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Induced Interactions and the Superfluid Transition Temperature in a Three-Component Fermi Gas

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We study many-body contributions to the effective interaction between fermions in a three-component Fermi mixture. We find that effective interactions induced by the third component can lead to a phase diagram different from that predicted if interactions with the third component are neglected. As a result, in a confining potential a superfluid shell structure can arise even for equal populations of the components. We also find a critical temperature for the BCS transition in a ^6Li mixture which can deviate strongly from the one in a weakly interacting two-component system.

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By using Feshbach resonances to change the effective interaction between ultracold atoms several groups have probed the crossover from the Bardeen-Cooper-Schrieffer (BCS) superfluid to a Bose-Einstein condensate of molecules [1–10]. Studies of the crossover have provided important insights into fermionic superfluids around the unitarity limit of strong interactions. Importantly, it has become experimentally feasible to study also more complicated mixtures than Fermi gases with two different atomic internal states. Bose-Fermi mixtures [11,12] and Bose-Einstein condensates with many components have been created using many different setups [13,14]. Also, heteronuclear Fermi-Fermi mixtures [15,16] and even heteronuclear Fermi-Fermi-Bose mixtures [17] have been recently demonstrated. In yet another breakthrough a three-component Fermi mixture of atoms in the three lowest hyperfine states of ^6Li [18,19] has also been demonstrated. Such multicomponent systems have some intriguing similarities with quark matter counterparts where color superconductivity may appear [20].

The purpose of this Letter is to explore how the induced interactions due to the third component modify the expected behavior of three-component mixtures. This is important because, depending on parameters, the many-body effects can change the effective interaction between atoms substantially and on occasion even change the relative magnitudes of couplings between different components. Such changes imply that phase diagrams predicted using only two-body scattering properties can be incorrect. Also, even when the corrections to the effective interactions are weak, they can cause large changes to the critical temperature for the BCS transition. Indeed, for a two-component system Gorkov and Melik-Barkhudarov (GM) showed [21] that the perturbative correction to the effective interaction can reduce the critical temperature by a constant factor of $(4e)^{1/3} \approx 2.22$ in the weak-coupling limit. Also in spin-density imbalanced systems such corrections have been

shown to have a considerable effect [22]. Here we will analyze the effects of analogous corrections on the three-component system and find important changes to the GM result. As an interesting consequence of these many-body corrections we predict in a spatially varying confining potential (typically harmonic trap) the appearance of superfluid shell structures even in the absence of population imbalance (polarization) of the components. These shell structures are due to *many-body effects only* and therefore fundamentally different from earlier predictions of shell structures due to population, mass, or trapping potential imbalance [23,24]. We also point out that many-body effects due to the third component provide a new way to tune the effective interaction between the two other fermions and that this contribution can dominate over the usual GM contribution.

Intiguing results have been found experimentally for the critical temperature of iron-based multiband superconductors [25] and degenerate three-component Fermi gases have been studied theoretically in a lattice [26,27]. Furthermore, pairing [23,28,29], stability [30], and

FIG. 1. Diagrams of second order in the interactions for the induced interaction between components 1 and 2. Solid lines represent atoms and the dashed lines interactions between them, and numbers indicate the fermionic component. The interactions are taken to be antisymmetrized with respect to interchange of spins and consequently interactions between atoms of the same species are absent.

breached pairing [31] have recently been studied in a three-component fermionic mixtures. However, these theoretical approaches did not consider situations directly relevant to ongoing experiments and also did not study how the many-body effects due to the presence of the third component influence the properties of the other two components. Some aspects of the many flavor problem were discussed by Heiselberg *et al.* [32].

The relevant second order diagrams which give rise to induced interactions between fermions of type 1 and 2 are shown in Fig. 1. In these diagrams the arrows are the component propagators and the dashed line is a contact interaction with strength $U_{\alpha\beta}$ between components labeled by $\alpha, \beta \in \{1, 2, 3\}$ ($\alpha \neq \beta$). These couplings can be expressed in terms of the scattering lengths $a_{\alpha\beta}$ through $U_{\alpha\beta} = 2\pi\hbar^2 a_{\alpha\beta}/m_{\alpha\beta}$, where $m_{\alpha\beta} = (1/m_\alpha + 1/m_\beta)^{-1}$ is the reduced mass. Of the diagrams shown, the diagram (a) is relevant in the case of a two-component system with a contact interaction between unlike fermions [33] and for equal mass fermions gives rise to the GM correction mentioned earlier. In a three-component system the diagram (b) describes the induced effect of the third component. Similar loop diagrams with the mediating fermion in component 1 or 2 are forbidden in the s -wave scattering channel for symmetry reasons. More formally the diagram (a) indicates the induced interaction

$$V^G(\mathbf{p}, \mathbf{p}') = -U_{12}^2 \sum_{\mathbf{k}} \frac{f[\xi_1(\mathbf{k} + \mathbf{q}/2)] - f[\xi_2(\mathbf{k} - \mathbf{q}/2)]}{\xi_1(\mathbf{k} + \mathbf{q}/2) - \xi_2(\mathbf{k} - \mathbf{q}/2)},$$

where $\mathbf{q} = \mathbf{p} + \mathbf{p}'$ and (b) describes the induced interaction

$$V^{3c}(\mathbf{p}, \mathbf{p}') = U_{13}U_{23} \sum_{\mathbf{k}} \frac{f[\xi_3(\mathbf{k} + \mathbf{q}'/2)] - f[\xi_3(\mathbf{k} - \mathbf{q}'/2)]}{\xi_3(\mathbf{k} + \mathbf{q}'/2) - \xi_3(\mathbf{k} - \mathbf{q}'/2)},$$

with $\mathbf{q}' = \mathbf{p} - \mathbf{p}'$. In these formulas $\xi_\alpha(\mathbf{k}) = \hbar^2 k^2/2m_\alpha - \mu_\alpha$ are the free atom dispersion relations and $f(\epsilon)$ is the Fermi distribution.

In the weak-coupling limit, the scattering processes around the Fermi surfaces dominate and to find the effective coupling the induced interactions are averaged over the Fermi surfaces. In this way we find that the effective coupling between fermions of types 1 and 2 becomes

$$U_{12}^{\text{eff}} = \frac{4\pi\hbar^2 a_{12}}{m_1} \left\{ 1 + \frac{2}{\pi} \left[a_{12} k_{F,1} F(1) - \frac{a_{13} a_{23}}{a_{12}} \right. \right. \\ \left. \left. \times \frac{(m_3 + m_1)^2}{4m_1 m_3} \left(\frac{k_{F,3}^3}{k_{F,1}^2} \right) F\left(\frac{k_{F,1}}{k_{F,3}}\right) \right] \right\}, \quad (1)$$

where $k_{F,\alpha}$ is the Fermi wave vector for the component α and we have assumed that fermions 1 and 2 both have a mass m_1 while the third component has a mass m_3 . The function $F(y)$ is given by the integral

$$F(y) = \int_0^y dw 2w \left[\frac{1}{2} + \frac{(1-w^2)}{4w} \ln \left(\frac{1+w}{1-w} \right) \right], \quad (2)$$

whose analytical solution is given by

$$F(y) = \frac{1}{6} \left[-y(y^2 - 3) \log \left| \frac{y+1}{y-1} \right| + 2(y^2 + \log|y^2 - 1|) \right].$$

The effective interactions in other channels can be found in the same way.

In Eq. (1) the first term describes the two-body scattering in the absence of Fermi seas, the second term gives rise to the GM correction, and the third term describes the effect of the interactions with the third component and its Fermi sea. The correction due to the second term always suppresses the critical temperature for the BCS transition. However, the last term is proportional to the product $a_{23}a_{13}$ and can have either sign. Therefore, the presence of the third component can either suppress or enhance the critical temperature.

Since three-component systems have been demonstrated using ^6Li atoms, let us now investigate these many-body effects using the coupled channel scattering data for ^6Li [34]. In Fig. 2 we show the scattering lengths between different components of the ^6Li mixture ($|1\rangle, |2\rangle, |3\rangle$ refer to the states $|F, m_F\rangle = |1/2, 1/2\rangle, |1/2, -1/2\rangle, |3/2, -3/2\rangle$, respectively). It can be seen that, in the absence of many-body corrections, the 1-3 channel has the most negative scattering length for weaker magnetic fields, while at magnetic fields above the Feshbach resonances the 1-2 channel eventually becomes dominant. In the simple mean-field picture one would infer that these channels are also the ones with highest critical temperatures. However, when we include induced interactions, density dependencies appear in the effective coupling strengths and change the simple picture in which the third component is neglected.

Let us first explore the case where all ^6Li components have the same density. In Fig. 3 we show the dominant coupling channel in the magnetic field-density plane. It can

FIG. 2 (color online). Scattering lengths in units of the Bohr radius between ^6Li atoms as a function of magnetic field. (The figure is taken from Ref. [34].)

FIG. 3 (color online). The dominant interaction channel including induced interactions for an equal-density ^6Li mixture. The solid black lines indicate phase boundaries. Below the (red) dashed line both $|k_F a_{13}|$ and $|k_F^2 a_{23} a_{12}|$ are less than one, while below the (red) dot-dashed line both $|k_F a_{12}|$ and $|k_F^2 a_{23} a_{13}|$ are less than one. In (a) we show the interesting regions at low magnetic fields and in (b) the behavior at higher magnetic fields.

be seen that below the Feshbach resonance the 1-3 channel dominates for smaller densities, but for densities higher than about $2 \times 10^{14}/\text{cm}^3$ there is a possibility that the 1-2 channel becomes dominant. At higher magnetic fields we find the possibility of a dominant 1-3 coupling in the region where the scattering lengths would predict the 1-2 channel. In experiments the atoms are trapped, and applying a local density approximation (which has been sufficient to describe many experiments) with our results suggests the interesting possibility of different superfluid phases appearing in different parts of the cloud even in the absence of polarization or unequal trapping potentials or masses [23]. This possibility is a many-body effect caused by the induced interactions only; for a balanced system at zero temperature, simple mean-field theory would not predict the shell structures that arise from the density dependence of the GM correction as shown here.

It is important to investigate what these results imply for the critical temperature. In attractive dilute Fermi gases, the critical temperature for the BCS transition in the weak-coupling limit is $k_B T_c / \epsilon_F \propto \exp[-\pi / (2k_F |a^{\text{eff}}|)]$ where $a^{\text{eff}} = U^{\text{eff}} N(\epsilon_F) \pi / (2k_F)$ is the effective scattering length and $N(\epsilon_F)$ is the density of states at the Fermi level. We now use this result to estimate the many-body correction to T_c in a three-component ^6Li system. For simplicity we use the above functional dependence in all regions where the coupling is attractive, but indicate the regions where $|k_F a| > 1$ in the figures. In those regions the weak-coupling formula is only suggestive. In Fig. 4 we show the fraction $T_c / T_{c,0}$ for the equal-density ^6Li mixture ($T_{c,0}$ is the critical temperature in the absence of induced interactions). As can be seen, the correction to T_c is often very different from the $1/2.22 \approx 0.45$ GM result and shows a nontrivial behavior as a function of the magnetic field and density due to complicated variation of the scattering lengths. This also makes it possible that the pairing channel is changed due to many-body effects. At high fields above the Feshbach resonances it is possible that the critical

FIG. 4 (color online). The highest fraction $T_c / T_{c,0}$ of the critical temperatures with and without induced effects for the equal-density ^6Li mixture. The fraction is only computed in the regions where the dominant effective interaction is attractive and set to zero elsewhere. Below the yellow line $|k_F a^{\text{eff}}| < 1$. Regions where the optimal pairing channel is changed are visible as kinks in the fraction $T_c / T_{c,0}$. The inset shows a close-up into the region of high magnetic fields and low densities.

temperature is enhanced since the effective scattering length there becomes more negative due to induced interactions. However, this happens in the region of stronger interactions where our results are not necessarily quantitatively accurate.

In Fig. 5 we demonstrate another possibility for changing the critical temperature: the use of density imbalance. In Fig. 5(a) we show an example of how T_c in the 1-3 channel is changed as the density of the component 2 is varied. It is again clear that the result deviates substantially from the GM prediction, but T_c is nevertheless suppressed by the component not involved in pairing. In Fig. 5(b) we show a similar result in the 1-2 channel which dominates at

FIG. 5 (color online). The fraction $T_c / T_{c,0}$ of the critical temperatures with and without induced effects for the ^6Li mixture. In (a) we show an example of how the critical temperature is changed in the 1-3 channel when the density of the component 2 is varied while in (b) we show the same for the 1-2 channel when the density of the component 3 is varied. We only focus on those magnetic fields where the pairing channel in question is the dominant one.

higher magnetic fields. Because of the different behavior of the scattering lengths, here the induced interactions can act to enhance T_c above the value predicted by the usual mean-field theory.

Finally, since heteronuclear fermionic mixtures are experimentally feasible [15,17] let us briefly discuss what our results imply in that case. A mass imbalance can be realized if the third component is a different isotope, but also if the third component experiences an optical lattice which changes its effective mass. In the latter case, for the formulas derived here to be valid, the filling fraction of all the components should be much less than one. For higher filling fractions the Fermi surface is no longer spherical and the result would change considerably [35]. We focus on a scenario with equal masses for atoms of type 1 and 2, since it is known that unequal mass of the interacting fermions suppresses the critical temperature [23,33] and for this reason the equal mass superfluidity appears more generic. The effective interaction between 1 and 2 is given by Eq. (2). Note that since $(m_3 + m_1)^2/4m_1m_3 > 1$, induced interactions become relatively stronger in an unequal mass mixture and mass imbalance can be used to enhance the role of many-body corrections. If $b = (a_{12}^2/|a_{13}a_{23}|)(k_{F,1}/k_{F,3})^3 F(1)/F(k_{F,1}/k_{F,3})$ and $b > 1$, the contribution of the third component to the induced interaction becomes larger than the GM contribution when $m_3/m_1 > (2b - 1) + 2\sqrt{b(b - 1)}$ or when $m_3/m_1 < (2b - 1) - 2\sqrt{b(b - 1)}$. If $b < 1$ the contribution due to the third component is dominant for all mass ratios. However, for mass ratios larger than about 13.6 other physics can come into play since then weakly bound diatomic molecules might become collisionally unstable [36].

In this Letter we have explored the induced interactions and their role in the BCS pairing in a three-component Fermi mixture. We found striking differences from physics ignoring these many-body corrections. In particular, we found that when the induced interactions are taken into account, the phase diagram can change drastically, that shell structures in traps can appear even without number, mass, or trap imbalance, and that the critical temperature for the BCS transition is strongly dependent on the induced interactions in the three-component systems.

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