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Universal Coherence-Induced Power Losses of Quantum Heat Engines in Linear Response

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We identify a universal indicator for the impact of coherence on periodically driven quantum devices by dividing their power output into a classical contribution and one stemming solely from superpositions. Specializing to Lindblad dynamics and small driving amplitudes, we derive general upper bounds on both the coherent and the total power of cyclic heat engines. These constraints imply that, for sufficiently slow driving, coherence inevitably leads to power losses in the linear-response regime. We illustrate our theory by working out the experimentally relevant example of a single-qubit engine.

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Heat engines are devices that convert thermal energy into useful work. A Stirling motor, for example, uses the varying pressure of a periodically heated gas to produce mechanical motion (Fig. 1). Used by macroscopic engines for two centuries, this elementary operation principle has now been implemented on ever-smaller scales. Over the past decade, a series of experiments has shown that the working fluid of Stirling-type engines can be reduced to tiny objects such as a micrometer-sized silicon spring [1] or a single colloidal particle [2–5]. These efforts recently culminated in the realization of a single-atom heat engine [6,7]. Thus, the dimensions of the working fluid were further decreased by 4 orders of magnitude within only a few years. In light of this remarkable development, the challenge of even smaller engines operating on time and energy scales comparable to Planck’s constant appears realistic for future experiments.

Quantum engines have access to a nonclassical mechanism of energy conversion that relies on the creation of coherence in their working fluid [8]; see Fig. 1. How does this additional freedom affect their performance? Having triggered substantial research efforts in recent years, this question constitutes one of the central problems in quantum thermodynamics; see, for example, Refs. [9–18]. However, the results now available are inconclusive. In fact, current evidence suggests that, depending on the specific setup, coherence can be either conducive [8,11,19–28] or detrimental [29–33].

Quantifying the role of coherence for a thermodynamic process requires a benchmark parameter that is sensitive to superpositions between the energy levels of the working medium. For operations involving nonselective measurements, several such figures were recently discussed; see, e.g., Refs. [34,35]. In this Letter, we put forward a universal coherence indicator for cyclic machines operating without external measurements. To this end, we describe the working fluid as a periodically driven $N$-level system with a time-dependent Hamiltonian $H_t$, which is embedded in a thermal environment. Provided that this reservoir is large, the system will settle to a periodic state $\rho_t$ after some transient time. The mean generated power per cycle of length $T$ is then given by [31]


FIG. 1. Classical and quantum engines. (Upper panel) Macroscopic Stirling cycle. In the first stroke, power is extracted by expanding the hot working fluid. Decreasing the temperature at constant volume in the second stroke leads to a reduction of pressure before the gas is compressed again in the third stroke. The cycle is completed by isochorically returning to the initial temperature. (Lower panel) Quantum Stirling cycle. The working fluid consists of a two-level system, whose Bloch vector at the beginning of each stroke is shown in the four diagrams. The energy eigenstates lie on the vertical axis and the radius of the circle indicates the level splitting. Two distinct control operations are applied during the work strokes: the level splitting is changed and superpositions are created, i.e., the Bloch vector is rotated away from the vertical axis. In linear response, the energy content of these coherences cannot be regained by the controller; it is dissipated during the thermalization strokes, while the level population adapts to the temperature of the environment.
\[ P = -\frac{1}{T} \int_0^T dt \text{tr}\{\mathcal{H}_t q_t\}. \]  

Using the spectral decomposition
\[ H_t \equiv \sum_n E_n^t |n_t\rangle \langle n_t| \]  

of the time-dependent Hamiltonian, this quantity can be divided into two contributions corresponding to the different mechanisms of work extraction illustrated in Fig. 1. First, the classical power
\[ P^d \equiv -\frac{1}{T} \int_0^T dt \sum_n \dot{E}_n^t |n_t\langle q_t|n_t\rangle \]  
generated by changing the energy levels of the working fluid depends only on the diagonal elements \( q_t \) with respect to the instantaneous energy eigenstates. Second, the coherent power
\[ P^c \equiv P - P^d = \frac{1}{T} \int_0^T dt \sum_n \langle \dot{n}_t| \{ H_t, q_t \}|n_t\rangle \]  
arises from creating superpositions between these states [36]. Accordingly, \( P^c \) vanishes when \( q_t \) commutes with \( H_t \) throughout the cycle or when the eigenvectors of \( H_t \) are time independent.

We note that the separation of the power operator \( \dot{H}_t \) into a diagonal and an off-diagonal part has been discussed in the context of adiabatic processes [37] and for a specific model of a quantum Otto engine [38]. Here, we obtained the identifications (3) and (4) without making any assumptions on the time scale of the driving, the driving protocol or the system-reservoir coupling. In fact, they follow directly from the expression (1) for the total generated power, which can be regarded as a consequence of the first law applied to the compound system of working fluid and environment. Therefore, the coherent power (4) qualifies as a universal indicator for the impact of coherence on periodic power generation, which, besides heat engines, could also be applied to other types of devices such as feedback engines [12,23,39,40].

As a first key application of this concept, we will explore how coherence affects the power of slowly driven heat engines in linear response. Our analysis thereby builds on the well-established theory of open quantum systems [41,42] and a recently developed thermodynamic framework describing periodically driven systems [31,43], which has already proven very useful in the classical realm [44–47]. To describe a quantum heat engine, we augment the setup discussed so far with a heat source, which periodically injects thermal energy into the environment at a rate much slower than its internal relaxation time. The working fluid then effectively feels the time-dependent temperature
\[ T_t \equiv T + f^c_t, \quad \text{with} \quad f^c_t \geq 0. \]  

Work is extracted through a periodic driving field \( f^{\text{wt}}_t \), which couples linearly to the system degree of freedom \( G^\alpha \). The Hamiltonian \( H_t \) thus assumes the form
\[ H_t \equiv H + f^{\text{wt}}_t G^\alpha. \]  

For uniqueness, the field \( f^{\text{wt}}_t \) is chosen to be dimensionless and with a vanishing average over one period \( T \).

Deriving constraints on the coherent power \( P^c \) requires us to further specify the dynamics of the working fluid. To this end, we first consider the equilibrium situation, i.e., \( f^{\text{eq}}_t = f^{\text{wt}}_t = 0 \). Assuming weak system-reservoir coupling and applying a coarse graining in time to wipe out memory effects and fast oscillating contributions to the state \( q_t \), then yields the Markovian master equation
\[ \partial_t q_t = -\frac{i}{\hbar} \{ H, q_t \} + D q_t, \]  

where the dissipator
\[ DX = \sum_\sigma \frac{\gamma_\sigma}{2} \left( [V_\sigma X, V^\dagger_\sigma] + [V^\dagger_\sigma X V_\sigma] \right) \]  
accounts for the effective influence of the thermal environment [41,42,48–50]. Here, \( \hbar \) denotes Planck’s constant and \( \{ \gamma_\sigma \} \) is a set of positive rates with corresponding Lindblad operators \( \{ V_\sigma \} \). Owing to microreversibility, these quantities are constrained by the quantum detailed balance relation, which can be expressed compactly in terms of the formal identity [48,51]
\[ D e^{-\beta H} = e^{-\beta H} D^\dagger. \]  

Here, \( \beta \equiv 1/(k_B T) \), where \( k_B \) denotes Boltzmann’s constant, and the adjoint dissipator is given by [42]
\[ D^\dagger X = \sum_\sigma \frac{\gamma_\sigma}{2} \left( V^\dagger_\sigma X V_\sigma + [V^\dagger_\sigma X V_\sigma] \right). \]  

In the adiabatic regime, where the driving is slow compared to the coarse-graining time scale used in the derivation of Eq. (7), finite driving can be included in this framework by allowing the rates and the Lindblad operators to be time dependent and replacing \( H \) and \( T \) with \( H_t \) and \( T_t \), respectively, in Eqs. (7)–(10) [41]. Solving the resulting master equation with time-dependent generator by treating \( f^{\text{eq}}_t \) and \( f^{\text{wt}}_t \) as first-order perturbations then yields the explicit expressions [52]
\begin{align*}
P^d &\equiv -\frac{1}{T} \int_0^T dt \int_0^\infty dt' f^{\text{eq}}_t \left( \dot{C}_{\text{det}}^{\text{eq}} f^{\text{eq}}_{t-t'} + \dot{C}_{\text{det}}^{\text{eq}} f^{\text{eq}}_{t'-t} \right), \\
P^c &\equiv -\frac{1}{T} \int_0^T dt \int_0^\infty dt' f^{\text{wt}}_t \dot{C}_{\text{cor}}^{\text{eq}} f^{\text{eq}}_{t-t'} \
\end{align*}

for the classical and the coherent power, respectively [53], in the following notation. We abbreviate with \( C_{\text{cor}}^{\text{eq}} \) the Kubo correlation function [54].
where \( t \geq 0 \) and each of the indices \( a \) and \( b \) can assume the values \( d \), \( c \), \( q \). Hats indicate Heisenberg-picture operators satisfying the adjoint master equation \([42]\)

\[
\dot{\hat{X}}_i = \frac{i}{\hbar} [H, \hat{X}_i] + D^\dagger \hat{X}_i, \quad \text{with} \quad \hat{X}_0 = X.
\]  

Angular brackets denote the thermal average throughout, i.e., \( \langle X \rangle \equiv \text{tr}\{X e^{-\beta H}\}/\text{tr}\{e^{-\beta H}\} \). Finally, we have defined the operator \( G^d \equiv -\partial_\Omega T \) and split \( G^w \) into a diagonal and a coherent part,

\[
G^d \equiv \sum_n |n\rangle \langle n|G^w|n\rangle \langle n| \quad \text{and} \quad G^c \equiv G^w - G^d,
\]

where the vectors \(|n\rangle\) correspond to the eigenstates of the unperturbed Hamiltonian \( H \).

As a first key observation, we note that the expression \((11)\) for \( P^c \) is independent of the temperature profile \( f[]
which leads to $P = L_t^qF_q/4$ for any temperature profile $f_q^t$ and for sufficiently short operation cycles [57]. Furthermore, using relation (20), the upper bound (18) can be expressed in a physically transparent form,

$$L_t^q = \frac{\lambda \langle H^2 \rangle - \langle H \rangle^2}{k_BT^3}. \quad (22)$$

Hence, the strength and the decay rate of the energy fluctuations in equilibrium essentially determine the maximum power of a cyclic $N$-level engine in the linear-response regime. A similar result was obtained only recently for classical engines [43,45].

We will now explore the quality of our bounds under practical conditions. To this end, we consider a two-level engine with the time-dependent Hamiltonian

$$H_t = \frac{\hbar \omega}{2} \sigma_z + \hbar \omega f_q^t \left(r \sigma_z + (1 - r) \sigma_x\right). \quad (23)$$

Here, $\sigma_{x,z}$ are the usual Pauli matrices, and the dimensionless parameter $0 \leq r \leq 1$ determines the relative weight of the classical and coherent parts, $G^d = r (\hbar \omega/2) \sigma_z$ and $G^c = (1 - r) (\hbar \omega/2) \sigma_x$, of the control variable $G^w$. The corresponding equilibrium dissipator (8) involves two Lindblad operators, $V_\pm = (\sigma_\pm \pm i \sigma_z)/2$, acting at the rates $\gamma_\pm \equiv \gamma e^{\pm \kappa}$, respectively, where $\kappa \equiv \hbar \omega \beta/2$. This setup lies within the range of forthcoming experiments using a superconducting qubit to realize the system and ultrafast electron thermometers for calorimetric work measurements [58–60]. Its coherent and total power are subject to the bounds

$$P^c \leq -\frac{\hbar \omega \lambda}{2} r^2 g_\Omega F_w^c \quad \text{and} \quad P \leq \frac{\hbar \omega \lambda}{8} \frac{g}{1 + \psi_{\Omega} T^2},$$

with $\psi_{\Omega} = \frac{(1 - r)^2 \sinh 2 \kappa}{2r^2} \frac{\Omega^2}{4 \kappa^2} + \omega^2 + \lambda^2/4$. \quad (24)

$g \equiv \kappa / \cosh^2 \kappa$, and $\lambda \equiv 2 \gamma \cosh \kappa$, which follow from Eqs. (15) and (17) [55].

To assess the quality of these constraints, we choose a temperature profile $f_q^t$ that mimics the Stirling cycle illustrated in Fig. 1 and a work protocol satisfying

$$2f_q^{\tau} = -\Omega (f_q^{\tau} - \bar{f}_0^q)/T + f_q^d/T. \quad (25)$$

both of which are shown in Fig. 2. This choice renders the amplitude and shape of $f_q^c$ independent of the cycle frequency $\Omega$. In Fig. 2, the resulting coherent power is plotted as a function of $\Omega/\lambda$ for $r = 1/2$. If the level splitting $\omega$ is significantly smaller than the dissipation rate $\lambda$, it decays monotonically while closely following its upper bound (24). With an increasing $\omega$, a resonant dip emerges close to $\Omega = \omega$. This feature is not reproduced by our bound, which is, however, still saturated in the limit $\Omega/\lambda \rightarrow 0$ and, formally, also for $\Omega/\lambda \rightarrow \infty$. For $r = 1$,
through coherence, e.g., squeezed reservoirs [11] or collective behavior [18,27], and features leading to coherence-induced power losses like quantum friction; the last phenomenon was observed earlier in various models describing the working fluid as an interacting spin system [29,30,38,63,64].

We conclude by stressing that our key expressions (3) and (4) are valid for an arbitrary driving strength and speed and for any type of system-reservoir coupling. The coherent power (4) can therefore be used as a universal control, Sci. Rep. 5, 14870 (2015).


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[36] To make these definitions unique, we understand that the time-dependent energy eigenvalues are arranged in increasing order, i.e., \( E_0^t \leq E_1^t \leq \cdots \leq E_N^t \) for all cases where \( t \in \mathbb{R} \), where we assume that \( N \) is finite for simplicity. Furthermore, we note that the expressions (3) and (4) are unaltered if the instantaneous energy eigenstates are multiplied by arbitrary time-dependent phase factors.

[37] H. T. Quan, P. Zhang, and C. P. Sun, Quantum heat engine with multilevel quantum systems, Phys. Rev. E 72, 056110 (2005).


[52] To obtain Eq. (11) from Eqs. (3) and (4), standard linear-response theory is applied. This derivation exploits the fact that, due to the detailed balance relation (9), the space of all system operators commuting with the unperturbed Hamiltonian \( H \) is invariant under the action of the superoperators \( D \) and \( D^\dagger \); for details, see Ref. [31].

[53] The detailed balance relation (9) implies that the set of Lindblad operators \( \{ V_\alpha \} \) is self-adjoint; see Ref. [51] and A. Kossakowski, A. Frigerio, V. Gorini, and M. Verri, Quantum detailed balance and KMS condition, Commun. Math. Phys. 57, 97 (1977)]. Additionally, we assume here that this set is irreducible such that \( \lambda = 1 \) is the only solution of \( D^\dagger X = 0 \) [H. Spohn, An algebraic condition for the approach to equilibrium of an open N-level system, Lett. Math. Phys. 2, 33 (1977)]. Under this condition, the improper integrals showing up in Eq. (11) are well defined [31].


[56] The Floquet-Lindblad equation can be derived in a similar way as the adiabatic master equation [41] used in this Letter; see, for example, Ref. [50] and R. Alicki, D. A. Lidar, and P. Zanardi, Internal consistency of fault-tolerant quantum error correction in light of rigorous derivations of the quantum Markovian limit, Phys. Rev. A 73, 052311 (2006)]. However, in the Floquet-Lindblad approach, the coarse-graining time scale is larger than the period of the external modulation. Thus, by carrying out the coarse graining, the driving is effectively averaged in time. Combining this method with the concept of coherent power would therefore require us to adapt the expressions (3) and (4) to the time coarse-grained picture.

[57] For long cycles, the reduced temperature profile \( f^\ell / f^q \) would oscillate slowly and thus be either positive or negative over substantial time ranges. Consequently, integrating Eq. (21) would yield a driving protocol \( f^\ell / f^q \) with a large amplitude, which violates the linear-response condition underlying the derivations leading to Eqs. (18) and (21).

