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Finding low-tension communities

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Abstract

Motivated by applications that arise in online social media and collaboration networks, there has been a lot of work on community-search and team-formation problems. In the former class of problems, the goal is to find a subgraph that satisfies a certain connectivity requirement and contains a given collection of seed nodes. In the latter class of problems, on the other hand, the goal is to find individuals who collectively have the skills required for a task and form a connected subgraph with certain properties.

In this paper, we extend both the community-search and the team-formation problems by associating each individual with a profile. The profile is a numeric score that quantifies the position of an individual with respect to a topic. We adopt a model where each individual starts with a latent profile and arrives to a conformed profile through a dynamic conformation process, which takes into account the individual’s social interaction and the tendency to conform with one’s social environment. In this framework, social tension arises from the differences between the conformed profiles of neighboring individuals as well as from differences between individuals’ conformed and latent profiles.

Given a network of individuals, their latent profiles and this conformation process, we extend the community-search and the team-formation problems by requiring the output subgraphs to have low social tension. From the technical point of view, we study the complexity of these problems and propose algorithms for solving them effectively. Our experimental evaluation in a number of social networks reveals the efficacy and efficiency of our methods.

1 Introduction

A large body of work in social and collaboration networks focuses on solving variants of two classes of problems: community-search and team-formation. The high-level goal of both classes is to discover a connected subgraph, but they differ in the input query. In the community-search problem [7, 18, 19, 20], the input query consists of individuals who should participate in the subgraph. In the team-formation problem [1, 2, 8, 15, 3, 11, 14, 16, 17], on the other hand, the input query consists of a set of skills that need to be covered by the solution.

These two problem classes have applications primarily in online social media and collaboration networks. For example, solutions to the community-search problem can be used to identify a set of individuals who are the most appropriate group to organize a social gathering. Similarly, solutions to the team-formation problem can be used by project managers or human-resource (HR) departments of companies and universities in order to identify a well-functioning team of employees for a project or to make a competitive cluster hire. Such hiring and resource-allocation decisions can prove important for the well-functioning and the success of an organization.

In all existing variants of the community-search and the team-formation problems the goal is to discover a connected subgraph; Different variants of the problem impose different requirements on the structure of the solution subgraph. For example, in some settings one asks to find high-density communities [19], while in others the objective is to find small-diameter communities [18]. From the computational point of view, different requirements lead to different problems that have different complexities and require the design of different algorithms. As usual, the choice of the appropriate problem formulation depends on the application domain.
In contrast to the static requirements imposed by existing work, in this paper we incorporate the dynamics of social interactions in the community-search problem. We do so by associating profiles to the individuals in the network.

Regarding any subject of matter, individuals have their own opinion or preference. However, in a social context, such as a working group or team, the opinion that is publicly held or enacted by each individual is influenced by his peers. Diverging from the opinion of peers generates a social cost bearing on the individual. Simultaneously, expressing an opinion that does not match one’s own beliefs creates an inner conflict and an associated internal cost. These constitute important factors impacting the operation of the community as a social group. Therefore, in this paper, we extend the community-search and the team-formation problems to take them into account, adding a new and realistic perspective.

We consider a social network between individuals, each one of which is associated with a profile. The profiles of individuals model their interests, skills, preferences, opinions, and so on, with respect to different aspects or topics. For example, the profile may represent the interest of an individual in discussing about politics, or the working style of an individual — e.g. whether he is a morning person or a night owl. Since profiles may cover a number of different aspects or topics, we assume that they are represented as multi-dimensional vectors.

We further assume that the profiles of individuals change due to the social influence they receive from other individuals in the network. We model this change through what we call conformation process, which is a dynamic process. Motivated by existing work on models of opinion formation and social influence [5, 6, 9, 13, 10] we assume that the conformation process is a repeated-averaging process. In our work, we generalize this process from one-dimensional opinions to multi-dimensional profiles. The effect of this process is that the initial profiles of individuals, which we call latent profiles, get re-enforced or altered through synthesis and aggregation of different viewpoints of the network participants.

Given this process, the latent profiles of individuals and a social network that represents their social interaction, social tension arises because of the differences between the conformed profiles of neighboring nodes and between the conformed and latent profiles of the nodes themselves. In this context, our goal is to identify communities and teams that are not only connected and qualified, but also exhibit low social tension. We refer to these problems as the $T$-COMM and the $T$-TEAM problems respectively. To the best of our knowledge we are the first to define and study these problems in the light of profile-conformation processes in a social network.

In terms of technical results, we show that both the $T$-COMM and the $T$-TEAM problems are NP-hard and we design graph-theoretic algorithms for solving them. A key difficulty that we overcome while designing these algorithms is that we do not run the conformation process for every step of these algorithms; indeed, this would be computationally very demanding. Rather, we create effective proxies of this process that allow our algorithm to scale. Our experiments with real-world data demonstrate the efficiency and the efficacy of our algorithms for both problems.

Applications. Both the $T$-COMM and $T$-TEAM problems have numerous potential applications. For example, when analyzing social networks or social media it is often useful to identify connected groups of users who have similar profiles with respect to a topic or an idea (i.e. they are in agreement). Note, however, that minimizing the social tension does not necessarily imply looking for highly homogeneous communities but, rather, favoring communities that are able to bridge opinion gaps at low social and communication costs. Groups with low social tension can be recommended as ideal in order to form a group to organize or to invite at a social event. Such problems also arise in human-resource management, when searching for groups of workers who can collaborate together in a conflict-free manner in order to successfully complete a project. Being able to identify a group of people who are not going to experience high social tension is particularly useful when considering cluster hires in universities or start-up companies, where the investment is high and the human factor risk needs to be minimized.

Roadmap. The rest of the paper is organized as follows. In Section 2 we give a thorough overview of the related work and identify the connections to our own work. After describing our notation and modeling assumptions in Section 3, we proceed to Section 4 where we define the community-search variant of our low-tension mining problem, prove that it is NP-hard and present algorithms for solving it in practice. We discuss other problem variants in Section 5. Our experimental results are described in Section 6 and we conclude the paper in Section 7.

A short version of this paper appeared in SDM’17. In this extended version, we discuss the team-formation problem variant, beside the original community-search problem, and include additional experimental results.
2 Related Work

To the best of our knowledge, we are the first to combine the problem of identifying a group of nodes from an input graph with an underlying dynamic conformation process of opinion formation at play over the graph. However, while the T-Comm and T-Team problems as we define them are novel, our work is related to existing work on graph mining and opinion dynamics. We summarize the related literature below.

2.1 Community search

Given a graph and a subset of its nodes as a seed, community-search problems ask for a set of nodes which is a superset of the seed nodes and induces a connected subgraph in the original graph. Since this class of problems was initially introduced [20], different instances of the community-search problem appeared. Each one of them imposes a different requirement on the graph-theoretic properties of the reported subgraph. For example, Sozio and Gionis [19] focus on finding a subgraph that connects the chosen seed members and has maximum density, Ruchansky et al. [18] solve the same problem with the objective to minimize the sum of pairwise shortest path distances among the nodes participating in the subgraph. Faloutsos et al. [7] study the problem of finding a subgraph that links two seed nodes while maximizing the quality of connection of the subgraph, for a generic measure of quality. Tong et al. [20] later extended their work to an arbitrary number of seed nodes.

Although these problems are related to the T-Comm problem we study here, the novelty of our problem comes from the fact that we associate nodes with profiles and that we assume a profile-conformation process that takes place over the network. Our objective function is directly related to this process as it aims to minimize the social tension in the reported community. As a result, the technical results we obtain for T-Comm are different from the other community-search problems.

2.2 Team formation

Team-formation is another class of graph-mining problems where the goal is to find a set of individuals in a network, who together possess the required skills while minimizing the communication costs between the members of the team. Since the introduction of this problem in the domain of data mining by Lappas et al. [15], a number of variants have been studied, from varying the communication cost function [3, 11, 14, 16, 17], to considering the workload distribution [1], designing algorithms for online team formation [2], and maximizing the influence of the team over the network [8].

At a high level, the T-Team problem we define is a team-formation problem. However, all existing work on team-formation problems aims to identify teams that optimize some static measure (e.g. the density or the diameter) of the subgraph that the team induces. However, the existence of a dynamic conformation process over the network makes our problem definition and setting quite distinct. Our goal is to minimize the social tension, which is much more complicated as it is caused by the difference in the profiles which arise from the interplay between the conformation process and the network structure. Therefore, existing algorithms for team formation cannot be applied to our problem.

2.3 Opinion dynamics

Starting in the 1970s, models have been built that try to capture the opinion-formation processes in groups and networks [5, 6, 9, 13, 22, 10]. For example, voter models, pioneered by Clifford and Sudbury [5] and Holley and Liggett [13] are stochastic models of opinion formation where at each step a node is selected at random and adopts the opinion of one of its neighbors, also selected at random. In DeGroot’s averaging model [6], each node updates its opinion, by the weighted average of its own opinion and its neighbors’ opinion at the previous time step. Friedkin and Johnsen [9, 10] introduced a model where every node has an immutable inner opinion and a changeable expressed opinion. Each node forms its expressed opinion in a repeated-averaging process involving its own inner opinion and the expressed opinions of its neighbors. Given the popularity of this model we also adopt it for modeling our profile-conformation process, and extend it to multiple dimensions.

The Friedkin and Johnsen model has been used in recent works by Bindel et al. [1] and by Gionis et al. [12]. Bindel et al. focus on the price of anarchy in terms of the tension achieved through local repeated averaging and global opinion coordination. Gionis et al. aim at identifying the set of nodes whose opinions need to be changed
so that the overall positive opinion in the network is maximized. Although our work builds upon Friedkin and Johnsen’s ideas to model the profiles and their conformation process, none of the above-mentioned works addresses the question of identifying a subgraph with low social tension as we do.

Extending our work to other times of opinion-dynamics models, i.e. beyond the model of Friedkin and Johnsen, is a promising direction for future research.

3 Preliminaries

Throughout the paper we consider a social network \( G = (V, E) \) where the nodes in \( V \) correspond to individuals and the edges in \( E \) represent the interactions between these individuals. For simplicity of exposition, we present the case of an unweighted and undirected graph, but the problems and algorithms we discuss can naturally be extended to weighted and directed graphs.

The set of neighbors of node \( i \) in \( G \) is denoted by \( N_G(i) \). Given a subset of vertices \( U \subseteq V \), we let \( E(U) \) denote the set of edges of \( G \) induced by \( U \), i.e. \( E(U) = \{(i, j) \in E, i, j \in U\} \), and \( G(U) \) denote the corresponding induced subgraph \( G(U) = (U, E(U)) \).

3.1 Profiles

Evidently, each individual has their own set of preferences (e.g. style, habits, biases, and opinions). We refer to this personal set of preferences as a profile. For now, let us assume that profiles only reflect preferences regarding a single aspect (e.g. working style), that is, profiles consist of a single attribute. We assume that profile attribute values are represented by a real number in the interval \([0, 1]\). For instance, a value between 0 and 1 may represent an individual’s preference towards a certain software tool, or his preference to work in a team or in isolation respectively.

The key characteristic of our model is that it captures the interaction between the user profiles and their social connections. This is done by assuming that each individual \( i \) has a latent profile and a conformed profile, denoted by \( x_i \) and \( f_i \) respectively. The latent profile of an individual represents the individual’s own true preference. However, individuals may choose not to act in accordance with their latent profiles as they try to minimize peer pressure by conforming their preferences to those of their peers. The conformed profile represents these adjusted preferences.

For simplicity, we first describe the model for single-attribute profiles, but later on we discuss how to extend it to multi-attribute profiles. We summarize the latent and conformed profiles of all \( n \) individuals with respect to a single attribute using vectors \( x \) and \( f \) respectively.

3.2 Measuring tension

Due to the underlying social structures and mechanisms, the conformed profile \( f_i \) of a node can differ from its latent profile \( x_i \). In such case, the node will bear an inner tension caused by the difference between its own latent and conformed profiles. On the other hand, the difference between the node’s conformed profile and between the conformed profiles of its neighbors will cause cross tension. Hence, the total tension on node \( i \) is:

\[
T_i(G, x, f) = (x_i - f_i)^2 + \sum_{j \in N_G(i)} (f_i - f_j)^2.
\]

Then, the social tension of the network is simply the sum of the individual tensions, defined as

\[
T(G, x, f) = \sum_{i \in V} T_i(G, x, f) = \sum_{i \in V} \left( (x_i - f_i)^2 + \sum_{j \in N_G(i)} (f_i - f_j)^2 \right),
\]

which can alternatively be written as the sum of the overall inner and cross tensions

\[
T(G, x, f) = \sum_{i \in V} (x_i - f_i)^2 + \sum_{(i,j) \in E} 2 (f_i - f_j)^2.
\]

4
3.3 Conformation process

But how do nodes arrive at their conformed profiles? Consider a repeated averaging process where at each step each node adjusts its conformed profile by setting it to the average of its latent profile and the conformed profile of its neighbors. Formally, denoting as \( f_i(t) \) the conformed profile of node \( i \) at step \( t \), we have:

\[
 f_i(t + 1) = \frac{x_i + \sum_{j \in N_G(i)} f_j(t)}{1 + |N_G(i)|}.
\] (2)

Computing the conformed profiles according to the repeated averaging model is equivalent to choosing \( f_i \) to minimize \( T_i(G, x, f) \). That is, if each node aims to minimize its tension, the repeated averaging model provides an optimal choice for the conformed profiles. In that sense, using the repeated averaging model yields a Nash equilibrium for the tension, not a social optimum [4].

A practical consideration is that in this model the latent profiles are assumed to be known, while the conformed profiles are the output of the conformation process. But, in practice, we have access to the conformed profiles while the latent profiles cannot be observed. This, however, does not constitute a problem for our model. One can swap the known and unknown variables of the system and solve for the latent profiles, given the conformed profiles and the original network.

The model adopted here for how conformed profiles emerge is a well-studied opinion-formation and social-influence model that has been introduced by sociologists [6, 9, 10]. In particular, the work of Friedkin et al. [10] validates this model by conducting a set of controlled experiments in which they observe how interactions between individuals in small groups influence their expressed opinions. The study demonstrates that the repeated-averaging model can both predict the opinions that individuals converge to, as well as the rate of convergence. Others have studied the mathematical properties of this model. For instance, it has been shown [4, 12] that the process converges in polynomial time to a fixed-point solution. In fact, the final conformed profiles can be computed by a matrix inversion [12], but actually repeating the averaging process leads to much faster computation.

3.4 Multi-attribute profiles

Our assumption so far has been that profiles reflect the preferences of individuals with respect to a single attribute. But our notion of latent and conformed profiles can be easily extended to the case of multiple attributes, leading to multi-attributes — i.e. multi-dimensional — profiles. Assume there are \( m \) aspects or topics of interest, each associated to an attribute. The latent and conformed profiles can be simply extended from a single real number to real-valued vectors of dimensionality \( m \), where each entry corresponds to one of the attributes. We summarize the latent and conformed profiles of \( n \) individuals on \( m \) attributes using \( n \times m \) matrices \( X \) and \( F \) respectively. Note that each column of these matrices, denoted by \( x_a \) and \( f_a \) for \( a = 1, \ldots, m \), corresponds to a single attribute. In the multi-attribute case, the conformed profiles can be computed as before by applying Equation (2) in a column-wise fashion. Similarly, the social tension \( T(G, X, F) \) is defined as the sum of the social tensions across all \( m \) attributes.

4 Finding low-tension communities with seed nodes

In this section, we introduce the \( T\text{-Comm} \) problem, study its complexity and provide algorithms for solving it.

4.1 Definition of the \( T\text{-Comm} \) problem

At a high level, the \( T\text{-Comm} \) problem aims to find a connected, low-tension community that involves a chosen subset of members. Formally, this intuitive statement is captured by the following problem definition:

**Problem 1 (\( T\text{-Comm} \))** Given a network \( G = (V, E) \), latent profiles \( X \) and a set of seed nodes \( Q \subseteq V \), find \( V' \subseteq V \) such that \( Q \subseteq V' \), the graph \( G' \) induced by \( V' \) on \( G \) is connected and \( T(G', X, F) \) is minimized, where \( F \) is computed by the repeated averaging model on \( G' \).

Note that when defining the \( T\text{-Comm} \) problem, we assume that the social tension is computed as in Equation (1), \( X \) is the matrix containing the latent profiles of the nodes in \( V' \) and \( F \) contains the conformed profiles.
of individuals, computed using the repeated averaging model described in the previous section (see Equation 2) over the subgraph induced by $V'$. Given these assumptions, we can make the following observations with respect to the requirements imposed on the solution of $T$-Comm: as the number of edges in the resulting subgraph and the differences in the conformed profiles across these edges decrease, so does the social tension. In particular, the complete absence of edges results in no tension at all. However, the requirement that the output subgraph should be connected forbids such solutions. From the application point of view, connectivity is important as it guarantees communication among the community members. One can see that minimizing tension and guaranteeing connectivity leads to an interesting trade-off between the density of edges and the homogeneity of the profiles of nodes in the reported subgraph. Communities should consist of individuals who share similar profiles or individuals who have divergent profiles but are needed to guarantee connectivity, these latter ones being preferably very sparsely connected with the rest of the community members.

with respect to the computational complexity of the $T$-Comm problem, we obtain the following result.

**Proposition 4.1** The $T$-Comm problem cannot be solved optimally in polynomial time even with a single attribute (i.e. $m = 1$) unless $P = NP$.

**Proof 1** (Sketch) We prove the hardness of $T$-Comm with a reduction from the problem of exact cover by 3-sets. The exact cover by 3-sets problem, $X3C$ for short, asks the following question. Given a universe of elements $B = \{b_1, \ldots, b_p\}$, where the number of elements $p$ is a multiple of 3, and a collection $S = \{s_1, \ldots, s_q\}$ of 3-elements subsets of $B$, is there a collection $S' \subseteq S$ such that every element in $B$ occurs in exactly one member of $S'$?

Given an instance of the $X3C$ problem, we construct an instance of the $T$-Comm problem with one-attribute profiles (i.e. $m = 1$) as follows. To each element $b_i$ in $B$ we associate a node, called an “element-node”, with latent profile 0, and to each set $s_j$ in $S$ we associate a node, called a “set-node”, with latent profile 1. Each set-node is connected to the three nodes that represent the elements it contains. In addition, we make all the $q$ set-nodes part of a larger clique $D$ of $o + q$ nodes with latent profiles equal to 1. Also, we make each element-node $b_i$ part of a larger clique $A_i$ containing $o$ nodes with latent profiles 0. Finally, we assume that all the nodes in our construction are seed nodes except for the $q$ set-nodes. This construction is illustrated in Figure 1.

We prove that if the given instance of $X3C$ has an exact 3-set cover, then the minimum tension subgraph solution of $T$-Comm will contain the set-nodes of this exact cover.

The main idea of the proof is based on the following observation. Selecting a large (yet polynomial in $p$) value for $o$, increases the size of all cliques in our construction (i.e. clique $D$ and all $A_i$ cliques). As a result, the conformed profile of all the nodes (including the set-nodes) in $D$ will be very close to 1, and the conformed profiles of all nodes in the $A_i$ cliques will be very close to 0. Thus, the only non-negligible source of tension will be the tension across the edges that connect element-nodes to set-nodes. Each such edge would increase the tension by almost a unit.

Note that since each $b_i$ is a seed node, it has to be connected to the $o$ seed nodes in $D$. This can be achieved only by going through a set-node. Thus, the solution to $X3C$ has to pick a subset of set-nodes that cover all the element-nodes to ensure connectivity. Now, if $x$ set-nodes are included in the subgraph then the tension would be roughly $3x$. Obviously, the solution that minimizes the tension is the solution that picks the smallest number of set-nodes. This is achieved by selecting an exact cover, if such cover exists.


\textbf{Input:} Network \(G = (V, E)\), latent profiles \(X\), seed nodes \(Q \subseteq V\), path length function \(\text{len}()\)

\textbf{Output:} Community nodes \(V'\)

1. \(H \leftarrow \) complete weighted graph over nodeset \(Q\), such that
   - the weight of edge \((i, j)\) is \(\lambda(i, j) = \min_{p \in P_{ij}} \text{len}(p)\)
     with \(P_{ij}\) the set of paths in \(G\) between \(i\) and \(j\)

2. \(K \leftarrow \) minimum spanning tree of \(H\)

3. \(V' \leftarrow \) expand \(K\) by replacing edges by their corresponding shortest path in \(G\)

4. \textbf{return} \(V'\)

Figure 2: The \texttt{CTree} algorithm for solving \textit{T-Comm}.

\subsection{4.2 Algorithms for the \textit{T-Comm} problem}

While the objective of the \textit{T-Comm} problem is to minimize the social tension in the solution graph, \(T(G', X, F)\), obtaining the conformed profiles through the repeated-averaging process is costly; thus, it is not feasible to compute the social tension on a large number of candidate subgraphs. A possible alternative is to compute the conformed profiles by applying the repeated-averaging process on the whole graph once and use these profiles when evaluating the tension in the candidates later on. However, while designing our algorithms, we observed that this is a poor choice. Intuitively, the presence of a node with a latent profile that departs greatly from its neighbors’ might significantly sway their conformed profiles. During the search, this node will likely be removed from the candidate set early on, but its effect would remain.

In order to avoid such effects, but also avoid the repeated computations of social tension we use the following trick in all our algorithms: for a pair of neighboring nodes \(i\) and \(j\) we assign to edge \((i, j) \in E\) weight \(w_{ij} = |x_i - x_j|\). We then use this weight as a way of quantifying the contribution of this edge in the overall social tension. Although \(w_{ij}\) is just a proxy of the edge’s contribution, the trick appears to perform well in practice and leads to significant speedups. Once our algorithms obtain the set of nodes to report, we apply the repeated-averaging process on the induced subgraph in order to evaluate the social tension of the solution.

We propose two approaches for finding good candidate solutions for this problem.

\textbf{Spanning-tree approach.} This approach connects the seed nodes by building a spanning tree between them and is based on the 2-approximation algorithm for the Steiner tree problem \cite{21}. It works as follows: first, it computes the shortest path between every pair of seed nodes. Next, it constructs a complete graph over the seed nodes such that the weight of the edge between two nodes corresponds to their shortest path distance in the original graph. Then, it considers the minimum-spanning tree from this complete graph, e.g. obtained with Prim’s algorithm, and replaces each edge of the spanning tree with the associated original shortest path. The resulting subgraph constitutes the output of our tree-based algorithm. A sketch of this algorithm, which we call \texttt{CTree}, is shown in Figure 2. Note that this approach, searching for the best spanning tree, lacks any control over the induced edges that will be included in the solution. In this sense, it is an optimistic strategy.

We obtain different variants of this algorithm depending on the measure used to evaluate the length of a path. Having experimented with various options, we focus on two variants, where the length of the path is either

\(i\) the number of edges involved, or

\(ii\) the sum of weights of the edges along the path.

In other words, if \(p_{ij} = (i, v_1, v_2, \ldots, v_k, j)\) is a path between \(i\) and \(j\), we have \(\text{len}(p_{ij}) = k + 1\) in the first variant, and \(\text{len}(p_{ij}) = w_{iv_1} + w_{v_1v_2} + \cdots + w_{v_ik}\) in the other. We denote these variants as \texttt{CTree(e)} and \texttt{CTree(s)} respectively.

The main step in our algorithm is the computation of the shortest path between all pairs of seed nodes by running the Dijkstra algorithm from each seed node in turn. The running time of our algorithm is thus \(O(|Q|(|V| + |E| \log |E|))\).

\textbf{Top-down approach.} The second algorithm is a top-down approach which starts with the full graph and iteratively removes nodes until it is no longer possible to continue without disconnecting the seed nodes. The pseudo-code for this algorithm, called \texttt{CPeel}, is given in Figure 3.
Input: Network $G = (V,E)$, latent profiles $X$, seed nodes $Q \subseteq V$, node scoring function $\text{score}()$  
Output: Community nodes $V'$  
1: $V' \leftarrow \emptyset$; $K \leftarrow V$  
2: while $K \neq \emptyset$ do  
3: $v \leftarrow \arg \max_{i \in K} \text{score}(i, V' \cup K)$  
4: $K \leftarrow K \setminus \{v\}$  
5: if $Q$ is not disconnected in $G(V' \cup K)$ then  
6: $K \leftarrow \{i \in K, N_{G(V' \cup K)}(i) \neq \emptyset\}$  
7: else  
8: $V' \leftarrow V' \cup \{v\}$  
9: return $V'$  

Figure 3: The CPeel algorithm for solving $T$-Comm.

Again, we obtain different variants, this time by varying the $\text{score}()$ function for choosing the next node to remove. We selected the following three scores:

(i) The score is a number assigned randomly to each node when initializing the algorithm, which determines the order in which the nodes are peeled. This random variant is denoted as CPeel(r).

(ii) The score is the sum of the weights of the remaining edges adjacent to the node, i.e.

$$\text{score}(i, U) = \sum_{j \in N_{G(U)}(i)} w_{ij}.$$

(iii) The score is the largest weight among the remaining edges adjacent to the node, i.e.

$$\text{score}(i, U) = \max_{j \in N_{G(U)}(i)} w_{ij}.$$

The second and third scores are similar but the former uses a sum where the latter takes the maximum, resulting in variants CPeel(s) and CPeel(m), respectively. Nodes with larger scores are considered first (line 3 in Figure 3). Meanwhile, in all three variants, nodes that get isolated are pruned away (line 5). Rather than favoring good connections, this second approach removes nodes that might generate large costs, and thus follows a pessimistic strategy.

In practice, we can find a minimum connecting tree using the strategy described previously. Then, when we are about to remove a node, we check whether it belongs to the current tree, in which case we need to look for an alternative tree that does not involve this node. Only if such a tree can be found can we safely remove the node. In the worst case, we would have to recompute the spanning tree for each node, resulting in a running time $O(|V| (|V| + |E| \log |E|))$.

5 Problem Variants

In this part, we discuss some variants of the $T$-Comm problem that make different assumptions with respect to the input, while the output is always a connected subgraph with low social tension.

5.1 Finding low-tension teams with chosen skills

In many team-formation problems, there is a universe $S$ of skills that individuals may possess and each individual $i$ is associated with a set of skills $S_i \subseteq S$. Note that the skills are different from the attributes of the profiles: skills are Boolean features associated with individuals (i.e. an individual has the skill or not) and do not change because of social interactions. In contrast, profiles are real-valued and the conformed profiles are subject to change due to social pressure.
Definition of the \textit{T-Team} problem. Given a project requiring a subset \( P \subseteq S \) of skills, the problem is to find a subset of individuals \( V' \subseteq V \) such that for every skill required by the project there is at least one individual in \( V' \) who possesses it. When individuals are organized in a network, it is also required that the subgraph induced by \( V' \) satisfies certain properties that capture the ease of communication within the team, e.g. small diameter, low weight spanning tree, small density, etc. \cite{11,2,15}.

In our setting, we consider a version of the team-formation problem where the goal for \( V' \) is to be such that the required skills are covered, the graph \( G' \) induced by \( V' \) is connected and the social tension in \( G' \) is minimized. This problem can be formally defined as follows.

Problem 2 (\textit{T-Team}) Given a network \( G = (V,E) \) and latent profiles \( X \), as well as a universe of skills \( S \), a set of skills \( S_i \subseteq S \) for every \( i \in V \) and a project requiring a subset of skills \( P \subseteq S \), find \( V' \subseteq V \) such that \( P \subseteq \bigcup_{i \in V'} S_i \), the graph \( G' \) induced by \( V' \) is connected and \( T(G',X,F) \), computed by the repeated averaging model on \( G' \), is minimized.

Algorithms for the \textit{T-Team} problem. This problem is an NP-hard problem – as a generalization of the Set Cover problem. However, we can adapt the algorithms we designed for \( T\text{-Comm} \) to solve this problem as well. If we denote by \( \text{Alg} \) an algorithm for solving \( T\text{-Comm} \), then we can use \( \text{Alg} \) to solve \( T\text{-Team} \) by applying the following two-step procedure: First, we construct an extended graph \( H \) that contains \( G = (V,E) \) as well as one additional node \( s \) for every skill \( s \in S \). Also, every skill node \( s \) is connected to every node \( i \) such that \( s \in S_i \). \( \text{Alg} \) is applied to solves the \( T\text{-Comm} \) problem on this extended graph, using the skill nodes that correspond to skills in \( P \) as the seed nodes.

The result of this first step is a subgraph of \( H \) that contains all the skill nodes required by the project as well as individuals that cover all these skills. One could think that reporting the individuals in this subgraph would provide an adequate solution. However, this is not the case because removing the skill nodes from the subgraph might disconnect the subgraph induced by the individual nodes. For this reason, we need to apply \( \text{Alg} \) a second time, now on the original graph \( G \) and using as seed nodes the individuals participating in the solution reported by the first step.

With this strategy, we can directly reuse the algorithms designed for the \( T\text{-Comm} \) problem to solve this variant. The runtime time of such a two-step procedure will depend on the running time of the algorithm used as a subroutine.

5.2 Finding low-tension communities with fixed size

Another variant of the \( T\text{-Comm} \) problem is one where there are no seed nodes provided as input but the restriction is put on the size of the output community instead.

Using a construction similar to the one we used for proving Proposition \ref{prop:11}, we can prove that this cardinality variant of our problem is again NP-hard, i.e. the problem cannot be solved optimally in polynomial time unless \( P = NP \).

Although this cardinality version of the problem seems natural, our experiments demonstrated that it is rather useless in practice. When the value of \( k \) is rather small, the best strategy is to pick one of the many possible subsets of size \( k \) which are rather sparsely connected. Thus, in the absence of guidance provided in the form of seed members or skills there are a lot of candidate solutions that are practically equivalent and reporting one of them is not necessarily interesting.

From the algorithmic point of view, the cardinality version of the problem cannot be solved using some adaptation of the algorithms discussed in Section \ref{sec:12}. For this problem, we obtained the best performance, i.e. lowest tension solutions, using a greedy algorithm that constructs a connected subgraph of cardinality \( k \) by repeatedly adding a node that minimally increases the tension of the candidate.

6 Experiments

We now turn to the evaluation of our proposed algorithms for the two problem variants, \( T\text{-Comm} \) and \( T\text{-Team} \).
6.1 Experimental setup

In our setting, each dataset consists of a network together with latent profiles for the nodes. Hence, we first introduce the networks, before explaining our approach for obtaining latent profiles. Then, we present the evaluation measures used throughout our experiments.

Networks. Of our collections of networks, two consist of subgraphs extracted from the DBLP co-authorship database, where vertices represent researchers, and edges represent co-authorship relations. Specifically, we extracted the ego-nets of radius 2 for some selected high-profile computer scientists. The resulting ego-nets form the collection denoted as DBLP.E2. We also consider the subgraphs induced by researchers who have published in the ICDM and KDD conferences, respectively, constituting the DBLP.C collection.

Our third collection of networks consists of subgraphs extracted from the Internet Movie Data Base, where vertices represent actors and edges connect actors who played together in at least one movie. Specifically, we constructed actor networks from this database by considering some well-known directors and production companies, such as Francis F. Coppola or the Warner Bros. Entertainment Inc., and extracting the network induced by their movies. The resulting networks form the collection denoted as IMDB.

In our problem, we are looking for connected subgraphs. If the seed nodes in $T$-Comm belong to different components, there is obviously no solution. Hence, in our experiments we consider only the largest component of each of the networks.

The statistics of a sample of the networks from these three collections can be found in Table 2.

Profiles. Besides links, the co-authorship and the co-acting networks contain additional information which we exploit to derive structured profiles, as follows.

In the co-authorship networks, we associate nodes, i.e. researchers, to the title keywords of papers they authored. We then turn these keywords into profiles by considering the eigenvectors associated to the largest eigenvalues of the incidence matrix of keywords to nodes, scaled to the unit interval. Vice-versa, we also consider the incidence matrix of researchers to conferences, i.e. indications of which author published in which conference and derive another set of profiles by computing the eigenvectors of that matrix. Neighboring researchers in these networks are collaborators who have published papers together. Hence, they have keywords in common, published in some of the same conferences, and more generally share similar research interest. Intuitively, they will therefore be assigned similar profile values.

In the co-acting networks, we consider the genres of the movies each actor played in and turn this information into profiles, once again by computing the eigenvectors associated to the largest eigenvalues of the obtained incidence matrices.

The distributions of latent profiles values ($x_i$) over nodes and of squared weights ($w_{ij}^2$) over edges in the DBLP.C ICDM network for latent profiles derived from keywords and from conferences are shown in Figures 4a and 4b respectively.

Evaluation measures. For a solution nodeset $V'$, our main evaluation measure is $T(G(V'), X, F) — T(V')$ for short — the social tension in the subgraph induced by $V'$ with conformed profiles obtained by applying the repeated averaging process over that subgraph.

Two main properties contribute to a solution subgraph having a low social tension (see Equation (1)). On one hand, finding a subgraph with few edges results in fewer terms in the sum. On the other hand, finding a subgraph with low tension edges results in small values in the sum. Thus, we compute two auxiliary values that provide insight into the nature of the solutions obtained. Namely, for a solution $V'$ we compute the number of edges in the solution, $|E(V')|$, and the average of the squared edge weights in the solution, $\overline{w^2}(V') = \frac{1}{|E(V')|} \sum_{(i,j) \in E(V')} w_{ij}^2$.

Solutions obtained for different seed sets are hardly comparable. Thus, to make the evaluation possible, we standardize the measured values before aggregating them. Specifically, we use the number of edges in the minimum spanning tree connecting the seed nodes, $e_b$, as a comparison basis (and lower bound) for the number of edges in the solution subgraphs, and divide $\overline{w^2}(V')$ by the corresponding average over the whole graph.

Given a solution $V'$, we take the following evaluation measures, for which lower values are more desirable:

\[ http://dblp.uni-trier.de/xml/ \]
\[ http://www.imdb.com \]
Figure 4: Distribution of latent profile values, $x_i$, over nodes (top) and of squared weights, $w_{ij}^2$, over edges (bottom) for the DBLP.C ICDM network with various latent profiles.

Figure 5: Results for the $T$-Comm problem on the C.Papadimitriou with single-attribute (left) and ten-attribute (right) latent profiles derived from conferences.

(i) the standardized social tension (main measure)
$$\tau(V') = \frac{T(V')}{2e_b \cdot \overline{w^2}(V)}$$,

(ii) the standardized solution size (auxiliary measure)
$$\varepsilon(V') = \frac{|E(V')|}{e_b}$$, and

(iii) the standardized average edge weight (auxiliary measure)
$$\sigma(V') = \frac{\overline{w^2}(V')}{\overline{w^2}(V)}$$.

6.2 Community search with seed nodes

We now compare the different variants of our proposed algorithms, CTree and CPeel, for solving the $T$-Comm problem (see Section 4).
Generating sets of seed nodes. For each dataset, i.e. each pair of network and latent profiles, we run each algorithm with a number of different sets of seed nodes $Q$. Here we restrict ourselves to sets of seven and four seed nodes, i.e. $|Q| = 7$ and $|Q| = 4$, for the co-authorship networks and the co-acting networks respectively, as representative scenarios for the community-search problem.

As we expect the distance between the nodes in the seed set to have an impact on the behavior of the algorithms, we want to sample seed sets across the range of possible distances and to group them based on this criterion. Thus, we generate a thousand seed sets and look at the maximum pairwise distance within each set. We then select at most 30 seed sets from the 10-33%, 33-66%, and 66-90% percentiles of the distance distribution. The resulting three groups of seed sets are denoted as $D_1$, $D_2$ and $D_3$, from tight seed sets to more dispersed ones. Results in Figure 5 are presented aggregated according to these distance groups.

Single-attribute and multi-attribute profiles. For each latent profile construction scheme, i.e. whether derived from keywords, conferences or movie genres (see Section 6.1), we can either consider the column vectors separately, thereby obtaining several single-attribute profiles, where each node is associated to a single profile value, or consider the entire matrix at once thereby obtaining one multi-attribute profile, where each node is associated to a multi-attribute profile vector.

In our experiments, we take the first four columns of the matrix as four separate single-attribute profiles and the entire matrix as one multi-attribute profile (6 and 21 attributes for the DBLP and IMDB datasets, respectively). Results for the DBLP.E2 C.Papadimitriou for single-attribute (left) and multi-attribute (right) latent profiles derived from conferences are shown in Figure 5. Similarly, results for the IMDB F.F.Coppola network for single-attribute (left) and multi-attribute (right) latent profiles derived from movie genres are shown in Figure 6.

We observe that the algorithms that exploit the profiles of individuals outperform the other two variants in (almost) all cases. Recall that neither $\text{CTree(e)}$ nor $\text{C Peel(r)}$ consider the profiles of individuals. All they can do is minimize the social tension by finding a small subgraph to connect the seed nodes. Our results show that they are indeed quite effective at minimizing the number of edges in the reported solutions, typically achieving the lowest values of $\varepsilon(V')$ (middle column in each block of Figure 5 and 6). However, $\text{CTree(s)}$, $\text{C Peel(m)}$, and $\text{C Peel(s)}$, the profile-aware variants, find solutions with lower edge weights, i.e. achieving lower values for $\sigma(V')$ (right hand side column), at the cost of including extra edges. This gives them an advantage for minimizing social tension, obtaining lower values for $\tau(V')$ (left hand side column).

This pattern clearly holds whether we consider single-attribute or multi-attribute profiles.
Comparison to finding dense communities. In addition to our proposed algorithms, we also obtain communities by applying the GreedyFast algorithm of Sozio and Gionis [19], denoted here as Cocktail, on the networks with the same sets of seed individuals. The aim of this algorithm is to find a subgraph that connects the seeds while maximizing the minimum degree among selected nodes. This algorithm considers neither profiles nor social tension; Nevertheless, we can compute the tension of the community that consists of the nodes returned as a solution and compare it to the communities obtained with our algorithms. Furthermore, Cocktail requires the user to set a value for the upper bound on the size of the solution. We set this value to $k = 200$, as it seems to result in reasonable runtimes while allowing the algorithm to construct a solution in most cases. The cases where the algorithm fails to return a solution are left out from our statistics.

On the DBLP.E2 C.Papadimitriou network the solutions returned by Cocktail are comparable in size to those of our CTree(s) algorithm (middle column in each block of Figure 5). The quality of edges selected in its solutions is on par with CTree(e) and CPeel(r) (right hand side column), which are also oblivious to the profiles of individuals. On the IMDB F.F.Coppola network, the solutions returned by Cocktail tend to be much larger than any of our algorithms (middle column in each block of Figure 6). Expectedly, Cocktail appears poorly suited for the task of finding low-tension communities.

Impact of the profiles distribution. We observed that the CTree(s) variant appears to typically achieve the best performance, followed closely by CPeel(m). Yet, variations can be observed depending on the network structure and on the properties of the opinions distribution.

Thus, we next investigate the impact of the distribution of profile values on the behavior of the algorithms. In order to do so, we construct random latent profiles under different sampling distributions. More specifically, beside the uniform distribution, we either sample from the exponential distribution with parameter $\lambda$, or we set the profile of a fraction $\alpha$ of the nodes to zero, while the rest is sampled uniformly at random, obtaining a thresholded distribution.

For this experiment, we focus on the DBLP.C ICDM network and single-attribute latent profiles. The distributions of latent profiles values ($x_i$) over nodes and of squared weights ($w_{ij}^2$) over edges in this network for various latent profiles are shown in Figure 4. Latent profiles were sampled following each of the three random distributions, in addition to considering latent profiles derived from the keywords and from the conferences respectively. The narrow blue bars show the distributions for the first four single-attribute profiles considered separately, while the broader purple bars show the distributions for the ten-attribute profiles resulting from the combination of the respective single-attribute profiles.

The results obtained for these different profile construction schemes, each time applying the algorithms to the first four single-attribute profiles considered separately are shown in Figure 7.

Looking at random profiles (Figures 7c, 7d and 7e) versus eigenvector profiles (Figures 7a and 7b), the gap between the profile-oblivious variants (i.e. CTree(e) and CPeel(r)) and the profile-aware variants increases. Indeed while the profile-aware variants generally tend to pick edges with lower tension at the cost of involving more edges, as discussed earlier, this tendency is more pronounced when handling eigenvector profiles as compared to the random distributions. This shows that the profile-aware variants are clearly suited to exploit the structure present in eigenvector profiles.

On the other hand, while variations can be observed between the different random distributions, these differences appear to be limited when contrasted with the gap that exists between random and eigenvector profiles. This indicates that the distribution of profile values has a limited impact compared to the presence of structure.

Impact of the network structure. The profile-aware variants typically achieve the best performance. Yet, variations in their behavior can be observed depending for instance on the network structure.

This is clearly evidenced by Figure 7f, showing results on the Fb.1684 network with single-attribute profiles sampled at random from the uniform distribution. Fb.1684 is an ego-net representing friend lists from Facebook and has a high density, $\delta = 18.07$. In such high density network, favoring low tension paths as done by CTree(s) can result in many more edges in the induced subgraph, yielding a significantly higher social tension and actually hurting the performance.

[^3]: http://snap.stanford.edu
Figure 7: Results for the T-Comm problem on the DBLP.C ICDM and Fb.1684 networks with single-attribute latent profiles.
Table 1: Average running times (in seconds) of the algorithms on the DBLP.C ICDM network with latent profiles derived from conferences for solving the T-COMM problem and the T-TEAM problem using keywords as skills (± standard deviation).

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Solve T-COMM</th>
<th>Solve T-TEAM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>latent profiles</td>
<td>latent profiles</td>
</tr>
<tr>
<td>CTree(s)</td>
<td>0.2 (±0.0)</td>
<td>0.7 (±0.3)</td>
</tr>
<tr>
<td>CTree(s)</td>
<td>3.5 (±0.5)</td>
<td>2.8 (±1.2)</td>
</tr>
<tr>
<td>CPeel(s)</td>
<td>147 (±26.6)</td>
<td>337 (±236)</td>
</tr>
<tr>
<td>CPeel(s)</td>
<td>107 (±24.9)</td>
<td>261 (±204)</td>
</tr>
<tr>
<td>CPeel(m)</td>
<td>37.0 (±12.1)</td>
<td>91.5 (±64.5)</td>
</tr>
<tr>
<td>CPeel(r)</td>
<td>0.2 (±0.0)</td>
<td>0.9 (±0.1)</td>
</tr>
<tr>
<td>CPeel(r)</td>
<td>3.3 (±0.3)</td>
<td>3.1 (±0.9)</td>
</tr>
<tr>
<td>CPeel(r)</td>
<td>164 (±28.1)</td>
<td>423 (±156)</td>
</tr>
<tr>
<td>CPeel(r)</td>
<td>134 (±29.7)</td>
<td>351 (±120)</td>
</tr>
<tr>
<td>CPeel(r)</td>
<td>38.9 (±11.2)</td>
<td>108 (±40.9)</td>
</tr>
</tbody>
</table>

Running times. Indicative running times of the different algorithms for the T-COMM problem on the DBLP.C ICDM network with single-attributes and multi-attributes latent profiles are shown in Table 1 (top). We observe that, also as expected, going from single-attribute to multi-attribute profiles hardly has any impact on the running times of our algorithms.

Indicative running times of the algorithms on networks of varying sizes and densities, with multi-attribute profiles are listed in Table 2. As expected, the tree-based algorithms are significantly faster than the top-down algorithms, up to two orders of magnitude, and scale much better.

6.3 Team formation with chosen skills

Next, we turn to the T-TEAM problem, the problem variant where we are given a project requiring a set of skills and asked to find a team which is connected, has low social tension and covers the chosen skills.

Generating sets of skills. For the experiments with the T-TEAM problem we consider the co-authorship networks from the DBLP.C and DBLP.E2 collections. Here, we take the conferences to be the skills associated with individuals. Specifically, each conference represents a skill which a researcher is considered to possess if he has published at least four papers in that particular conference. A project then consists of a subset of conferences. Thus, forming a team to fulfill the project can be thought of as finding a group of researchers that span the sub-areas of computer science represented by these conferences.

For each network in DBLP.C and DBLP.E2, we randomly sampled 80 projects (subsets of 3 to 13 conferences present in that network). Following the two-step procedure described in Section 5, we then used our five algorithm variants to solve the T-TEAM problem for each dataset (a network and its associated keyword-based latent profiles) and each project. Results are presented in Figure 8.

Characteristics of the reported teams. We observe the same general trend as for T-COMM (Section 6.2). That is, profile-aware variants CPeel(s), CPeel(m), and CTree(s) outperform the other variants, although CPeel(r) and CTree(e) report subgraphs with fewer edges. In the DBLP.C networks, however, we see an increased variability in the behavior of the CTree(s) variant and a notable performance of CPeel(m). This latter algorithm now appears to return solutions that have both small size and low edge weights.

Running times. Indicative running times of the different algorithms for this problem are shown in Table 1 (bottom). To allow comparison against the running times for T-COMM reported in the top half of the table, we include
times restricted to runs with projects requiring seven skills, i.e. $|P| = 7$. Note, that while this implies that the number of seed nodes in the first run equals seven, the number of seed nodes in the second run might vary.

While the running times for the two-step procedure are expectedly larger than those of the basic algorithms, the variance is also much greater. In particular, this is because while two runs are often necessary to find a connected solution, it can happen that a single run suffices but such cases cannot be easily identified at the outset.

7 Conclusions

Problems related to community search and team formation have multiple applications in online social media and collaboration networks. In this paper, we add a new modeling angle to these two classes of problems. The key characteristic of our model is that each node of the social network is not only characterized by its connections and its skills, but also by its profile. These profiles, which change dynamically through a conformation process, give rise to social tension in the network. Given this model, we define the $T$-Comm and $T$-Team problems, where the goal is to identify a set of connected individuals that define a low-tension subgraph. Such problems arise both in social network and social media mining as well as in human-resource management, where the goal is to find a set of workers who are not only connected, but also will have a potentially fluid collaboration. The contributions of our paper include the formal definition of these problems and the design of algorithms for solving them effectively in practice. Our experimental results with real data from social and collaboration networks highlight the characteristic behavior of the different algorithms variants and illustrate the effect of network structure and profile distribution on the algorithms’ relative performance. Finally, our work enables future research combining subgraph mining with dynamic processes occurring among the nodes.

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| Network                        | | | | | | | | | |
|-------------------------------|---|---|---|---|---|---|---|---|
|                               | $|V|$| $|E|$| $\delta$| CTree(e)| CTree(s)| CPeel(s)| CPeel(m)| CPeel(r) |
| IMDB Kassovitz                | 96| 262| 2.73| 0.0 (±0.0)| 0.0 (±0.0)| 0.1 (±0.0)| 0.1 (±0.0)| 0.0 (±0.0) |
| IMDB J.Cameron               | 278| 898| 3.23| 0.0 (±0.0)| 0.1 (±0.0)| 0.5 (±0.1)| 0.4 (±0.1)| 0.1 (±0.0) |
| IMDB WarnerBros 1970s        | 225| 1599| 7.11| 0.0 (±0.0)| 0.1 (±0.0)| 0.9 (±0.1)| 0.7 (±0.1)| 0.1 (±0.1) |
| IMDB Forman                  | 395| 1638| 4.15| 0.0 (±0.0)| 0.2 (±0.0)| 1.3 (±0.2)| 1.0 (±0.2)| 0.2 (±0.1) |
| IMDB Eastwood 2000s          | 449| 2307| 5.14| 0.0 (±0.0)| 0.2 (±0.0)| 2.2 (±0.4)| 1.8 (±0.4)| 0.4 (±0.2) |
| IMDB F.F.Coppola              | 678| 6306| 9.30| 0.0 (±0.0)| 0.7 (±0.1)| 8.8 (±1.6)| 6.4 (±1.2)| 1.4 (±0.6) |
| IMDB WarnerBros 2000s        | 1032| 7279| 7.05| 0.1 (±0.0)| 0.7 (±0.0)| 14.4 (±2.5)| 10.1 (±2.2)| 2.4 (±1.1) |
| DBLP.E2 E.DeMaine            | 2234| 7701| 3.45| 0.1 (±0.0)| 2.8 (±0.3)| 75.7 (±12.3)| 60.9 (±13.0)| 19.2 (±6.2) |
| IMDB Paramount 2000s          | 1097| 8469| 7.72| 0.1 (±0.0)| 0.8 (±0.1)| 17.8 (±3.0)| 12.1 (±2.5)| 3.2 (±1.2) |
| DBLP.E2 C.Papadimitriou       | 2613| 9472| 3.62| 0.1 (±0.0)| 3.2 (±0.3)| 114.6 (±20.0)| 91.6 (±22.0)| 31.6 (±9.6) |
| DBLP.C ICDM                  | 2795| 10280| 3.68| 0.2 (±0.0)| 3.3 (±0.3)| 163.9 (±28.1)| 133.9 (±29.7)| 38.9 (±11.2) |
| DBLP.C KDD                   | 2737| 11072| 4.05| 0.2 (±0.0)| 3.5 (±0.2)| 166.8 (±27.8)| 136.7 (±28.1)| 36.0 (±13.0) |
| DBLP: E2 P. Yu               | 4596| 13250| 2.88| 0.2 (±0.0)| 4.4 (±0.3)| 291.3 (±56.3)| 242.3 (±43.1)| 68.6 (±20.5) |
| IMDB Paramount                | 1952| 28992| 14.85| 0.3 (±0.0)| 2.6 (±0.1)| 116.2 (±17.3)| 49.3 (±11.1)| 17.7 (±7.8) |
| IMDB WarnerBros              | 2111| 32166| 15.24| 0.3 (±0.1)| 3.0 (±0.2)| 139.1 (±19.7)| 57.2 (±13.0)| 22.8 (±9.1) |
| IMDB WB+Paramount+Fox         | 5758| 178741| 31.04| 1.4 (±0.2)| 15.4 (±1.0)| 2192.3 (±346.6)| 670.5 (±168.1)| 281.6 (±101.6) |

References


