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Korolyuk, Anna; Massel, Francesco; Törmä, Päivi

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Probing the Fulde-Ferrell-Larkin-Ovchinnikov Phase by Double Occupancy Modulation Spectroscopy

Anna Korolyuk, Francesco Massel, and Päivi Törmä*

Department of Applied Physics, P.O. Box 15100, 00076 Aalto University, Finland

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In this Letter we consider a spin-imbalanced two-component attractive Fermi gas loaded in a 1D optical lattice in the presence of an harmonic confining potential. We propose that the observation of the change in the double occupancy with respect to a lattice depth modulation can provide clear evidence of the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase. Simulating the time evolution of the system, we can characterize the double occupancy spectrum for different initial conditions. In particular, we numerically observe a striking narrowing of the width of the spectrum for increasing imbalance. Using Bethe-ansatz equations in the strongly interacting limit, we show that the width relates directly to the FFLO wave vector q .

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Ultracold atoms trapped in optical lattices have become an important tool to mimic strongly correlated condensed matter systems, leading to the possibility to explore regimes unattainable within the traditional solid-state framework. Recently, a considerable experimental effort [1,2] has been devoted to the analysis of two-component spin-imbalanced Fermi gases. Theoretical investigations [3–12] have revealed that, in the characterization of 1D spin-imbalanced Fermi gases, a major role is played by the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state [13]. This state has been under investigation for half a century, and it is considered of capital relevance for the understanding of fermionic superfluidity both in condensed matter and in particle physics [14].

The key idea in the definition of this phase is that the pairs carry a finite center-of-mass momentum given by the FFLO wave vector $q = k_{F\uparrow} - k_{F\downarrow}$ (where $k_{F\uparrow}$ and $k_{F\downarrow}$ are the Fermi vectors of the majority and minority component, respectively). Pairing with a single q constitutes an idealization. In fact, in 1D systems where the FFLO state has been predicted to be greatly stabilized compared to higher dimensions, the FFLO wave vector corresponds to a maximum in the momentum distribution of the pairs.

In a solid-state context the FFLO phase has been investigated in heavy fermions systems [15], with techniques ranging from heat capacity to nuclear magnetic resonance measurements, even though conclusive evidence of its existence is still missing. In ultracold gases, even though it has been suggested that its presence can be detected through various measurements such as noise correlation [7,9,16], radio-frequency spectroscopy [10], collective modes analysis [12], and local density profile measurement [11], no direct experimental evidence of the FFLO phase has been found in these systems. Nevertheless, in the experiment conducted at Rice University [2], the density profile of each component has been measured, exhibiting a behavior compatible with the theoretical analyzes focusing on the characterization of the FFLO phase. Compared to

previous theoretical suggestions, our approach relies on a simple experimental setup and at the same time provides unequivocal signature of the FFLO phase.

In particular, we propose that a clear experimental evidence of the FFLO phase in a 1D optical lattice can be provided by the measurement of the double occupancy (DO)—i.e., the number of sites populated by two atoms—with respect to a periodic lattice modulation of the initial state at different frequencies (DO modulation spectrum, as proposed in [17]). This technique has been employed to observe the appearance of the Mott gap in a repulsive two-component Fermi gas [18] and it has been suggested as a possible tool to detect the antiferromagnetic phase in such systems [19,20]. Performing the same kind of experiment for an attractive gas is well within reach of the current experimental techniques and, as we will show here, can provide clear evidence of the FFLO phase through a reduction of the width in the DO spectrum directly related to the FFLO vector q . The underlying physics is simple: the existence of a collective momentum q restricts the available momentum states of the excitations, thus narrowing the spectrum.

In the presence of a parabolic confining potential, we assume that the system is described by the Hubbard Hamiltonian

$$H = H_J + H_U + \sum_i^L V_i(n_{i\uparrow} + n_{i\downarrow}), \quad (1)$$

where $H_J = -J \sum_{i,\sigma=\uparrow\downarrow}^L c_{i\sigma}^\dagger c_{i+1\sigma} + \text{H.c.}$, $H_U = -U \sum_i^L n_{i\uparrow} n_{i\downarrow}$, $V_i = V(i - \frac{L}{2})^2$, J is the hopping amplitude, $-U$ is the on-site attractive interaction, V the global confining potential, and the polarization P , defined as $P = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$, with $N_\sigma = \sum_i n_{i\sigma}$.

The lattice depth modulation proposed in [17,18] can be modeled by the modulation of the hopping amplitude $J(t) = J + \delta J \cos(\omega t)$ [19,20]. Since we are interested in excitations which lead to pair breaking, it is natural to focus on modulation frequencies close to the energy U ,

related to pairing; i.e., we concentrate on the transition between the first and the second Hubbard band. Intuitively, the process we are interested in might be understood as the transition between the ground state and a state where a pair has been broken by the hopping modulation.

Our approach to the problem is twofold. We first perform numerical simulations of the ground state and of the dynamical evolution of the system. We then move to the analysis of the results in terms of Bethe-ansatz (BA), in the limit $U/J \rightarrow \infty$. The numerical simulations, both for the ground-state calculation and for the time evolution, are performed with the aid of a time-evolving block decimation (TEBD) code [21], which can be regarded as a quasiexact method for the analysis of 1D quantum systems. In the spin-polarized case, for the range of parameters that we have considered ($U = -10$, $J = 1$, $L = 80$, $N_\uparrow + N_\downarrow = 40$, $P \geq 0.04$, $V = 0.005$) the ground state consists of a central region where $\langle n_{i\uparrow} \rangle > \langle n_{i\downarrow} \rangle > 0$ and an outer, fully polarized region ($\langle n_{i\uparrow} \rangle > 0$, $\langle n_{i\downarrow} \rangle = 0$). Moreover, in the central region of the trap $\langle n_{i\uparrow} \rangle \simeq \langle n_{i\downarrow} \rangle$ implying that due to the strong interaction considered here all minority particles $n_{i\downarrow}$ are paired (Fig. 1). We have considered a system large enough ($L = 80$) to exclude the presence of any particles on the edges, hence avoiding finite-size effects.

The periodic spatial dependence of $n_{i\uparrow} - n_{i\downarrow}$ suggests the presence of the FFLO state [22]. In order to give a quantitative estimate, we extract the value of the FFLO wave vector q from the pair correlation function $\langle c_{j\uparrow}^\dagger c_{j\downarrow}^\dagger c_{i\uparrow} c_{i\downarrow} \rangle$ and its Fourier transform $n_{\text{pair}}(k)$ (Fig. 1). The maximum of $n_{\text{pair}}(k)$ is found to be, approximately due to the harmonic confinement [8], $q \simeq \pi/(L(N_\uparrow - N_\downarrow)) = \pi\rho P$, where $\rho = (N_\uparrow + N_\downarrow)/L$.

We first examine the properties of the system for $P = 0$. After calculating the ground state for the Hamiltonian, we turn on the modulation of the hopping amplitude. At each time step we calculate the total DO spectrum $D_\omega(t) = \sum_{i=1}^L \langle n_{i\uparrow} n_{i\downarrow} \rangle$, where the modulation frequency ω is centered around the value of the interaction strength $|U|$.

In Fig. 2, the DO spectrum $\bar{D}_\omega(t)|_{t=50}$ is plotted for frequencies $\omega/|U| \in [0.5; 1.8]$, where $\bar{D}_\omega(t)|_{t=50}$ is the

FIG. 1 (color online). Left panel: particle and pair densities and the difference $n_{i\uparrow} - n_{i\downarrow}$ for the polarization $P = 0.5$ ($N_\uparrow = 30$, $N_\downarrow = 10$). Right panel: pair momentum distribution $n(k)$ for the polarizations $P = 0$, $P = 0.04$, $P = 0.5$. The maxima are at $q = 0$, $\simeq \pi/10$, $\simeq \pi/2$, respectively.

average between the local maxima and minima in the small-time dynamics of the DO. The deviations of the extrema from the averaged DO are approximately 1%, comparable to the experimental accuracy given in, e.g., [18]. The spectrum shows a bandlike structure with $\omega_{\min}/|U| \simeq 0.68$ and $\omega_{\max}/|U| \simeq 1.5$. As we will later show, the band in Fig. 2 can be explained in terms of the excited states within the second Hubbard band.

We will now turn our attention to the numerical results for the double occupancy spectrum of the spin-polarized case (Fig. 2). The first important aspect is the decrease in the reduction of the DO as the number of paired particles is decreased (see Fig. 2: reduction of 0.34 for $P = 0$, 0.24 for $P = 0.04$, and 0.04 for $P = 0.5$). This feature can be easily understood considering that the lattice modulation at frequencies close to U affects the paired component of the gas only and hence the number of broken pairs is reduced accordingly. However, the most prominent feature of the spectrum in the spin-polarized case is the reduction of the width of the band. In particular, while the position of its upper limit is independent of the polarization, the lower

FIG. 2 (color online). Top: double occupancy $\bar{D}_\omega(t)$ as function of frequency ω for times $t = 10, 15, 20$ for the balanced case $N_\uparrow = N_\downarrow = 20$. Bottom. Normalized double occupancy $\bar{D}_\omega(t)/N_\downarrow$ as a function of frequency ω at $t = 10$ in a trap for three cases with polarizations $P = 0, 0.04, 0.5$ corresponding to $N_\uparrow = 20, 22, 10$. The vertical lines correspond to E_{high} (black line), and to E_{low} for different polarizations. As mentioned in the text, the depth of the local minimum for $\bar{D}_\omega(t)$ decreases for a decreasing number of pairs. Specifically it is located at 19.66, 21.76, 9.96 corresponding to a depth of 0.34, 0.24, and 0.04 for $N_\downarrow = 20, 22, 10$, respectively.

limit depends strongly on P . The main goal of our analysis is to show that the width of the band $\Delta\omega$ can be described by the relation $\frac{\Delta\omega}{U} = \frac{4J}{U}(1 + \cos q)$, where q is the FFLO wave vector, calculated from the ground-state value of $n_{\text{pair}}(k)$. We thus claim that the determination of the DO modulation spectrum in an imbalanced gas allows the direct determination of the q vector characteristic of the FFLO phase.

The physical situation depicted here can be analyzed in terms of the mapping between the attractive and the repulsive Hubbard model. Changing $U \rightarrow -U$, the single-site basis states can be mapped according to the scheme $|\uparrow\downarrow\rangle \leftrightarrow |\uparrow\rangle$, $|\emptyset\rangle \leftrightarrow |\downarrow\rangle$. For repulsive interaction, the hopping modulation results in an increase of the DO, since, in that case, the modulation causes the formation of a doubly occupied and an empty site [19,20]. In the case analyzed here the opposite process takes place: a doubly occupied or empty site “pair” is broken. However, as a consequence of the mapping, the bandwidth for the two processes is the same.

In order to explain the results obtained we will consider here the BA solution for the open-boundary conditions (OBC) Fermi-Hubbard model in the limit $U/J \rightarrow \infty$. In the case $U > 0$, it is possible to prove that the excitations of the system can be described in terms of $N = N_\uparrow + N_\downarrow$ spinless fermions with energy and momenta given, respectively, by

$$E = -2J \sum_{j=1}^N \cos k_j, \quad k_j = \frac{\pi}{L+1} I_j, \quad I_j \in \mathbb{N}, \quad (2)$$

with $j = [1, \dots, N]$. Equation (2) can be directly obtained from the $U/J \rightarrow \infty$ limit of the BA equations (see supplementary information [23]). The distribution of I_j should correspond to a condition where energy is minimized. In the half-filled case, the energy minimization condition is given by $I_j = [1, \dots, L]$, leading to $E = -2J \sum_{j=1}^L \cos k_j = 0$, $p = \sum_{j=1}^L k_j$.

We now turn to the analysis of the attractive interaction case. Following the $U \rightarrow -U$ mapping, the total number of up spins $N_\uparrow$ in the repulsive case maps to the total number of pairs $N_{\uparrow\downarrow}$ and $N_\downarrow$ to the number of empty sites $N_\emptyset$, leading to $N = N_\emptyset + N_{\uparrow\downarrow}$. In the strongly attractive regime, we can assume that all down particles are paired, leading to $N_{\uparrow\downarrow} = N_\downarrow$. $N_\emptyset$ is the number of sites which are neither occupied by a pair ($N_{\uparrow\downarrow} = N_\downarrow$) nor by an unpaired majority atom ($N_\uparrow - N_{\uparrow\downarrow}$), and hence $N = L - (N_\uparrow - N_{\uparrow\downarrow})$, considering the symmetry properties of $\cos k_j$ ($\sum_{j=1}^{L-N_\uparrow+N_{\uparrow\downarrow}} \cos k_j \rightarrow \sum_{j=1}^{N_{\uparrow\downarrow}-N_\uparrow} \cos k_j$), we have

$$E = -2J \sum_{j=1}^{N_{\uparrow\downarrow}-N_\uparrow} \cos k_j, \quad k = \sum_{j=1}^{N_{\uparrow\downarrow}-N_\uparrow} k_j. \quad (3)$$

For a schematic representation of the Fermi sea, see sup-

plementary material. From Eq. (3), it is possible to relate the Fermi momentum for the spinless fermion gas k_F^s to the polarization P , namely, consider $k_F^s = \pi(N_{\uparrow\downarrow} - N_\uparrow)/(L + 1)$ and then observe that the FFLO momentum, defined as $q = \pi\rho P$ with $\rho = (N_{\uparrow\downarrow} + N_\downarrow)/L$, coincides with k_F^s (if $L \simeq L + 1$). Obviously, for a half-filled system $N_\uparrow + N_\downarrow = L$ and hence $k_F^s \simeq \pi P = q$.

The effect of the hopping modulation is to create two fermionic excitations (corresponding to the up and down fermions originating from the breaking of the pair) above the Fermi energy of the spinless fermions given in Eq. (3). The change of the kinetic energy imposed by the presence of these two excitations with respect to the ground state is given by $\Delta E_{\text{kin}} = -2J(\cos k_1 + \cos k_2)$ with $-1 \leq \cos k_{1,2} < \cos q$. In addition to ΔE_{kin} , the pair breaking also involves a change in the interaction energy $\Delta E_{\text{int}} = U$. The total energy difference associated to the breaking of the pair can be thus expressed as $\Delta E = -2J(\cos k_1 + \cos k_2) + U$. We then expect that in a DO modulation experiment for an imbalanced gas the pair breaking band will lie between $E_{\text{low}} = U - 4J \cos q$ ($k_1 = q, k_2 = q$) and $E_{\text{high}} = U + 4J$ ($k_1 = \pi, k_2 = \pi$), leading to a bandwidth

$$\frac{\Delta\omega}{U} = \frac{4J}{U}(1 + \cos q). \quad (4)$$

In addition, the finite value of U/J in the numerical simulations implies a shift in the DO spectrum $U \rightarrow U^*$ leading to $E_{\text{low}} = U^* - 4J \cos q$, $E_{\text{high}} = U^* + 4J$, keeping the value of $\Delta\omega$ unchanged. This shift is connected to the shift of the ground-state energy within the first Hubbard band induced by the finite value of the ratio U/J . The explicit calculation of the Hubbard spectrum for a two-site system allows us to get a qualitative understanding of the physical reason behind this phenomenon. More specifically, the finite value of U/J implies a lowering of the ground-state energy with respect to the case $U/J \rightarrow \infty$, along with a removal of the degeneracy connected to the spin degree of freedom (see supplementary information [23]). The relation given by Eq. (4) as well as the values E_{low} and E_{high} are nevertheless still valid. In Fig. 2 it is possible to observe how the numerical results correspond to our analytical description.

Intuitive understanding of the BA results can be provided by considering a related example, namely, the inelastic scattering of particles (Fig. 3). A restriction imposed on the possible values of the momenta—in our case dictated by the FFLO wave vector q of the initial state—implies a reduction of the bandwidth associated with the scattering process. If the particles considered have initial momenta and final momenta k_1, k_2 and k'_1, k'_2 , respectively, the maximum kinetic energy change in the scattering process will be (assuming the lattice dispersion relation) $\Delta E_{\text{max}} = 4J + 4J \cos q$ (for $k_1 = q, k_2 = q, k'_1 = \pi, k'_2 = \pi$), and the minimum will be given by

FIG. 3 (color online). (left) Representation of the scattering process between two particles with initial momentum $k_1 = q$ and $k_2 = q$ and final momentum $k'_1 = \pi$ and $k'_2 = \pi$, corresponding to the kinetic energy transfer $\Delta E_{\max} = 4J + 4J \cos q$. The pair momentum q restricts the available initial states. (right) Scattering process between the states $k_1 = \pi + q$, $k_2 = -\pi + q$ and $k'_1 = 0$ and $k'_2 = 0$.

$\Delta E_{\min} = -4J - 4J \cos(q)$ (for $k_1 = \pi + q$, $k_2 = -\pi + q$, $k'_1 = 0$, $k'_2 = 0$), hence, the largest possible difference in the kinetic energy associated to the scattering process will be given by $\Delta E_{\max} - \Delta E_{\min} = 8J(1 + \cos q)$. This simple example illustrates how, in general, the limitation of the accessible momentum states is reflected in the reduction of the bandwidth.

Since we are addressing a possible experimental setup to detect the FFLO phase in 1D, it is necessary to address the role of temperature. In [9,24], the temperature stability of the FFLO in 1D traps has been considered. In particular in [9] the transition temperature T_c between a phase-separated FFLO + normal $\rightarrow$ normal phase is discussed, leading to $T_c \approx 0.2T_F$. This result is obtained within a mean-field picture, providing an approximate upper limit of the temperatures needed to observe the FFLO phase. Recent quantum Monte Carlo calculations give $T_c \gtrsim 0.11$ for a trapped system [25]. For instance in [2] experimental temperatures $\approx 0.1T_F$ have been reported, suggesting that the FFLO phase could be observed in the near future. Note that the bandwidth reduction given by Eq. (4) is independent of the system size ($q = \pi\rho P$); thus one can immediately estimate the reduction in an experimentally realistic case. It is clear that, e.g., $q = \pi/2$ leading to 50% reduction should be observable. Furthermore, the typical observed bandwidths [18] lie in the kHz range and suggest that even 10% reductions ($q \approx 0.14\pi$) would be observable, although 1% ($q \approx 0.045\pi$) probably not.

Through a combination of numerical simulations and analytical results expressed in terms of BA equations, we have been able to relate the DO modulation spectrum to the presence of a FFLO state, giving a quantitative estimate of the bandwidth narrowing in terms of the wave vector q . Our analysis establishes the first simple clear experimental tool to detect and quantitatively characterize the FFLO phase in ultracold gases in quasi-1D optical lattices. It also shows, on more general grounds, how a collective (pair) momentum can be related to observable quantities in 1D systems in a simple and clear manner.

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*paivi.torma@hut.fi

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