to house two quartz tuning forks. The copper container itself acted as a buffer volume to ensure that the liquid in the cell was always under saturated vapor pressure, and that the forks were properly immersed in liquid. The cell was installed in a glass dewar, which could be filled with liquid \(^4\)He and then pumped to reach temperatures down to about 1.6 K. The temperature of the glass dewar was adjusted by combination of throttling the pumping and a computer controlled heater. Temperature was monitored with two carbon resistors: one was placed directly on the cell, while the other was fixed on the support structure. Cell and bath pressure were measured using Pfeiffer Vacuum PCR 280 pressure gauges. The carbon resistors were calibrated against the vapor pressure during pure \(^4\)He measurements.

\(^3\)He –\(^4\)He mixtures were prepared at room temperature. We started with 25 mmol of commercial quality pure \(^4\)He and systematically added mixture of known molar concentration (6%, 11%, or 24%) to obtain the desired composition, finally ending up with 94 mmol of 9% concentration mixture. Afterwards, we conducted measurements using mixture taken directly from 6%, and 11% storage tanks to calibrate our quartz tuning fork to cross-check the concentrations of the earlier used mixtures.

#### B. Quartz tuning fork

We used a ECS-.327-8-14X 32.768 kHz quartz tuning fork oscillator, which was driven by a function generator, and the signal was detected by a lock-in amplifier. We also had another fork with a different resonance frequency and larger physical size installed, but it behaved erratically, and it then could not be used to produce any meaningful experimental data.

We measured the fork in so-called tracking mode [9], in which a computer program determines the resonance frequency and the width of the resonance from a single measurement point close to the actual resonance frequency, assuming that the shape of the resonance is Lorentzian. The tracking mode enables us to repeat the measurement every few seconds instead of minutes, as it would take if we were to record entire resonance spectra with sufficient precision.

Quartz tuning forks do not respond to the temperature of the liquid directly, rather they sense it through the change in viscosity and density of the fluid due to temperature. This means that a standing second sound wave is invisible to the fork, as temperature oscillations cannot cause direct piezoelectric response.

But second sound can create first sound. Such driven first sound has the same wave characteristics as the second sound wave that is generating it, but as pressure wave, it can push on the fork altering its resonance response. When the first sound decouples from second sound, the standing second sound wave is no longer able to drive first sound, and the anomaly in the fork resonance behavior disappears.

These anomalies, caused by first sound driven by second sound, are usually simply called second sound resonances, even if it is slightly inaccurate. The velocity of second sound ranges approximately between 0 and 40 m/s in the temperature and concentration range of our experiment. Largest values are obtained at low temperatures and at high concentrations, while near \(T_s\) it tends to zero, as second sound ceases to exist [9,24–27]. Since the second sound velocity has a significant temperature and concentration dependence, there exist numerous possible standing sound wave modes that can be observed with a quartz tuning fork [28].

We do not need to know the exact shape of the standing wave mode within the liquid surrounding the fork. This would be rather difficult to determine as the geometry of the fork is nontrivial. The quantity of interest here is the coupling between first sound and second sound, and this is independent from the quartz oscillator. The fork geometry only determines how many second sound resonances we are able to see, at which temperature and concentration they appear, and how strong an anomaly they create.

Sound emission from the quartz tuning fork in superfluid helium is likely not just pure first sound, but rather some combination of first and second sound. Second sound contribution generated by the fork can couple back through single sound conversion (second sound to first sound), and first sound contribution through double sound conversion (first sound to second sound and back to first).

Alternatively, if there were to exist a direct coupling between the fork and second sound, the amplitude of the fork resonance anomalies so created should not depend on the coupling strength in sound conversion, since the coupling would no longer play any role. However, it appears that the direct coupling contribution is negligible.

In a single sound conversion, the coupling factor is just \(\alpha\) [Eq. (5)], but in a double sound conversion the amplitude of the resonating first sound mode is proportional to the product of the coupling factors \(\alpha\beta\) [Eqs. (5) and (6)]. But since only the coupling factor \(\alpha\) has a significant temperature and \(^3\)He concentration dependence, it chiefly determines the...
coupling/decoupling behavior. The weak temperature dependence of $\beta$ meant that we were not able to discern whether the pressure wave affecting the fork had come about through single sound conversion from second sound generated by the fork, or through double sound conversion from first sound generated by the fork. The bottom line is, nevertheless, that the determining factor, in both cases, is the coupling coefficient $\alpha$. When it becomes very small, in either case, second sound can no longer drive first sound, and there would then no longer exist a first sound mode that can couple back to the fork altering its resonance behavior.

IV. RESULTS

Temperature sweeps were carried out from the $\lambda$ point down to about 1.6 K and back while measuring the resonance frequency and resonance width of the quartz tuning fork. Near $T_\lambda$ the sweep rate had to be quite slow, 0.5 mK/min, since there were many small second sound anomalies there. Below 2 K we could increase the rate to 1.5 mK/min, as the anomalies became more infrequent and wider in temperature. Figure 3 shows the resonance frequency of the 32 kHz quartz tuning fork versus resonance width at different $^3$He concentrations, while Fig. 4 takes one of those traces and shows it separately to illustrate key features as well as temperatures along the sweep.

In these presentations, each anomaly caused by second sound appears as a loop, and its magnitude, when changing concentration, is proportional to the coupling strength between first and second sound. The loops in each case showed up at the same temperature independent of the direction of the temperature sweep, to better than $\pm 0.6$ mK. Pure $^4$He was measured multiple times, both before and after the mixture measurements, from which we could estimate the reproducibility to be about $\pm 3$ mK. This spread is due to uncertainty in our temperature determination with the carbon resistors, rather than any variation in the sound properties.

Starting from 2.1% $^3$He concentration, there appears a horizontal feature near the $\lambda$ point, which is caused by the mixing of the helium isotopes after condensation. They do not mix properly until near $T_\lambda$, after which the fork resonance frequency changes rapidly to a new value, whereafter the behavior is retraceable. This is shown more clearly in Fig. 4, where we see that, during the initial cooldown, the shape characteristic to the superfluid transition begins to form at a different location than where it eventually occurs at equilibrium.

The 1.1% data set is shifted with respect to others due to some unknown unrepeatable phenomenon, possibly related to impurities sticking to the fork. We reached this conclusion since the problem did not repeat itself after we had warmed our cooling system back to room temperature between measurements. These data can be made compatible with the others by simple shift, bringing the kink at the $\lambda$ point to the appropriate position. Obviously similar phenomenon, albeit at much smaller scale, has caused some of the offset between the two pure $^4$He runs illustrated in the inset of Fig. 3. We emphasize that Fig. 3 displays the raw data as measured, with no adjustment or post processing.

Since the sound mode coupling in pure $^4$He is caused only by the very small thermal expansion, the magnitude of the loops is also quite small. Remarkably, they become even smaller as some amount of $^3$He is added. The addition of $^3$He initially weakens the coupling between the sound modes, as was predicted by our calculations of the coupling factor $\alpha$ in Sec. II. The second sound anomalies seem to vanish somewhere between 1.1% and 2.1% concentrations, which is indicative of the decoupling between the two sound modes. As the $^3$He concentration is further increased, the anomalies reappear, eventually becoming significantly larger.

![FIG. 3. Resonance width ($\Delta f$) of the 32 kHz quartz tuning fork versus its resonance frequency ($f$) with number of $^3$He concentrations.](image-url)
FIG. 4. 4.2% $^3$He concentration fork trace with key features highlighted. Temperature values at different points are shown next to the curve. Arrows indicate the direction of the temperature sweep: first down from 2.53 to 1.71 K, and then back up to 2.36 K. Dashed line represents the linear fit to the decline below $T_\lambda$, which was used in concluding the concentration value.

than they were in pure $^4$He, because now the $^3$He contribution to the coupling is dominant. Such clear decoupling behavior confirms that the direct coupling between the fork and second sound is indeed very small, as we had assumed.

What is more, at 2.1% concentration, there appears a new set of second sound anomalies at the low temperature end of the sweep. These were absent in pure $^4$He, due to the second sound velocity leveling out to about 20 m/s at around $1.8 K \approx 0.83T_\lambda$ in it. Temperature sweeps between pure $^4$He and 2.1% $^3$He concentration, except for 1.1% measurement, were not extended down to the lowest reachable temperature, since we had not initially expected to find anything there. Since the 1.1% sweep continued to a lower temperature than the ones next to it, we can conclude that these anomalies had not yet appeared at this concentration.

Because the horizontal shifting of the fork traces in Fig. 3 is a result of changing $^3$He concentration, we can determine the concentration values by making a linear fit to the characteristic decline in each temperature sweep below $T_\lambda$ (see Fig. 4), and then comparing the fitting parameters to those of the 6.0% and 11.0% measurements, as their mixture was taken directly from storage tanks with known concentrations. The values shown in Fig. 3 were obtained this way. The uncertainty of all readings is estimated to be $\pm 0.3$ percentage points. The concentration values obtained from the tuning fork analysis were systematically somewhat lower, but still within 0.5 percentage points from the concentrations estimated while preparing the gas mixture at room temperature.

In Fig. 5 we take a closer look of one set of second sound anomalies illustrating their behavior near the decoupling region. Even though their shape changes as the $^3$He concentration is increased, we can still identify the corresponding features because they always appear in the same sequence—a larger anomaly flanked by two smaller anomalies, plus a number of tinier ones, in this example. When the coupling is at its weakest, only the largest anomaly remains barely visible, and it too would seem to disappear somewhere between 1.7% and 2.1% concentrations. Even if the fork resonance width of the 1.1% measurement set is in slightly different range than the

FIG. 5. Closer view of one set of second sound anomalies (the ones shown in the inset of Fig. 3) followed through the decoupling region. The color of the line changes according to temperature, showing that the anomalies move to a higher relative temperature as the $^3$He concentration increases. Even though the shape of the features changes, we can still identify the three biggest anomalies of pure $^4$He also in 4.2% mixture, labeled as 1–3.

FIG. 6. Locations of the second sound anomalies followed through the decoupling region in $^3$He concentration–relative temperature plane, as well as their amplitude (represented by the size of the circle). The set of anomalies that were presented in Fig. 5, up to 4.2% concentration, are indicated by the arrow. The dashed line corresponds to the empirical fit of Fig. 8, which separates the regions before and after the decoupling.
FIG. 7. Amplitude $a$ of a typical second sound anomaly as a function of $^3$He concentration. Amplitudes were normalized to the value at 9% concentration. Fits at either side of the minimum were used to evaluate the decoupling concentration. Solid circles indicate the points where we could say with confidence that the anomalies had not yet disappeared, which were then used as the error bars of Fig. 8.

others (see Fig. 3), when also considering temperature, that data set fits in quite well with the others.

We determined the amplitude for each second sound anomaly as the maximum deviation from the fork trace’s linear decline. The temperature for each anomaly was defined to be the point of the maximum deviation. These are shown in Fig. 6. Not all second sound anomalies of Fig. 3 are included, but rather the selected few we were able to follow through the decoupling region with sufficient confidence. Near $T_s$, or at high concentrations, the anomalies appear almost at the same relative temperature, but otherwise they tend to bend to lower temperatures.

The amplitude data allow us to determine the locations where the anomalies would practically disappear by making a linear fit to the logarithmic amplitude plotted as a function of $^3$He concentration, on either side of the decoupling point, as shown in Fig. 7, and then determining the temperature of this point through interpolation. The decoupling conditions obtained this way are shown in Fig. 8, where they are compared against the decoupling behavior calculated in Sec. II. The confidence limits (error bars) were determined from concentrations and temperatures where the anomalies had definitely not yet disappeared, or had clearly appeared again (see Fig. 7). We also extrapolated the lowest temperature second sound anomaly data to find where they would disappear, and since they were not visible in pure $^4$He, their error bars extend all the way to zero concentration. Decoupling points determined from our measurements lie systematically at higher $^3$He concentrations than the calculated values. This would suggest that either the $^3$He contribution to the coupling is slightly smaller than the calculations indicated, for example, due to our assumption of ideal mixture of $^3$He and $^4$He, or, alternatively, that there exists some additional contribution to the coupling, e.g., due to the mutual friction between normal and superfluid component.

When we identify corresponding second sound anomalies at different $^3$He concentrations, we are following the same particular standing second sound wave modes. Since these modes occur at specific second sound velocities, we can obtain a constant second sound velocity graph. This is presented in Fig. 9, which contains the same points as Fig. 6. We have only omitted the circles that represented the amplitude of the anomaly, connected the datapoints with spline fits, and labeled the curves with estimated sound velocities.

V. CONCLUSIONS

We studied coupling between first sound and second sound in $^3$He–superfluid $^4$He mixtures, down to 1.6 K temperature under saturated vapor pressure. Velocity of second sound is such that it can form standing waves around a quartz tuning fork immersed and oscillating in superfluid. Second sound drives first sound with the same waveform, and this first sound perturbation can be detected by the fork as an anomalous resonance behavior. Since the specific second sound anomalies
always appear under the same conditions due to the nature of standing waves, they can be used, for example, as reference points of temperature with good accuracy [11].

We confirmed, that at certain concentrations and temperatures, these second sound anomalies disappear, before reappearing again as the $^3$He concentration is increased. This behavior is a result of the competing contributions to the coupling between the two sound modes. When the sound modes become decoupled, the standing second sound wave can still exist, but it can no longer create first sound, and hence it becomes invisible to the quartz tuning fork. Since the decoupling behavior is quite pronounced, it also suggests that the direct coupling between the quartz tuning fork and second sound is very small, and the sound mode detected by the fork has come about mainly through sound conversion phenomenon. Our calculation, which revised the results presented earlier by others [12], predicted this kind of decoupling behavior, but our analysis projected the decoupling to occur at somewhat lower $^3$He concentration.

To better understand the changing shape of the second sound anomalies with changing concentration, more complete modeling of the phenomenon is required.

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