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Published in:
Physical Review Letters

DOI:
[10.1103/PhysRevLett.102.165301](https://doi.org/10.1103/PhysRevLett.102.165301)

Published: 24/04/2009

Document Version
Publisher's PDF, also known as Version of record

Please cite the original version:
Koponen, T. K., Pasanen, J., & Törmä, P. (2009). Fermi condensates for dynamic imaging of electromagnetic fields. *Physical Review Letters*, 102(16), 1-4. [165301]. <https://doi.org/10.1103/PhysRevLett.102.165301>

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Fermi Condensates for Dynamic Imaging of Electromagnetic Fields

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(Received 30 September 2008; published 24 April 2009)

Ultracold gases provide micrometer size samples whose sensitivity to external fields may be exploited in sensor applications. Bose-Einstein condensates of atomic gases have been demonstrated to perform excellently as magnetic field sensors in atom chips. Here we propose that condensates of fermions can be used for noninvasive sensing of time-dependent and static magnetic and electric fields, by utilizing the tunable energy gap in the excitation spectrum as a frequency filter. Perturbations by the field create collective excitations and quasiparticles. The latter requires the frequency of the perturbation to exceed the gap. The frequencies of the field may be selectively monitored from the amount of quasiparticles which is measurable, e.g., by rf spectroscopy. We analyze the method by calculating the density-density susceptibility and discuss its sensitivity and spatial resolution.

DOI: 10.1103/PhysRevLett.102.165301

PACS numbers: 67.85.De, 67.85.Lm

The density of an ultracold alkali gas is sensitive to spatially varying magnetic fields due to the Zeeman effect. This is the principle behind magnetic trapping: atoms in low field seeking states are trapped at the minima of the field. On the other hand, it can be used for sensing since magnetic perturbations leave marks on the density of the gas. Such magnetic field imaging has been experimentally demonstrated [1,2] with Bose-Einstein condensates in microtraps [3,4]. As such, they offer a combination of resolution and sensitivity presently unattainable by other methods [2]. The microkelvin sample of atoms is magnetically trapped at about 5–200 μm distance from the room temperature chip surface. Any additional perturbing magnetic field δB will displace the center of the trapping potential and this can be measured by absorption imaging, either *in situ* or after ballistic expansion. The change in the trapping potential V is directly proportional to the additional field, i.e. $\delta V \propto \delta B$. Similarly, electric fields can be sensed using the Stark effect, $\delta V \propto \delta E$ [2]. The principle of using density perturbations of an ultracold atomic gas for sensing can be extended, for instance, to gases that are trapped optically, that are not on atom chips but brought in the vicinity of the sample by other means, or that consist of fermionic atoms instead of bosons. We propose to utilize the pairing gap present in a fermionic superfluid for temporally and spatially resolved imaging of magnetic (or electric) fields. Superfluids of Fermi gases have been recently observed, for reviews see, e.g., [5–7]. Also degenerate Fermi gases in microtraps have been realized [8].

In the proposed method, the Fermi condensate is trapped, magnetically or optically, near the sample of interest. Magnetic fields in the sample, generated for instance by electric currents or even spin, cause density perturbations to the Fermi gas. The perturbations, providing energy and momentum to the gas, lead to collective or quasiparticle excitations. The sensing is initiated by having a high value for the excitation gap Δ . Only frequencies

above 2Δ will be able to break pairs; see Fig. 1. The gap Δ can be controlled with a Feshbach resonance (or by changing the density). Gradually changing the gap allows the isolation of individual frequencies: every time 2Δ crosses a frequency present in the magnetic field, the measured amount of quasiparticle excitations increases abruptly. The quasiparticles can be detected by rf spectroscopy [9–12].

For spatial imaging of static fields, the following variant of the method can be used: The spatial dependence of the static perturbation provides momenta for the gas but no energy. Energy is given by modulating the gas uniformly in space, with a frequency corresponding to the pair breaking. In other words, the static perturbations serve as nucleation centers for quasiparticles under time-periodic modulation.

FIG. 1 (color online). A Fermi condensate is trapped near the sample of interest. A gap opens around the Fermi level of the superfluid and sets the minimum energy of single particle excitations to the value of the order parameter, Δ . Magnetic fields with certain frequency and location in the sample cause density perturbations in the condensate. Only if the frequency exceeds 2Δ , quasiparticles are created, which allows sensing perturbations of different frequencies by tuning the gap.

Within linear response, the density response is $\delta\rho(\mathbf{q}, \omega) = \chi(\mathbf{q}, \omega)\delta V(\mathbf{q}, \omega)$, where we calculate the susceptibility χ with the generalized random phase approximation, following [13]. We solve $\chi(\mathbf{q}, \omega)$ numerically from the most general form given in [13], without making the approximation of weak coupling strength.

We assume a two-component (pseudospins $\uparrow$ and $\downarrow$) Fermi gas in a superfluid state described by the standard Bardeen-Cooper-Schrieffer (BCS) theory, given by the Hamiltonian $H = \sum_{\mathbf{k}} (k^2 - \mu)(c_{\uparrow\mathbf{k}}^\dagger c_{\uparrow\mathbf{k}} + c_{\downarrow\mathbf{k}}^\dagger c_{\downarrow\mathbf{k}}) + \Delta c_{\uparrow\mathbf{k}}^\dagger c_{\downarrow-\mathbf{k}}^\dagger + \Delta c_{\downarrow-\mathbf{k}} c_{\uparrow\mathbf{k}}$. The order parameter Δ and the chemical potential μ are obtained by iteratively solving the self-consistent crossover equations $1 = \frac{2|k_F a|}{\pi} \times \int_0^{k_C} \frac{k^2(1-2n_F(E_k))}{E_k} - 1 dk$ and $1 = \frac{3}{2} \int_0^{k_C} \frac{(k^2 - \mu)}{E_k} [2n_F(E_k) - 1] + 1) k^2 dk$, where $E_k = \sqrt{(k^2 - \mu)^2 + \Delta^2}$ is the BCS quasiparticle dispersion, $n_F(\epsilon) = 1/(1 + e^{\beta\epsilon})$ is the Fermi function, k_C is the cutoff, and $k_F a$ is the dimensionless coupling constant.

The magnetic field is taken to be of the form $B = \sum_i A_i \delta(\omega - \omega_i) \varphi_i(\mathbf{q})$, where A_i is the amplitude. The momentum part, $\varphi_i(\mathbf{q})$, is due to the geometry of the perturbation and we assume it is independent of frequency. Then $\delta\rho(\mathbf{q}, \omega) = \chi(\mathbf{q}, \omega) \varphi(\mathbf{q}) \sum_i A_i \delta(\omega - \omega_i)$. All the relevant information is embedded in $\chi(\mathbf{q}, \omega)$, or rather its imaginary part, the dynamic structure factor: $S(\mathbf{q}, \omega) = -1/\pi \text{Im}\chi(\mathbf{q}, \omega)$. The dynamic structure factor has two parts: Anderson-Bogoliubov (AB) phonon which is a collective mode with frequency below 2Δ , and quasiparticle excitations with frequencies above 2Δ ; see Fig. 2. The results are in qualitative agreement with those in [13–20].

The strong dependence of the qualitative behavior of the dynamic structure factor on momentum, Fig. 2, allows to focus on perturbations of a chosen length scale. The AB-phonon, or the collective modes of a harmonically trapped gas, may be used for detecting spatially large scale perturbations.

FIG. 2. Dynamic structure factor S as a function of frequency, with two momenta, $q = 0.2k_F$ (solid line) and $q = 0.4k_F$ (dashed line). The smaller momentum case shows the AB phonon as a clear peak, and the quasiparticle continuum above $\omega = 2\Delta \approx 0.13$, whereas when q approaches k_F the phonon merges with the quasiparticle continuum. Both curves are scaled to unity for readability.

Here we concentrate on perturbations of small size [$1/q \sim 1/(2k_F)$] which cause a strong quasiparticle response near and above the pair breaking frequencies. For sizes smaller than $1/(2k_F)$ the quasiparticle threshold loses its dependence on Δ and approaches the free particle dispersion $\omega \propto q^2$.

In Fig. 3 we show the response for different values of the gap Δ , in a case where the perturbation contains four different frequencies, with $A_i = 1$ for all. The response is the sum of dynamic structure factors for the four frequencies. The frequencies show up as prominent features in the amount of quasiparticles when the gap is varied. Note that for both momenta ($0.4k_F, 1k_F$), the peaks caused by quasiparticle formation are very similar. Thus for a realistic perturbation geometry, whose Fourier transform contains several momenta, the signal should still be well resolved as long as the perturbation is roughly of the size $1/k_F$.

To detect the quasiparticles, rf pulses transferring atoms in one of the components $\uparrow$ or $\downarrow$ to a third internal state are applied with zero and/or negative detunings (or positive if there are strong Hartree contributions [21]), avoiding detunings which would break pairs. In this way only the quasiparticles produced by the magnetic field perturbation are observed [22]. The quasiparticle response could be calibrated by experiments with known perturbations, e.g., microfabricated current carrying structures. Moreover, the static structure factor of a Fermi gas can be measured by Bragg spectroscopy [19] which is also useful for calibration.

The gases are typically confined by a harmonic potential; therefore, the density and the gap are not uniform throughout the gas. In order to account for the effects caused by the harmonic trapping, we have used the local density approximation (LDA) to average the signal over the trap. Using the local chemical potential $\mu(r) = \mu_0 - \frac{1}{2}m\omega^2 r^2$, where μ_0 is the chemical potential at the center of the trap, we calculate $S(\mu(r))$ at distance r as for a uniform system. The result is given by $S_{\text{LDA}} \propto \int_0^\infty S(\mu(r)) r^2 dr = \frac{1}{2} \left(\frac{2\mu_0}{m\omega^2}\right)^{3/2} \int_{-\infty}^1 \sqrt{1 - \mu} S(\mu) d\mu$, where μ is in the units of μ_0 . Figure 4 shows that the threshold type behavior disappears when the trapping potential has been taken into account, but the frequency of the perturbation is still visible as a maximum. However, we found that such smoothed response allows to isolate only a few, not very closely spaced, frequencies, unlike in the homogeneous case [23].

With tomographic techniques [21,25], the rf spectroscopy can be spatially resolved in three dimensions. In our proposed method, spatially resolved rf spectroscopy could be used for accurate determination (without smoothing by the trap averaging) of the perturbation frequencies and, naturally, for resolving the perturbation spatially (also in the static version of the method). Furthermore, the nonuniform density profile of the trapped gas simultaneously provides experiments with different gap values, which could be utilized when the perturbation is, e.g., a long thin wire.

FIG. 3. Dynamic structure factor as a function of the pairing gap, summed for four frequencies, 0.03, 0.06, 0.09, and 0.12, with two different momenta, $0.4k_F$ (top) and $1k_F$ (bottom). This corresponds to the amount of quasiparticles caused by a perturbing field with these four frequencies. The solid line shows the data with the AB phonon suppressed: for the momenta considered, the dynamic structure factor mainly corresponds to quasiparticle creation as the solid line closely follows the full result. The dashed line shows the result at a finite temperature, $T = 0.01E_F$, which is of the order $0.5T_c$ for the Δ considered.

We used interaction parameters in range $0 > k_F a > -0.66$, resulting in pairing gaps Δ up to $0.1E_F$. All our calculations are at zero temperature except the dashed line in Fig. 3. We used the maximum $k_F a = -0.66$ which is in the BSC limit just in order to be able to do the finite temperature calculation within simple BCS theory. The method itself is by no means limited to weak interactions. The frequencies that can at the present be resolved with the method are limited by the experimentally demonstrated gap values to the order of 10 kHz. At the unitarity limit, the gap becomes proportional to the Fermi energy, thereby higher particle numbers allow higher frequencies. Note that while the AB phonon is a signature of superfluidity, the quasiparticle creation does not require a superfluid. Therefore a gas at temperatures above T_c but having a pseudogap [26] could serve as well but the response would be smoothed due to the lack of sharp features in the density of states [17,26].

Within linear response, the sensitivity is basically given by the time available for the measurement. When estimating the time available for the experiment, one should consider not only the lifetime of the gas which can be easily 100 ms–1 s or even longer, but also the diffusion time of quasiparticles if high spatial resolution is aimed at. According to the measurements in [25], no significant diffusion happened during 5 ms. Therefore we take 10 ms and 1 s as the lower and upper bounds for the time τ available when estimating the sensitivity. The rate for producing excitations with potential energy V applied for duration τ is proportional to $|V/h|^2 \tau$. Assuming the probability needed for a detectable signal (minimum number of excited particles) is at best 0.01 and at worst 1, the minimum potential energy V is between $0.1h/\sqrt{\tau}$ and $h/\sqrt{\tau}$. With the 10 ms–1 s time scales given above the potential energy sensitivity lies between $h \times 0.1 \text{ Hz}^{1/2}$ and $h \times 10 \text{ Hz}^{1/2} \equiv h\sqrt{\nu}$. The potential experienced by a neutral atom in the hyperfine state m_F is $V = g\mu_B m_F B$, where μ_0 is the vacuum permeability, $\mu_B = e\hbar/2m$ is the Bohr magneton, and $g \approx 2$ is the Landé factor. Therefore, assuming that the potential energy sensitivity is $h\sqrt{\nu}$, the magnetic field sensitivity is $h\sqrt{\nu}/2\mu_B m_F \approx 8 \times$

$10^{-12} \text{ T/Hz}^{1/2}$ for $m_F = 9/2$. With the limits for ν given above, the sensitivity is between 10^{-12} and 10^{-9} T .

Even the detection of a *single spin* is in principle possible. The magnitude of the magnetic field due to the spin of an electron is approximately $B(r) = \mu_0 \mu_B g / 4\pi r^3$, where μ_0 is the vacuum permeability, and r is the distance from the electron. Therefore, the required sensitivity to be able to detect a single spin has an upper bound of $\delta V = \frac{\mu_0 \mu_B m_F}{\pi r^3}$. Conversely, assuming the potential energy sensitivity of $h \times 1 \text{ Hz}$ (from our estimated range of 0.1–100 Hz), the maximum distance at which the detection is possible is $(\mu_0 \mu_B m_F / h \pi \text{ Hz})^{1/3} \approx 0.6 \text{ } \mu\text{m}$ for $m_F = 9/2$. The $\sim 0.6 \text{ } \mu\text{m}$ distance is not possible due to noise and heating of the gas for samples at room temperature [3] but may be for those at cryogenic temperatures [27,28] or for ones utilizing photonic band gap materials [29].

FIG. 4. The dynamic structure factor for a single frequency $\omega = 0.03$ and $q = 1k_F$, averaged for a harmonic confinement using LDA, as a function of the gap at the center of the trap Δ_0 . Since the pairing gap is always small at the edges of the trap, there is a finite response already for $\Delta_0 > \omega_0/2$. Decreasing Δ_0 allows quasiparticle creation in larger areas in the trap, increasing the response, but once the quasiparticles can be formed also at the center of the trap, such growth stops. This leads to a maximum of the response at $\Delta_0 = \omega_0/2$, shown by the vertical dashed line.

Using Feshbach resonances, the gas of Fermions can be also converted into a Bose-Einstein condensate of molecules [30,31]. Thereby, a setup used for the Fermi condensate sensor proposed here could be easily turned into one that functions as the Bose-Einstein condensate sensor [1,2] as well, only with double mass of the particles which increases the sensitivity. At the present, several other systems than BCS-type superfluids are being pursued with ultracold gases: the proposed method could be extended to other gapped systems and thereby to new frequency ranges. This could also allow higher spatial resolution: Quite naturally, the spatial resolution of a response that involves interparticle correlations is given by the interparticle distance $1/k_F$, which for typical trapped Fermi gases is about $1/(2k_F) \sim 0.5 \mu\text{m}$ as discussed above. It can, however, be smaller in optical lattices [5,32] and, especially, the self-assembled crystals of ultracold polar molecules proposed in [33] could offer interparticle distances and thus resolutions in the nanometer scale.

In summary, we have proposed to use an ultracold Fermi gas in a gapped state as a sensor for time-dependent and static magnetic fields. The tunable gap works as a frequency filter, and the locations of the perturbation act as nucleation centers for quasiparticles measurable with rf spectroscopy.

We thank J. Hecker Denschlag for useful discussions. This work was supported by the National Graduate School in Materials Physics, Ellen and Artturi Nyyssönen foundation, Academy of Finland (Project No. 213362, No. 217045, No. 217041, No. 217043) and conducted as a part of a EURYI scheme grant [34].

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