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SPECIAL TOPICS

Modeling the spectral shape of InGaAlP-based red light-emitting diodes

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We have developed a spectral model for describing the shape of the emission spectrum of InGaAlP-based red light-emitting diodes (LEDs) with quantum-well structure. The model is based on Maxwell-Boltzmann distribution with junction temperature T_j and an experimental two-dimensional joint density of states (DOS). We model the DOS with a sum of two exponentially broadened step functions describing the two lowest sub-bands in semiconductor quantum well. The relative locations $\Delta E_1 = 0$ meV and $\Delta E_2 = 112.7$ meV above the band gap energy $E_g = 1.983$ eV and the ratio 2.13 of the step heights were fixed using an experimental DOS extracted from a LED spectrum measured at known T_j and driving current I . The model can then be fitted to other spectra of other LED samples at varied T_j and I by varying the fitting parameters E_g , T_j , and the broadening of the sub-band edges. The model was tested for three LED samples over $I = 200$ –370 mA and $T_j = 303$ –398 K. Junction temperatures obtained by modeling were compared with calibrated T_j obtained by the forward voltage method. The mean absolute difference was about 2.9 K (0.8%) over the whole region studied and the maximum difference was 8.5 K. The thermal coefficient measured for E_g was -0.509 meV K⁻¹. For the first and second sub-band edges, the thermal broadening coefficients were 18 μ eV K⁻¹ and 37 μ eV K⁻¹, respectively. © 2015 AIP Publishing LLC.

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I. INTRODUCTION

The usable lifetime of a light-emitting diode (LED) is highly dependent on its junction temperature. As the LED degrades, its internal quantum efficiency decreases, and a larger proportion of the electrical driving current is transformed to heat increasing the junction temperature and not to producing photons as intended. High junction temperatures may damage the chip structure, especially when driven at high current densities. Due to this, manufacturers¹ state upper limits for the junction temperature at around 125–135 °C. Also, the polymer materials covering the LEDs may get damaged. Monitoring the junction temperature, especially in the case of high-power LEDs, helps in improving the thermal management and the current control of the LED chips and thus increases their luminous efficacy and lifetime.^{2–4} Measuring the junction temperature directly using, e.g., a thermocouple is challenging due to the small size of the LED chip. A contact measurement would change the thermal properties of the LED component typically lowering the measured junction temperature.

In principle, the junction temperature of a LED can be determined optically from the spectral shape of its emitted light. Such a method is an attractive alternative for continuous monitoring of the junction temperature, especially for the integrated LEDs, whose electrical contacts are inaccessible. The spectrum is a product of the joint density of states (DOS) of electrons and holes, with the highest contribution to the low energy wing of the spectrum, and the Maxwell-

Boltzmann distribution dominating the high energy wing of the spectrum.^{5,6} For bulk LEDs, the ideal three-dimensional DOS follows the free electron model with parabolic shape, whereas for quantum-well (QW) LEDs each sub-band of the ideal two-dimensional DOS produces a step function.^{5,6} Modern commercial high-power LEDs have a QW structure to achieve higher radiative recombination rate.⁷ InGaAlP-based (Indium Gallium Aluminum Phosphide) QW LEDs cover the wavelength region from 560 nm to 700 nm, whereas InGaN-technology (Indium Gallium Nitride) enables manufacturing QW LEDs with peak wavelengths between 400 nm and 590 nm.^{7,8} The peak wavelength of an InGaAlP LED can be tuned by varying the active layer thicknesses⁹ and the strain between the layers.¹⁰ Increasing the indium content shifts the peak wavelength of an InGaN LED towards longer wavelengths⁸ while degrading the total emission efficiency.⁷ The efficiency of InGaAlP LEDs decreases towards shorter wavelengths due to leakage currents and non-radiative carrier recombination outside the active region.

In practice, no experimentally measured QW LED spectra have sharp edges due to the broadening caused by the wavelength dependence of the reflectance of ohmic contact metallization, reabsorption by the active layers, and free-carrier absorption.^{7,9,11} In addition, metal layers used as reflective mirrors distort the LED spectra. Previous studies^{4,12,13} have shown that the low energy wing of the spectrum follows an exponential shape, indicating that the overall broadening of the DOS is exponential. Various methods, for determining the junction temperature optically, have been published previously.^{2,4,14} In the slope method,² a line is fitted to the high energy wing of the logarithmic spectrum. The slope of the fitted line is inversely proportional to the

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actual junction temperature. The model is applicable especially for red LEDs, since their junction temperature can be mathematically derived from the slope with $\sim 2\%$ accuracy. For LEDs with other colors, the junction temperature can be determined by using individual correction factors for each LED. The semi-empirical models^{4,12} successfully model the spectral shape of LEDs with any color. However, the DOS and the Maxwell-Boltzmann distribution are inseparable in the models, and thus only the models with individually calibrated parameters can be used for determining the junction temperature.

We have developed a new model for the determination of the junction temperature of InGaAlP-based QW LEDs, which describes both the high and low energy sides of the LED peak. The model is based on Maxwell-Boltzmann statistics describing the thermal equilibrium in the junction combined with a DOS modeled with two sigmoid functions with fixed relative distances ΔE_1 and ΔE_2 above the band gap energy E_g and the amplitudes A_1 and A_2 . The model DOS includes possible distortion by the absorption coefficients and the light traps of the layers. Free parameters to be fitted are the band gap energy E_g , the junction temperature T_j , and broadening of the sub-band edges $\Delta\sigma_1$ and $\Delta\sigma_2$.

Experimental part of this work is described in Section II. Spectra of three InGaAlP-based red QW LEDs were measured at varied known junction temperatures. The junction temperatures were adjusted with a temperature controlled LED holder¹⁵ which varies the heat sink temperature, and thus the forward voltage of the semiconductor junction. The junction temperature dependence on the forward voltage of each LED was characterized with the forward voltage method.¹⁶ In spectral measurements, this information was used to derive the reference junction temperatures from measured forward voltages. In order to study the effect of the current density on the spectral shape, the experiment was repeated at three different current levels. Section III introduces the developed spectral model. The model is validated in Section IV by comparing the modeled junction temperatures with the true measured junction temperatures of the spectra. In addition, the junction temperatures obtained by our model were compared with the estimated junction temperatures by two commonly used methods in Section V. Finally, conclusions are drawn in Section VI.

II. MEASURING SPECTRA AT KNOWN JUNCTION TEMPERATURES

Five commercial InGaAlP-based red LEDs¹ of type LA CPDP by Osram Opto Semiconductors were studied in this work. These LEDs labeled as R1–R5 were manufactured using company's specific thin film technology, which indicates that the LEDs have quantum-well structure.

The LEDs were characterized for their relationships between the junction temperatures and the forward voltages with the forward voltage method¹⁶ according to the standard¹⁷ JESD50–51. Each LED was separately submerged in a thermostatic bath. A water bath thermostat model Cal 2 by Huber GmbH was used at the temperature range of 303–323 K, and an oil bath thermostat Cal 4 by Huber

GmbH was used at higher temperatures up to 398 K. The LEDs were driven by $2\ \mu\text{s}$ electrical pulses with an adjustable amplitude and $19\ \mu\text{s}$ repetition time interval together with a synchronized forward voltage measurement. The wires were connected in a four-wire measurement configuration to compensate for the wire losses. This calibration setup is automated, and it can be controlled using an ordinary computer.

The $2\ \mu\text{s}$ electrical pulses were short enough not to rise the junction temperatures significantly. We estimate the uncertainty in T_j to be $\pm 0.2\ \text{K}$ within the temperature range of 303–398 K. The uncertainty arising from the voltage meter was $\pm 2\ \text{mV}$, and the uncertainty of the pulsed current source was $\pm 0.5\ \text{mA}$. Measurements were repeated over various current levels from 200 mA to 370 mA. Figure 1(a) shows the dependence between the measured junction temperatures and the forward voltages. The behavior is monotonous and can be approximated with a slope and offset. The slope does not significantly depend on the current level, but the offset term is a function of current. Similar behavior was also reported by Xi and Schubert.¹⁶ Figure 1(b) shows that both the slope and the offset term slightly vary with different LED specimens.

Based on the junction temperature–forward voltage characteristics in Fig. 1(b), the most deviating LEDs R1, R3, and R4 were selected for spectral measurements. The

FIG. 1. Junction temperature T_j as a function of the forward voltage U_f for one LED specimen (R3) at different driving currents (a) and for different LED specimens (R1–R5) at the driving current of 300 mA (b).

emission spectra of the selected LEDs were measured at varied temperatures with the setup shown in Fig. 2. Radiation of the LED being measured was aligned perpendicularly to a BaSO₄-coated integrating sphere to compensate possible angle dependence of the spectral shape. The diameter of the integrating sphere is 300 mm with the surface reflectance of 97%. The integrated spectrum was measured with a Konica Minolta CS 1000 spectroradiometer. The relative spectral throughput of the sphere was afterwards mathematically corrected from the measured LED spectra. To obtain the junction temperature, the forward voltage corresponding to the junction temperature was measured using a four-wire measurement. The temperature was adjusted with a custom-made temperature controlled LED holder.¹⁵ It utilizes liquid cooling with a water recirculator Haake WKL26 by Thermo Electron GmbH and heating with resistors. Both the heating and the cooling are controlled by a PID controller LFI-3551 by Wavelength Electronics. The accuracy of the adjusted junction temperature was estimated to be ± 2 K. Spectra of the LEDs were measured at various junction temperatures between 303 K and 398 K at three DC current levels of 200 mA, 300 mA, and 370 mA.

III. SPECTRAL MODEL

The LED emission spectrum corresponds to the radiative recombination distribution $r(E)$ inside the semiconductor junction defined as⁵

$$r(E) = \frac{1}{\tau_r} \rho(E) f_c (1 - f_v), \quad (1)$$

where τ_r is the electron-hole radiative recombination lifetime, $\rho(E)$ is the joint density of states of electrons and holes, and $f_c(1 - f_v)$ depicts the occupation probability of the energy states in the conduction and valence bands as the LED die strives to achieve thermal equilibrium by the Fermi-Dirac statistics.^{2,5,6} Since LEDs function with low-level excitation,⁵ the occupation probability of the energy states $f_c(1 - f_v)$ can be approximated with the Maxwell-Boltzmann distribution, and thus Eq. (1) becomes

$$r(E) = \frac{1}{\tau_r} \rho(E) \exp\left(\frac{E_{fn} - E_{fp}}{k_B T_j}\right) \exp\left(-\frac{E}{k_B T_j}\right), \quad (2)$$

where E_{fn} is the quasi-Fermi energy of electrons and E_{fp} is the quasi-Fermi energy of holes with the subscripts n and p referring to negative and positive charges, T_j is the junction temperature of the chip, and k_B is the Boltzmann constant.

Semiconductor materials used in the LED die typically allow different radiative band-to-band recombinations, i.e., multiple sub-bands. Ideally for a two-dimensional QW case, each sub-band contributes to the DOS with a step function

$$\rho(E) = \frac{m_r u(E - E_g)}{\pi \hbar^2 d}, \quad (3)$$

where m_r is the reduced mass of the electron-hole pairs, $u(E - E_g)$ is a unit step function with the band gap edge at E_g , and d is the thickness of the quantum well.

In practice, the determination of the DOS is challenging due to the spectral broadening of the energy bands. In addition, in the case of QW LEDs, the locations of the sub-bands depend on the material compounds,^{8,9} the related strain between the layers,¹⁰ and the thickness of the layers.⁹ Previously, Chen *et al.*⁵ have simulated the DOS of InGaAlP-based green-yellow and amber QW LEDs by a single step-function with Gaussian broadening. Later, for example, Karpov¹³ has proposed that the QW DOS seems to follow an exponentially broadened step function also known as the sigmoid function.

Our new approach utilizes empirically determined locations $E_g + \Delta E_1$ and $E_g + \Delta E_2$ and densities A_1 and A_2 for the two lowest energy sub-bands present in a QW LED. To achieve improved agreement in the low-energy side of the spectrum, the broadening of the sub-band edges is modeled as two sigmoid functions

$$\rho_{\text{est}}(E) = \frac{A_1}{1 + \exp\left(-\frac{E - (E_g + \Delta E_1)}{\sigma_1}\right)} + \frac{A_2}{1 + \exp\left(-\frac{E - (E_g + \Delta E_2)}{\sigma_2}\right)}, \quad (4)$$

where σ_1 and σ_2 are the broadening parameters. These exponential tails are known as Urbach tails,¹⁸ and their formation is mostly dependent on the materials used in the junction and the operating junction temperature. Higher junction temperatures in the semiconductor increase phonon-assisted transitions spreading the band gap edges. In addition, semiconductors with alloy compositions or high doping levels lead to band gap fluctuations which spread the Urbach tails.

The model DOS $\rho_{\text{est}}(E)$ in Eq. (4) was fitted to the experimental DOS $\rho(E)$ using least squares fitting on a logarithmic scale with the sum of squared residuals S to be minimized as

$$S = \sum_{i=1}^n (\log \rho(E_i) - \log \rho_{\text{est}}(E_i))^2, \quad (5)$$

where index $i = 1, 2, \dots, n$ runs over the energy values corresponding to the wavelengths of spectral measurements. We

FIG. 2. Spectral measurement system comprises a Konica Minolta CS 1000 spectroradiometer, two Agilent 34401A digital multimeters, an integrating sphere, a custom-made temperature controlled LED holder¹⁵ with its accessories, and a custom-made DC current source.

extracted the experimental DOS $\rho(E)$ from a LED spectrum measured with a driving current of 200 mA in a similar manner as Chen *et al.*⁵ and Wang *et al.*¹⁹ by dividing the spectrum with the Maxwell-Boltzmann distribution. We set $E_{\text{fn}} - E_{\text{fp}}$ as the peak energy of the experimentally measured E_{peak} of 1.988 eV and the T_j as the known junction temperature of 303 K, respectively. These parameter values depend on the studied LED and the ambient conditions of the spectral measurement. In addition to the broadening, the experimental DOS includes spectral distortion originating from the spectral absorption of the layer structure and the optical dome covering the LED chip.²⁰ Our model does not consider these distortions separately.

Figure 3 shows the modeled DOS (solid line) and the experimental DOS (circles). We set $\Delta E_1 = 0$ meV since the lowest sub-band edge locates close to E_g . The optimized DOS parameters for the LED R3 are $A_1/A_2 = 2.13$, $\sigma_1 = 8.86$ meV, $\sigma_2 = 22.68$ meV, $\Delta E_2 = 112.7$ meV, and $E_g = 1.983$ eV. Only the ratio A_1/A_2 is relevant from the point of view of the analysis method. The experimental data were measured with a 1-nm interval, but the circles have been spaced out for clarity. Although we do not know the exact layer structure and alloy mixing ratio, our experimentally determined $\Delta E_2 = 112.7$ meV matches closely with the spin-orbit splitting of 130 meV of $(\text{Al}_{0.7}\text{Ga}_{0.3})_{0.52}\text{In}_{0.48}$ P alloy theoretically modeled by Dawson and Duggan.²¹ One ideal unit step function would produce the spectral full width at half maximum (FWHM) of $0.7 k_B T_j$. Two broadened step functions in the DOS produce larger FWHM with dependence on current. The junction temperature dependence is $1.34 k_B T_j$ with a positive intercept term that varies from 13 meV to 16 meV between 200 mA and 370 mA, increasing systematically towards higher current levels.

By substituting the modeled DOS in Eq. (2) and approximating the semiconductor energy gap as $E_g \approx E_{\text{fn}} - E_{\text{fp}}$ due to the fact that the quasi-fermi levels locate within the conduction and valence band edges,⁵ we obtain

FIG. 3. Model function (solid line) fitted to the DOS (circles) extracted from an experimental LED spectrum measured at 303 K with 200 mA driving current (a) on a linear scale and (b) on a logarithmic scale. The ideal QW DOS (dotted line) has been drawn for comparison.

$$r(E) = \left[\frac{A_1}{1 + \exp\left(-\frac{E - E_g}{\sigma_1 + \Delta\sigma_1}\right)} + \frac{A_2}{1 + \exp\left(-\frac{E - (E_g + \Delta E_2)}{\sigma_2 + \Delta\sigma_2}\right)} \right] \exp\left(-\frac{E - E_g}{k_B T_j}\right), \quad (6)$$

where T_j , E_g , $\Delta\sigma_1$, and $\Delta\sigma_2$ are free parameters for fitting the equation to the measured spectrum, and A_1 , A_2 , σ_1 , σ_2 , and ΔE_2 are fixed parameters. When fitting, the spectra were normalized to the maximum value of 1. Due to the normalization, the model is for spectral shape only.

IV. VALIDATION OF THE MODEL

The model in Eq. (6) was fitted to the experimental LED spectra at varied junction temperatures. The modeling parameters A_1 , A_2 , σ_1 , σ_2 , and ΔE_2 were derived from the spectrum of the LED R3 measured at 200 mA and 303 K. These parameters were then fixed and used when modeling other spectra at varying current levels and junction temperatures. Parameters T_j , E_g , $\Delta\sigma_1$, and $\Delta\sigma_2$ were derived separately for each spectrum. Figure 4 shows the modeled spectra of the LED R3 at 300 mA (solid lines) fitted to the spectra (symbols) measured at varying junction temperatures. The figure legend states the junction temperatures obtained from the fitting and the measured true junction temperatures inside parentheses. As can be seen, the modeled and measured values of T_j agree within a standard deviation of 0.65 K over the temperature range of 95 K. Based on the results, the DOS modeled at 303 K works also for the spectra measured at other temperatures. Figure 4(c) reveals that the spectral model agrees within $\pm 5\%$ with the measurements throughout the spectrum except around 1.89 eV, where relative deviations are larger due to the fine structure in the DOS originating from the localized phonon-assisted radiative recombinations.²¹

Measurements at various temperatures were repeated at three current levels of 200, 300, and 370 mA. Table I presents the relative differences of the modeled junction temperatures from the measured temperatures for three different LEDs R1, R3, and R4. Changing the current does not increase the deviation in the modeled T_j . This is partly because the broadening parameters $\Delta\sigma_1$ and $\Delta\sigma_2$ compensate for the effect of the current. The mean absolute difference in the T_j is 2.9 K ($\sim 0.8\%$), indicating that the new model has at least as good accuracy as the slope method ($\sim 2\%$).²

Figure 5 shows the fitted parameters E_g , $\Delta\sigma_1$, and $\Delta\sigma_2$ as functions of the fitted junction temperature T_j . The peak energy E_{peak} also shown is not a fitting parameter, but can be compared with E_g . The temperature coefficients ($\mu\text{eV K}^{-1}$) of the fitted parameters can be approximated as

$$\alpha = \frac{\Delta x}{\Delta T_j}, \quad (7)$$

FIG. 4. Modeled spectra (solid lines) and measured spectra (symbols) for an InGaAlP-based red QW LED at varied T_j on a logarithmic scale (a) and on a linear scale (b). Figure (c) shows the relative differences. The figure legend states the modeled junction temperatures and the measured junction temperatures inside parentheses.

where x can be replaced with the desired parameter. The temperature coefficients are presented in Table II. The semiconductor band gap E_g and the spectral peak energy E_{peak} suppress equally, on the average with a temperature coefficient of $-0.509 \text{ meV K}^{-1}$. For the first energy band, the broadening coefficient is $18 \mu\text{eV K}^{-1}$, and for the split-off band it is $37 \mu\text{eV K}^{-1}$. Larger temperature coefficient for the split-off band, and the correspondingly shorter relaxation time, is most probably caused by the increase of the carrier-carrier and

TABLE I. Differences between the modeled and the measured junction temperatures. Abbreviation MD refers to the mean absolute difference.

LED	R1			R3			R4		
I/mA	200	300	370	200	300	370	200	300	370
T_j/K	Difference/K								
303	4.0	5.3	1.6	0.0	0.6	4.0	1.6	4.7	8.5
313	2.6	4.1	2.4	-1.3	0.3	2.8	2.1	4.2	6.2
323	2.6	3.2	2.6	-1.4	0.2	3.1	2.1	3.4	4.7
348	1.9	3.3	1.3	-3.0	-0.7	2.2	1.4	3.9	6.0
373	1.3	4.2	3.4	-2.8	1.4	4.4	1.4	4.9	7.6
398	1.9	4.4	—	-2.5	-0.2	1.9	0.3	3.0	3.6
MD	2.4	4.1	2.3	1.8	0.6	3.1	1.5	4.0	6.1

FIG. 5. Fitted parameters E_g , $\Delta\sigma_1$, and $\Delta\sigma_2$ as a function of the fitted junction temperature T_j for LED R3 at three current levels (a) and for three different LEDs R1, R3, and R4 at 300 mA (b). The peak energy E_{peak} is shown for comparison.

TABLE II. Temperature coefficients α of the fitted parameters.

LED	R1			R3			R4		
I/mA	200	300	370	200	300	370	200	300	370
	$\alpha/\mu\text{eV K}^{-1}$								
E_g	-515	-514	-509	-506	-503	-503	-512	-511	-511
$\Delta\sigma_1$	18	19	20	18	18	18	18	18	17
$\Delta\sigma_2$	38	39	39	38	37	37	37	36	36

carrier-phonon scattering probabilities at higher energy levels.²² Carrier-carrier scattering includes electron-electron, electron-hole, and hole-hole scattering processes.¹¹ In addition to the temperature induced broadening, the higher driving current increases the current density leading to increased carrier-carrier scattering induced broadening.²² The effect can be seen in Fig. 5(a) as biased broadening parameters $\Delta\sigma_1$ and $\Delta\sigma_2$ at higher current levels.

V. COMPARISON WITH OTHER METHODS

We evaluated our model for estimating the junction temperatures T_j of LED R3 spectra by comparing the temperature differences with the corresponding differences obtained using the slope method and the peak method. These results are presented in Table III. The slope method has the best accuracy with the mean absolute difference of 1.5 K for this specific LED alone. Our method has almost as good accuracy with the mean absolute difference of 1.8 K. The corresponding value of the peak method is 3.5 K.

The comparison was repeated by modeling the other two LED specimens R1 and R4 at varied temperatures and current levels. Using our method, the mean absolute difference in the junction temperatures for R1 was 2.9 K and for R4 it was 3.9 K. With the slope method, the corresponding values were 10.9 K for R1 and 5.6 K for R4. The mean absolute differences for LEDs R1 and R4 obtained with the peak method were 3.4 K and 12.7 K. Thus, our model works better when comparing different specimens of the LED. The slope method and the peak method do not account for the DOS and thus lose some of their accuracy when the LED specimen is changed.

When we used the slope method, a line with a slope and offset was fitted to the high energy tail of the logarithmic spectrum separated by 90 meV–150 meV from the peak energy E_{peak} . The inverse derivative temperature was then derived from the slope as

$$T_{\text{ID}}(\Delta) = \left[-k_B \frac{\delta \ln r(E)}{\delta E} \Big|_{\Delta} \right]^{-1}, \quad (8)$$

TABLE III. Comparison of our method with other methods using the data of LED R3. The spectrum measured at 303 K and 200 mA was used to optimize parameters both in our method and in the slope method. Two spectra measured at the junction temperatures of 303 K and 313 K at the current of 200 mA were used to fix parameters in the peak method. Abbreviation MD refers to the mean absolute difference.

I/mA T_j/K	Our method			Slope method ²			Peak method ³		
	200	300	370	200	300	370	200	300	370
	Difference/K								
303	0.0	0.6	4.0	0.0	1.2	4.2	0.1	-0.1	2.5
313	-1.3	0.3	2.8	-0.4	-0.1	2.5	0.1	0.3	3.0
323	-1.4	0.2	3.1	0.7	0.5	2.8	1.0	1.4	4.0
348	-3.0	-0.7	2.2	0.5	1.0	2.9	2.0	2.7	4.9
373	-2.8	1.4	4.4	-0.1	2.4	4.0	4.4	6.9	9.2
398	-2.5	-0.2	1.9	-1.9	-1.6	-0.5	6.2	6.7	8.2
MD	1.8	0.6	3.1	0.6	1.1	2.8	2.3	3.0	5.3

where $\Delta = E - E_{\text{peak}}$. The corresponding characteristic temperature T_s that is close to the true junction temperature T_j was then calculated as

$$T_s = \frac{\Delta}{k_B} \left(\sqrt{1 + \frac{2k_B T_{\text{ID}}(\Delta)}{\Delta}} - 1 \right). \quad (9)$$

For InGaAlP LEDs, the characteristic temperature differs from the true junction temperature only because of the offset term. We calibrated this offset to be

$$T_j = T_s + 8.6 \text{ K}, \quad (10)$$

using one characteristic temperature obtained from the spectrum of the LED R3 measured at 303 K and 200 mA. We then used this calibration when determining the junction temperatures for other currents and LEDs.

When we used the peak method, we approximated the relationship between the junction temperature and the peak energy as

$$T_j = -2250 \frac{\text{K}}{\text{eV}} \cdot E_{\text{peak}} + 4788 \text{ K}, \quad (11)$$

from two spectra of the LED R3 measured at 303 K and 313 K at the current level of 200 mA. We used this calibration when determining the junction temperatures from other spectra.

VI. CONCLUSIONS

We have developed a spectral model for describing the shape of the emission spectrum of InGaAlP-based red LEDs with quantum-well structure. We found that the experimental DOS extracted from the spectrum of such LEDs follows approximately a sum of two broadened step functions describing the two lowest energy bands of the semiconductor material. The relative heights and locations of the bands were calibrated by fitting two sigmoid functions to an experimental DOS of a LED with a known junction temperature and driving current. The optimized ratio of the heights between the first and second sub-bands was 2.13 with the energy split-off difference being 112.7 meV. The parameters describing the broadening of the sub-band edges were 8.86 meV and 22.68 meV. These parameters were fixed when modeling the spectra of different LED specimens at unknown junction temperatures by varying the broadening of the sub-band edges, the band gap energy, and the junction temperature.

The model was tested for three InGaAlP-based red LEDs and based on our results, it can be used to derive the junction temperature from a measured spectrum with a mean absolute difference of 2.9 K (0.8%). The semiconductor band gap suppressed with a temperature coefficient of $-0.509 \text{ meV K}^{-1}$, and for the first and second sub-bands, the thermal broadening coefficients were $18 \mu\text{eV K}^{-1}$ and $37 \mu\text{eV K}^{-1}$, respectively. The larger temperature coefficient for the second sub-band is most probably caused by the increase of the carrier-carrier and carrier-phonon scattering probabilities at higher energy levels.

We compared our model with two commonly used methods, the slope method and the peak method, for determining the junction temperatures from LED spectra. With the specific LED specimen, where the model parameters had been derived, the accuracy of the three methods did not deviate significantly. However, when deriving temperatures of two other LED specimens of the same type, our method worked significantly better. It seems that the derived DOS is valid for the specific LED type and enables temperature measurements on other specimens of this LED type.

The estimated parameters for the DOS are valid for InGaAlP LEDs with peak intensities close to those of this experiment, approximately from 610 to 650 nm since the carrier distribution depends on the material composition, the strain between the layers, and the thickness of the layers. Outside that range, the DOS has to be remodeled. In future, one could take these effects into account by creating a three-dimensional model of the chip layer structure to obtain a more detailed description for the density of states.

The method could be extended to other visible range QW LEDs by finding a suitable function to model their DOS. However, for example, InGaN-based blue LED chips have a spectral interference originating from the lattice mismatch and stronger strain between the layer compounds distorting the measurable spectrum. In addition, the spectral absorption of the materials should be taken into account as it increases towards the UV-blue region.

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