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Cooperative Energy Detection with Heterogeneous Sensors under Noise Uncertainty: SNR Wall and use of Evidence Theory

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Abstract—The analyzed system model in this paper is a distributed parallel detection network in which each secondary user (SU) evaluates the energy-based test statistic from the received observations and sends it to a fusion center (FC), which makes the final decision. Uncertainty in the noise variance at each SU is modeled as an unknown constant in a certain interval around the nominal noise variance. It is assumed that the SUs are heterogeneous in that the nominal noise variances and the uncertainty intervals can be different for different SUs. Moreover, the received signal power at each SU may be different. For the considered system model, the paper presents important results for two inter-related themes on cooperative energy detection (CED) in the presence of noise uncertainty (NU). First, the expressions for generalized SNR walls are derived for the classical CED fusion rule, i.e., sum of energies from all SUs. Second, a Dempster-Shafer theory (DST) based CED is proposed in the presence of NU with heterogeneous sensors. In the proposed scheme, the test statistic from each SU is the energy-based basic mass assignment (BMA) values, which are first discounted depending on the uncertainty level associated with the SU and then fused at the FC using the Dempster rule of combination to arrive at the global decision. It is shown that the proposed scheme outperforms the traditional sum fusion rule in terms of detection performance as well as the location of SNR wall.

Index Terms—Cognitive radio, cooperative spectrum sensing, data fusion, Dempster-Shafer theory, energy detection, noise uncertainty, SNR wall, SP wall.

I. INTRODUCTION

WITH the advent of internet of things (IoT), machine to machine (M2M) communication and 5G systems, billions of wireless devices performing simple to complicated tasks will be added to the existing crowded wireless spectrum. As a result, availability of good quality wireless spectrum is going to be a major bottleneck for such future wireless services and systems. In such a scenario, opportunistic spectrum access provided by cognitive radio (CR) will enable these devices to efficiently use the spectrum and enhance reliability in data transfer [1]–[5].

Spectrum sensing is a key enabler for flexible spectrum use and CRs (also called secondary users (SUs)) as it provides spectrum awareness crucial for maximizing the spectrum utilization while limiting the interference to the primary user

(PU) to minimum. Cooperative energy detection (CED), where several energy detection based CRs collaborate to detect the PU activity in a given spectrum, is an attractive choice because of its simplicity, low power consumption and ability to capture highly dynamic behaviour of the radio spectrum. It also has good sensing performance when noise variance is exactly known [6], [7]. However, in most of the cases, the noise variance is not known and has to be estimated. In the real physical world, the estimates of system parameters are always subjected to uncertainty. Typically, the uncertainty in the noise variance is ± 1 dB even if no external interferences are assumed. In a practical scenario, where external and unknown interferences from many different sources are present, the uncertainty in the noise variance can be significantly higher [8].

In the presence of noise uncertainty (NU), detection schemes based on the energy of the observed signal suffer from drastic performance degradation [9]. In fact, [9] is one of the first papers that studied the effect of NU in energy detection. The authors demonstrated that in the absence of perfect knowledge of noise power, detection of a spread-spectrum signal by a wideband radiometer is more difficult in practice than suggested by standard results. Another major issue with energy detection schemes in the presence of NU is the performance limitation of *signal-to-noise ratio (SNR) wall*, which is defined as the value of SNR at and below which a energy detector fails to achieve the expected performance even if the number of observations tends to infinity [10]–[12]. In [10], the SNR wall was derived for the local energy detector by considering a NU model such that the true noise variance lies within an upper and lower bounds where the bounds are defined by the nominal noise variance and the uncertainty factor. In [11], the author reconsidered the case of a log-normal approximated NU as suggested in [9] and presented close form expressions of the SNR wall that depends on the desired performances of the detector and also on the level of uncertainty. In [12], based on the estimated noise power, the conditions for the existence of the SNR wall were derived and the impact of noise power estimation on the performance of energy detector was studied. In [13], the authors studied the effects of NU in generalized energy detector and derived an expression of the SNR wall for the same. The papers from [9]–[13] deals with the effects of NU and formation of SNR in a single energy detector. However, in our work the prime focus is on studying the effects of NU on the CED.

SNR wall for cooperative spectrum sensing is addressed in

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[14]–[16]. In [14], the noise variance is modeled as a random variable with uniform distribution in particular interval while in [15], [16], noise variance is assumed to be an unknown value in some specified interval. Both these noise models are discussed in [10]. However, the works in [14]–[16] assume homogeneous SUs, where all the SUs have the same nominal noise-variances and the same uncertainty intervals. Contrary to that, we derive the SNR wall for a more general case of heterogeneous sensors, where each SU may experience a different noise variance as well as a different uncertainty interval. Note that the term *heterogeneous sensors* essentially means non-identical detectors and has been used in different contexts in sensing literature. In [17], the term *heterogeneous sensors* means employing different sensing algorithms whereas in this paper, the term means that the sensors may have different nominal noise variances and different uncertainty intervals.

Currently in spectrum sensing literature, design of most detectors including the sum fusion rule is based on the Bayesian probability theory. One drawback of the Bayesian probability theory is its inability to deal with uncertainty in the observed data. Dempster-Shafer theory (DST), also referred to as *evidence theory* or *theory of belief functions*, has the ability to mathematically represent uncertainty or ignorance [18]. Confidence values in the DST are associated with the elements of the power set instead of the sample space as in the probability theory. This allows for modeling ignorance and uncertainty in the observed data. DST also provides an upper and lower bound on the likelihood of an event. Furthermore, the theory provides Dempster's rule of combination for fusing data from various sources. Therefore, it has been widely used in several applications including safety-and-reliability modeling, artificial intelligence, object classification, target tracking, information fusion, and process engineering [19]–[21]. Please refer to [18]–[21] for more details on mathematical analysis and applications of DST. In this paper, we propose the use of DST as an efficient alternative to the traditional sum fusion rule [22] for CED in the presence of NU.

DST has been applied earlier to the problem of distributed detection in traditional networks [23] while in CR networks, it has been applied to CED in [24]–[28]. In [24], SU credibility was evaluated based on the imperfections in the decisions at the SU arising due to the channel conditions between the PU and the SU. In [25], it is assumed that the SNR values at the SUs are different and credibility for each SU is calculated to evaluate the degree of reliability of each local spectrum sensing terminal. The work in [25] is later extended in [26] by employing an effective quantizer for the sensing data based on the hypothesis distribution under different SNRs of the PU signal. In [27], a DST based CED scheme is proposed where the credibility and belief measures are calculated based on the local sensing result of the CRs, which are then combined at the fusion center (FC) using Dempster combination rule to arrive at the global decision. In [28], a double threshold method is used to evaluate the local spectrum sensing requirements and the DST based belief measures. Moreover, a node selection technique is proposed that removes the redundant sensors from participating in CED. However, the works in [24]–[28] assume

that the noise variance is perfectly known while our paper specifically targets the scenario where there is uncertainty in the noise variance at each SU.

There are a few papers [29]–[31] which have tried to improve the performance of CED in the presence of NU. In [29], a linear weighted gain combining scheme is proposed by maximizing the deflection coefficient in the presence of NU. In [30], a two-threshold method is employed for local detection where the thresholds are chosen according to the NU at each SU. In the CED scheme presented in [31], the FC employs two thresholds, which are dynamically changed based on the estimated NU factor and are toggled based on the predicted activity of the PU. However, the work in [29]–[31] do not employ DST which is the prime focus of this paper.

The contributions of this paper are as follows ¹:

- SNR wall expression is derived for the sum fusion rule for the scenario where the SUs have different NU parameters as well as different received signal power levels.
- A novel DST based CED scheme is proposed with heterogeneous SUs that experience NU. In the proposed scheme, basic mass assignment (BMA) values are evaluated for each SU based on the likelihood functions of the energy of the received signal, which is calculated from the received observations. Each SU sends its BMA values to the FC which combines them using Dempster fusion rule.
- When the noise variance is exactly known, it is shown that the Dempster fusion rule based on the proposed BMA, reduces to the optimal likelihood ratio (LR) based fusion rule under the assumption of conditional independence of observations at the SUs conditioned on either of the hypotheses.
- In the presence of NU, its effect is taken into account by discounting the BMA for each node. A method is proposed to evaluate the discount factor based on the width of the NU interval.
- The detection performance of the proposed DST based CED is compared to that of the traditional sum and maximal ratio combining (MRC) fusion rule for the performance parameters of probability of detection in additive white Gaussian noise (AWGN) and multipath fading channels.
- The performance of the DST and the sum fusion rules are also compared in terms of SNR wall phenomenon and their locations for different NU parameters in different channel conditions.

The paper is organized as follows: Section II first presents the traditional sum fusion rule based CED in the absence of NU and next in the presence of NU. Section III presents the derivation and discussion of the SNR wall for the sum fusion rule with heterogeneous sensors. Section IV briefly describes the basics of DST while section V presents the proposed DST based CED method. Section VI presents the simulation results and section VII concludes the paper.

¹Some preliminary results were presented in [32] at COMSNETS 2017

Figure 1: Considered CED model : SUs (or sensors) have non-identical NU parameters. Each SU evaluates energy from the observed data and sends it to the FC, where sum fusion rule is applied for decision making.

II. COOPERATIVE DETECTION USING SUM FUSION RULE

In this section, the system model for the sum fusion rule is presented followed by its performance in the absence of NU. Next, the modeling of NU and its effect on the detection performance of the sum fusion rule is presented.

A. System Model

Fig. 1 shows the considered CED model for the sum fusion rule, which consists of a PU, U number of energy detection based SUs and a FC. There are two hypotheses H_0 and H_1 , where H_0 is associated with noise only scenario and H_1 with the case where PU signal is present. Each SU evaluates the energy from the received observations and sends it to the FC. At the FC, assuming conditional independence between the received observations at the SUs, the sum fusion rule is employed to arrive at the final decision. The SUs are assumed to be heterogeneous in nature such that the true noise variances and related uncertainty intervals are different for different SUs. Here, the true noise variance σ_i^2 at the i^{th} SU is modeled as an unknown constant and takes values in an interval defined by the nominal noise variance σ_{ni}^2 and the parameter ρ_i setting the bounds of the interval. It is assumed that the information about NU parameters (σ_{ni}^2 and ρ_i) of the SUs is available at the FC.

The considered signal model is

$$\begin{aligned} H_0 : x_i[n] &= w_i[n] \\ H_1 : x_i[n] &= h_i[n]s[n] + w_i[n] \end{aligned} \quad (1)$$

for $n = 1, 2, \dots, N$, where $x_i[n]$, $w_i[n]$, and $s[n]$ are samples of the received signal, noise and PU signal respectively at the

i^{th} SU. Here, $h_i[n]$ represents the channel gain of a single-tap fast-fading multipath channel. The local observations at the SUs, are assumed to be independent of each other conditioned on either of the hypotheses. The noise sample $w_i[n]$ is assumed to be a complex circular symmetric Gaussian random variable with zero mean and variance σ_i^2 , i.e., $w_i[n] \sim \mathcal{N}_c(0, \sigma_i^2)$. It is assumed that $s[n]$ and $w_i[n]$ are independent of each other. Moreover, the noise samples $w_i[n]$ and channel gains $h_i[n]$ are assumed to be independent among sensors too. Several types of PU signals, including widely used OFDM signals, can be modeled with Gaussian distribution [5]. Therefore the PU signal $s[n]$ is also assumed as complex circular symmetric Gaussian random variable with zero mean and variance σ_s^2 . Similarly, the channel coefficient $h_i[n]$ is assumed to be a complex circular symmetric Gaussian random variable with zero mean and variance δ_i . Consequently, $|h_i[n]|$ is Rayleigh distributed. Note that the reporting channels are assumed to be error-free.

The received signal energy E_i can be evaluated from the N received samples by

$$E_i = \sum_{n=1}^N |x_i[n]|^2. \quad (2)$$

According to the central limit theorem [33], if the number of samples N is sufficiently large (e.g., ≥ 250 in practice) [34], the test statistic E_i is asymptotically Gaussian distributed, and its distributions at the i^{th} SU, under the two hypotheses H_0 and H_1 , are given in [22], [35] as

$$\begin{aligned} H_0 : E_i &\sim \mathcal{N}(\mu_{0i}, \sigma_{0i}^2), \\ H_1 : E_i &\sim \mathcal{N}(\mu_{1i}, \sigma_{1i}^2), \end{aligned} \quad (3)$$

where

$$\begin{aligned} \mu_{0i} &= N\sigma_i^2 ; \quad \mu_{1i} = N\sigma_i^2(1 + \text{SNR}_i) = N(\sigma_i^2 + P_i) \\ \sigma_{0i}^2 &= N\sigma_i^4 ; \quad \sigma_{1i}^2 = N\sigma_i^4(1 + \text{SNR}_i)^2 = N(\sigma_i^2 + P_i)^2. \end{aligned}$$

Here, $\text{SNR}_i = P_i/\sigma_i^2$ is the true SNR in linear scale with $P_i = \delta_i\sigma_s^2$ denoting the received PU signal power at the i^{th} SU. Corresponding SNR in dB is given by $\text{SNR}_i(\text{dB}) = 10 \log_{10} \text{SNR}_i$.

At the FC, the sum fusion rule is applied so that the test statistic is given by

$$T_{\text{sum}} = \sum_{i=1}^U E_i, \quad (4)$$

while the global decision is made by using

$$T_{\text{sum}} \underset{H_0}{\overset{H_1}{\geq}} \eta_{\text{sum}}, \quad (5)$$

where η_{sum} is the threshold of a Neyman-Pearson (NP) detector at the FC.

B. Performance with known noise statistics

In this paper, we have considered NP based detector, where the prime objective is to maximize the probability of detection (P_d) for a given probability of false alarm (P_{fa}). The threshold η_{sum} for a NP detector depends on the distribution of T_{sum}

under the null hypothesis H_0 and the constraint on the probability of false alarm $P_{fa} \leq \beta$. As T_{sum} in (4) is a linear combination of U independent Gaussian random variables, it is also Gaussian distributed under both the hypotheses with distributions given in [22] by

$$\begin{aligned} H_0 : T_{sum} &\sim \mathcal{N}(\mu_0, \sigma_0^2), \\ H_1 : T_{sum} &\sim \mathcal{N}(\mu_1, \sigma_1^2), \end{aligned} \quad (6)$$

where

$$\begin{aligned} \mu_0 &= N \sum_{i=1}^U \sigma_i^2 \quad ; \quad \mu_1 = N \sum_{i=1}^U (\sigma_i^2 + P_i); \\ \sigma_0^2 &= N \sum_{i=1}^U \sigma_i^4 \quad ; \quad \sigma_1^2 = N \sum_{i=1}^U (\sigma_i^2 + P_i)^2. \end{aligned} \quad (7)$$

Assuming that the knowledge of noise variance σ_i^2 and received signal power P_i is available for all SUs, the probability of false alarm (P_{fa}) and probability of detection (P_d) for a NP detector are expressed as [36]

$$P_{fa} = Q\left(\frac{\eta_{sum} - \mu_0}{\sigma_0}\right); \quad P_d = Q\left(\frac{\eta_{sum} - \mu_1}{\sigma_1}\right), \quad (8)$$

where $Q(\cdot)$ is the tail probability of the standard normal distribution. The threshold η_{sum} with false alarm constraint of β can then be calculated from (8) as given in [36]

$$\begin{aligned} \eta_{sum} &= Q^{-1}(\beta)\sigma_0 + \mu_0 \\ &= Q^{-1}(\beta)\sqrt{N \sum_{i=1}^U \sigma_i^4 + N \sum_{i=1}^U \sigma_i^2}, \end{aligned} \quad (9)$$

Note that the sum fusion rule given by (4) is also an optimal fusion rule for binary hypothesis testing problem in (6) when noise variance is perfectly known [22], [36].

C. Modeling noise uncertainty

Estimating the uncertainty in noise variance is a well-studied topic in radar and constant false alarm rate (CFAR) detectors [36]. There are a good number of methods for doing it using guard bands, auxiliary channels, on top of pilot signals, while PU is definitely not active (for example, calibration stage). There have also been a few attempts in estimating and modeling the noise variance uncertainty in the spectrum sensing literature in [10], [36], [37].

In this paper, the NU at each SU is modeled as in [10] by considering σ_i^2 to be an unknown constant that lies in the interval $\left[\frac{1}{\rho_i}\sigma_{ni}^2, \rho_i\sigma_{ni}^2\right]$, where σ_{ni}^2 is the nominal noise variance and $\rho_i \geq 1$ is the uncertainty parameter. Here, the subscript ni signifies that this is a nominal value for the i^{th} SU. This NU model is used to quantify the impact of misspecified noise level on the performance of the detector. Lower and upper bounds are used instead of Bayesian prior to characterize the impact of NU in the worst case scenario. Nominal SNR corresponding to the nominal noise variance σ_{ni}^2 is denoted as $\text{SNR}_{ni} = P_i/\sigma_{ni}^2$. Corresponding nominal SNR in dB is denoted by $\text{SNR}_{ni}(\text{dB}) = 10 \log_{10} \text{SNR}_{ni}$. As it is sometimes convenient to describe the uncertainty parameter

in dB, we denote the deviation in noise variance about the nominal value in dB for the i^{th} SU by $\Delta_i = 10 \log_{10} \rho_i$. Therefore, if the deviation is $\pm \Delta_i$ dB then the lower and upper bounds on the noise variance are given by

$$\begin{aligned} \sigma_{li}^2 &= \sigma_{ni}^2 \cdot 10^{(-\Delta_i/10)} = \frac{1}{\rho} \sigma_{ni}^2, \\ \sigma_{ui}^2 &= \sigma_{ni}^2 \cdot 10^{(+\Delta_i/10)} = \rho \sigma_{ni}^2. \end{aligned} \quad (10)$$

D. Performance in the presence of NU

In the presence of NU, (9) cannot be used to determine the threshold of NP detector as σ_i^2 is unknown. In such a case, to maintain constraint on the false alarm probability $P_{fa} \leq \beta$ for noise variance in the known interval $\left[\frac{1}{\rho_i}\sigma_{ni}^2, \rho_i\sigma_{ni}^2\right]$, we can set β to be the worst-case false alarm probability [10] corresponding to σ_{ui}^2 so that

$$\begin{aligned} \beta &= \max_{\sigma_i^2 \in \left[\frac{1}{\rho_i}\sigma_{ni}^2, \rho_i\sigma_{ni}^2\right]} Q\left(\frac{\eta'_{sum} - \mu_0}{\sigma_0}\right) \\ &= Q\left(\frac{\eta'_{sum} - N \sum_{i=1}^U \rho_i \sigma_{ni}^2}{\sqrt{N \sum_{i=1}^U \rho_i^2 \sigma_{ni}^4}}\right), \end{aligned} \quad (11)$$

where η'_{sum} denotes the threshold of NP detector at the FC in the presence of NU and can be evaluated from (11) as

$$\eta'_{sum} = Q^{-1}(\beta) \sqrt{N \sum_{i=1}^U \rho_i^2 \sigma_{ni}^4 + N \sum_{i=1}^U \rho_i \sigma_{ni}^2}. \quad (12)$$

The probability of detection for the worst case scenario is as given in [10]

$$\begin{aligned} P_d &= \min_{\sigma_i^2 \in \left[\frac{1}{\rho_i}\sigma_{ni}^2, \rho_i\sigma_{ni}^2\right]} Q\left(\frac{\eta'_{sum} - \mu_1}{\sigma_1}\right) \\ &= Q\left(\frac{\eta'_{sum} - N \sum_{i=1}^U \left(\frac{1}{\rho_i}\sigma_{ni}^2 + P_i\right)}{\sqrt{N \sum_{i=1}^U \left(\frac{1}{\rho_i}\sigma_{ni}^2 + P_i\right)^2}}\right). \end{aligned} \quad (13)$$

III. SNR WALL FOR THE SUM FUSION RULE WITH HETEROGENEOUS SUs

In this section, we first derive the SNR wall expression for CED under the general assumption that all the participating SUs are heterogeneous in nature. Later, it is shown that the traditional SNR wall expressions for local as well as CED with homogeneous sensors can be obtained as special cases of the generalized SNR wall proposed in this paper.

In the presence of NU, energy detector suffers from a performance limitation such that if the SNR at the SU is below a certain SNR threshold, called *SNR wall*, it fails to achieve the desired P_d and P_{fa} even if the number of samples N tends to infinity [10]. As such, the expression for the sample size N is required to find the SNR wall for the considered sum rule based CED scheme with heterogeneous SUs. Substituting

(12) in (13) we obtain the expression for sample size N for given P_{fa} and P_d values as

$$N = \frac{\left[Q^{-1}(P_{fa}) \sqrt{\sum_{i=1}^U \rho_i^2 \sigma_{ni}^4} - Q^{-1}(P_d) \sqrt{\sum_{i=1}^U \left(\frac{1}{\rho_i} \sigma_{ni}^2 + P_i \right)^2} \right]^2}{\left[\sum_{i=1}^U \left(\frac{1}{\rho_i} \sigma_{ni}^2 + P_i \right) - \sum_{i=1}^U \rho_i \sigma_{ni}^2 \right]^2}. \quad (14)$$

Therefore, to derive the SNR wall, where $N \rightarrow \infty$, equating the denominator of (14) to zero, we get

$$\frac{1}{U} \sum_{i=1}^U P_i = \frac{1}{U} \sum_{i=1}^U \sigma_{ni}^2 \left(\rho_i - \frac{1}{\rho_i} \right), \quad (15)$$

where both sides are divided by U for convenience. From (15), we can clearly observe that in the case of heterogeneous SUs having different nominal noise variances σ_{ni}^2 , defining a single SNR wall for all the SUs is not feasible, since each SU will have its own nominal SNR value. Therefore, instead of defining a SNR wall for each SU, we take the average signal power as the reference to coin a new term called *signal power (SP) wall* denoted by SP_{wall} . The expression for the SP wall is given from (15) as

$$SP_{\text{wall}} = \frac{1}{U} \sum_{i=1}^U \sigma_{ni}^2 \left(\rho_i - \frac{1}{\rho_i} \right). \quad (16)$$

If we denote $P_{avg} \triangleq \frac{1}{U} \sum_{i=1}^U P_i$ in (15), then (16) signifies that the average signal power, P_{avg} , required to achieve the target P_d and P_{fa} at the FC should always be greater than SP_{wall} .

A. SNR wall as a special case of SP wall homogeneous sensors and $P_i = P$

For $\sigma_{ni}^2 = \sigma_n^2$, (16) can be written as

$$SP_{\text{wall}} = \frac{\sigma_n^2}{U} \sum_{i=1}^U \left(\rho_i - \frac{1}{\rho_i} \right). \quad (17)$$

Using $SNR = P/\sigma_n^2$, we can rewrite the above equation as

$$SNR_{\text{wall}} = \frac{SP_{\text{wall}}}{\sigma_n^2} = \frac{1}{U} \sum_{i=1}^U \left(\rho_i - \frac{1}{\rho_i} \right). \quad (18)$$

For homogeneous SUs, where $\rho_i = \rho$ in addition to $\sigma_{ni}^2 = \sigma_n^2$, we can rewrite (18) as

$$SNR_{\text{wall}} = \frac{SP_{\text{wall}}}{\sigma_n^2} = \left(\rho - \frac{1}{\rho} \right). \quad (19)$$

The above expression given by (19) is the traditional SNR wall as shown in [10] for local energy detector and in [15], [16] for CED. From this we can conclude that cooperative spectrum sensing using traditional soft combining fusion rule does not contribute in lowering the SNR wall for energy detector. However, when $SNR_n > SNR_{\text{wall}}$, cooperation does help in reducing the sample size N at individual SU for

achieving the same detection performance. This can be seen from

$$N = \frac{[\rho Q^{-1}(P_{fa}) - (1/\rho + SNR_n) Q^{-1}(P_d)]^2}{U [(1/\rho + SNR_n) - \rho]^2}, \quad (20)$$

which can be obtained by using $\rho_i = \rho$ and $\sigma_{ni}^2 = \sigma_n^2$ in (14).

IV. DEMPSTER-SHAFFER THEORY (DST) OF EVIDENCE

Our proposed CED scheme is based on the DST of evidence. Although the theory of evidence is well studied and applied in different fields, it has not received sufficient attention in the field of wireless communication. Keeping this in mind, this section is dedicated for a brief overview of DST basics. This section also includes a toy example with two hypotheses θ_0 and θ_1 used for explaining the concepts of DST.

In DST, a set of mutually exclusive and exhaustive hypotheses are first defined. This initial set of hypotheses is called *frame of discernment* and denoted by Θ . For the toy example considered in this section, frame of discernment is given by $\Theta = \{\theta_0, \theta_1\}$ while the power set of Θ is given as $2^\Theta = \{\phi, \theta_0, \theta_1, \{\theta_0, \theta_1\}\}$, which basically represents all the possible subsets of Θ , including the empty set ϕ and Θ itself. Next, a function $m(\cdot)$, called BMA, is defined for the elements of power set such that $m : 2^\Theta \rightarrow [0, 1]$ and satisfies the following properties

$$m(\phi) = 0; \quad 0 \leq m(A) \leq 1; \quad \sum_{A \in 2^\Theta} m(A) = 1.$$

The quantity $m(A)$ is the basic mass or weight assigned to a proposition/set $A \in 2^\Theta$ denoting the measure of belief that is committed exactly to A and not to any subset of A . Here, it is important to note that in DST the basic masses are assigned not to the elements of Θ , but to the power set 2^Θ . This is a key difference between the Bayesian theory and DST. To elaborate this point consider the following hypothetical example. Suppose we are asked to assign weights for the propositions “whether extraterrestrial life exists or not”. We consider the same frame of discernment $\Theta = \{\theta_0, \theta_1\}$, where θ_0 stands for the hypothesis that there is no extraterrestrial life and θ_1 stands for the hypothesis that there is extraterrestrial life. From Bayesian theory, we may assign weights as $\{0.5, 0.5\}$, which basically accounts for the least informative scenario when our knowledge is null or minimal. However, with DST and based on the available evidence at our disposal, we can assign weights to all four possibilities $\{\phi, \theta_0, \theta_1, \{\theta_0, \theta_1\}\}$, for example, as $\{0, 0.1, 0.2, 0.7\}$. Here, $\{\theta_0, \theta_1\} = 0.7$ signifies our ignorance level or amount of uncertainty. In fact, if our evidence is null we can assign weights as $\{0, 0, 0, 1\}$, with $\{\theta_0, \theta_1\} = 1$, which basically means that our ignorance level is 100%. This grants DST more flexibility and allows for the inclusion of unquantified uncertainty, which is the most significant advantage of the DST over the Bayesian theory.

There are two more functions associated with BMA. They are belief function denoted as Bel and plausibility function designated as Pl, defined for all $A \subseteq \Theta$ is given in [18] as

$$\begin{aligned} \text{Bel}(A) &= \sum_{B \subseteq A} m(B), \\ \text{Pl}(A) &= \sum_{A \cap B \neq \emptyset} m(B). \end{aligned} \quad (21)$$

The function $\text{Bel}(A)$ describes the minimum support (lower bound) of one's belief that hypothesis A is true, while $\text{Pl}(A)$ denotes the maximum support (upper bound) or belief that A can be true if more evidence is available. Also from the above definition, the following relationships hold: $\text{Bel}(A) \leq \text{Pl}(A)$, $\text{Pl}(A) = 1 - \text{Bel}(\bar{A})$, $\text{Bel}(A) = 1 - \text{Pl}(\bar{A})$, where $\bar{A}$ is the complement set of A .

The Dempster rule of combination enables us to compute the orthogonal sum of several belief functions over the same frame of discernment but based on distinct bodies of evidence. If there are U independent sources based on the same frame of discernment with BMAs $m_1(\cdot), m_2(\cdot), \dots, m_U(\cdot)$, then the combined basic mass for an element A is given in [18] as

$$\begin{aligned} M(A) &= [m_1 \oplus m_2 \oplus \dots \oplus m_U](A) \\ &= \frac{1}{K} \left\{ \sum_{\substack{A_1, A_2, \dots, A_U \in 2^\Theta \\ A_1 \cap A_2 \cap \dots \cap A_U = A}} m_1(A_1) \dots m_U(A_U) \right\}, \end{aligned} \quad (22)$$

where

$$K = \sum_{\substack{A_1, A_2, \dots, A_U \in 2^\Theta \\ A_1 \cap A_2 \cap \dots \cap A_U \neq \emptyset}} m_1(A_1) \dots m_U(A_U).$$

The symbol $\oplus$ denotes the Dempster combination operator and K is the renormalization factor. For instance, for the toy example described above, consider two independent sources for the same frame of reference $\Theta = \{\theta_0, \theta_1\}$, with BMA $m_1(\cdot)$ and $m_2(\cdot)$, and basic masses as given in table I.

Table I: BMA of two sources

BMA OF SOURCE 1	VALUE	BMA OF SOURCE 2	VALUE
$m_1(\phi)$	0	$m_2(\phi)$	0
$m_1(\theta_0)$	0.1	$m_2(\theta_0)$	0.2
$m_1(\theta_1)$	0.2	$m_2(\theta_1)$	0.2
$m_1(\{\theta_0, \theta_1\})$	0.7	$m_2(\{\theta_0, \theta_1\})$	0.6

The combined BMA $M(\cdot)$ for hypothesis θ_0 and θ_1 based on DS combination rule can be expressed as

$$\begin{aligned} M(\theta_0) &= \frac{1}{K} \left\{ m_1(\theta_0)m_2(\theta_0) + m_1(\theta_0)m_2(\{\theta_0, \theta_1\}) + m_2(\theta_0)m_1(\{\theta_0, \theta_1\}) \right\}, \\ M(\theta_1) &= \frac{1}{K} \left\{ m_1(\theta_1)m_2(\theta_1) + m_1(\theta_1)m_2(\{\theta_0, \theta_1\}) + m_2(\theta_1)m_1(\{\theta_0, \theta_1\}) \right\}, \end{aligned}$$

where K is computed as

$$\begin{aligned} K &= m_1(\theta_0)m_2(\theta_0) + m_1(\theta_1)m_2(\theta_1) + m_1(\theta_0)m_2(\{\theta_0, \theta_1\}) \\ &\quad + m_2(\theta_0)m_1(\{\theta_0, \theta_1\}) + m_1(\theta_1)m_2(\{\theta_1, \theta_2\}) \\ &\quad + m_2(\theta_1)m_1(\{\theta_0, \theta_1\}) + m_1(\{\theta_0, \theta_1\})m_2(\{\theta_0, \theta_1\}). \end{aligned}$$

Figure 2: Framework for the proposed DST based CED. Each SU first evaluates the basic mass for different elements and then performs the discounting operation. The discounted BMA values are sent to the FC via reporting channels for fusion.

Using the above formulation we get $K = 0.94$, $M(\theta_0) = 0.2340$ and $M(\theta_1) = 0.3191$. At this stage, a decision can be made based on the obtained data, simply by comparing the combined basic masses. For this hypothetical example, since $M(\theta_1) > M(\theta_0)$, we can infer that there is a higher chance that extraterrestrial life exists.

V. PROPOSED DST BASED CED

Fig. 2 shows the framework for the proposed DST based CED. First step in this approach is estimating the BMA values for different elements of the power set based on the energy of the received signal. Next, the BMA values for each SU are discounted based on the NU interval. In the final step, these discounted BMA values are used in the DST fusion rule at the FC. These steps are explained in detail in this section. Towards the end of this section, we also provide a proof for the optimality of the proposed DST based CED in the absence of NU.

A. Proposed BMA method

In DST based spectrum sensing for CRs, assigning basic mass to the elements of the power set is a crucial part as the end decision depends on the correctness of how the basic masses are assigned. In DST, there is no single explicit rule for assigning basic masses to different elements. As such, in some scenarios, they are assigned by field-experts or formulated using an application specific equation. Here, we propose a novel BMA method, which is based on the energy of the received signal.

In [18], the author has discussed the idea of assigning support values to different hypotheses based on probabilistic

models. In this context, for a SU performing local sensing, consider the frame of discernment $\Theta = \{H_0, H_1\}$. The power set of Θ is given as $\{\phi, H_0, H_1, \{H_0, H_1\}\}$ where $\omega = \{H_0, H_1\}$ represents the uncertainty or ignorance set. Let $p(\mathcal{E}; H_j)$ for $j = 0, 1$, denote the class of likelihood functions on the set of energy values $\mathcal{E} \in \mathbb{R}_{\geq 0}$. Now, according to the DST, if we have an observation $E_i \in \mathcal{E}$, then E_i lends plausibility to a singleton $\{H_j\} \subset \Theta$ in strict proportion to the probability that $p(E_i; H_j)$ assigns to E_i . Moreover, $p(E_i; H_j)$ is further parameterized by the unknown noise variance σ_i^2 . Therefore, E_i should determine a plausibility function Pl_{E_i} obeying

$$\text{Pl}_{E_i}(H_j) = c \cdot p(E_i; H_j, \sigma_i^2), \quad (23)$$

where c is a constant and $\text{Pl}_{E_i} : 2^\Theta \rightarrow [0, 1]$. Note that c in (23) is a quantity that normalizes the plausibility values. Therefore, we propose c to be taken as

$$c = \frac{1}{p(E_i; H_0, \sigma_i^2) + p(E_i; H_1, \sigma_i^2)}. \quad (24)$$

It will be shown later in this section that this choice of c is optimal under the no NU assumption. Using (3) the likelihood functions under both the hypotheses are given as

$$\begin{aligned} p(E_i; H_0, \sigma_i^2) &= \frac{1}{\sqrt{2\pi}\sigma_{0i}} \exp\left(-\frac{(E_i - \mu_{0i})^2}{2\sigma_{0i}^2}\right), \\ p(E_i; H_1, \sigma_i^2) &= \frac{1}{\sqrt{2\pi}\sigma_{1i}} \exp\left(-\frac{(E_i - \mu_{1i})^2}{2\sigma_{1i}^2}\right). \end{aligned} \quad (25)$$

Now the belief or support function $S_{E_i} : 2^\Theta \rightarrow [0, 1]$ is given as in [18]

$$S_{E_i}(A) = 1 - \text{Pl}_{E_i}(\bar{A}), \quad (26)$$

for all proper subsets $A \subseteq \Theta$. Using equations (23), (24) and (26), the support function for the hypothesis H_0 at the i^{th} SU is obtained as

$$\begin{aligned} S_{E_i}(H_0) &= 1 - \text{Pl}_{E_i}(H_1) \\ &= 1 - \frac{p(E_i; H_1, \sigma_i^2)}{p(E_i; H_0, \sigma_i^2) + p(E_i; H_1, \sigma_i^2)} \\ &= \frac{p(E_i; H_0, \sigma_i^2)}{p(E_i; H_0, \sigma_i^2) + p(E_i; H_1, \sigma_i^2)}. \end{aligned} \quad (27)$$

Similarly, support functions for H_1 and ω at the i^{th} SU are obtained as

$$S_{E_i}(H_1) = \frac{p(E_i; H_1, \sigma_i^2)}{p(E_i; H_0, \sigma_i^2) + p(E_i; H_1, \sigma_i^2)}, \quad (28)$$

$$S_{E_i}(\omega) = 1. \quad (29)$$

Now, there is a one-one correspondence between the BMA function and support function, i.e., $m \leftrightarrow S_{E_i}$. The BMA values for hypotheses H_0, H_1 , and ω can be uniquely obtained from the support function S_{E_i} by means of the inversion formula [18], which is given as

$$m(A) = \sum_{B \subseteq A} (-1)^{|A-B|} S_{E_i}(B) \quad (30)$$

for all proper subsets $A \subseteq \Theta$. Here $A - B$ denotes difference of sets A and B (or the set of all elements of A that are not

in B) and $|A|$ denotes the cardinality of the set A . Therefore we have,

$$\begin{aligned} m_i(H_0) &= \sum_{B \subseteq H_0} (-1)^{|H_0-B|} S_{E_i}(B) \\ &= (-1)^{|H_0-H_0|} S_{E_i}(H_0) \\ &= S_{E_i}(H_0) \\ &= \frac{p(E_i; H_0, \sigma_i^2)}{p(E_i; H_0, \sigma_i^2) + p(E_i; H_1, \sigma_i^2)}, \end{aligned} \quad (31)$$

$$\begin{aligned} m_i(H_1) &= \sum_{B \subseteq H_1} (-1)^{|H_1-B|} S_{E_i}(B) \\ &= (-1)^{|H_1-H_1|} S_{E_i}(H_1) \\ &= S_{E_i}(H_1) \\ &= \frac{p(E_i; H_1, \sigma_i^2)}{p(E_i; H_0, \sigma_i^2) + p(E_i; H_1, \sigma_i^2)}, \end{aligned} \quad (32)$$

$$\begin{aligned} m_i(\omega) &= \sum_{B \subseteq \omega} (-1)^{|\omega-B|} S_{E_i}(B) \\ &= (-1)^{|\omega-\omega|} S_{E_i}(\omega) + (-1)^{|\omega-H_0|} S_{E_i}(H_0) + \\ &\quad (-1)^{|\omega-H_1|} S_{E_i}(H_1) \\ &= 1 - S_{E_i}(H_0) - S_{E_i}(H_1) \\ &= 1 - m_i(H_0) - m_i(H_1). \end{aligned} \quad (33)$$

B. BMA adjustment under NU

The BMA functions $m_i(\cdot)$ are formulated in such a way that in the absence of NU, each SU sends its BMA values to the FC as it is. As a result, the sum of $m_i(H_0)$ and $m_i(H_1)$ will always be one, i.e., $m_i(H_0) + m_i(H_1) = 1$ and consequently the basic mass for ω will be $m_i(\omega) = 0$. However, in the presence of NU, these BMA values may not be completely reliable. The DST provides an attractive way to discount these BMA values based on their reliability using the *discounting rule* of DST [18]. The discounting rule states that if we have a degree of trust of $1 - \alpha$ in the evidence as a whole, where $0 \leq \alpha \leq 1$, then α is adopted as a discount rate and reduce the degree of support for each proper subset A of Θ from $m(A)$ to $(1 - \alpha)m(A)$. So under NU conditions, the new BMA values for each SU will be

$$\hat{m}_i(H_0) = (1 - \alpha_i) m_i(H_0), \quad (34)$$

$$\hat{m}_i(H_1) = (1 - \alpha_i) m_i(H_1), \quad (35)$$

where α_i denotes discount rate for the i^{th} SU such that $0 \leq \alpha_i \leq 1$. Now, the BMA for ω is obtained as

$$\begin{aligned} \hat{m}_i(\omega) &= 1 - \hat{m}_i(H_0) - \hat{m}_i(H_1) \\ &= 1 - (1 - \alpha_i) [m_i(H_0) + m_i(H_1)] \\ &= 1 - (1 - \alpha_i) \\ &= \alpha_i. \end{aligned}$$

Thus we find that the BMA value for the set ω under NU, i.e., $\hat{m}_i(\omega)$ is same as the discount rate α_i . Therefore, when SUs' are subjected to NU, $\hat{m}_i(H_0) + \hat{m}_i(H_1) < 1$ and $\hat{m}_i(\omega) = \alpha_i > 0$.

The discounting factor α_i may be different for different SUs depending on their noise variance interval. However, since $0 \leq \alpha_i \leq 1$, it is to be ensured that under any NU interval and any arbitrary nominal noise variance, the α_i value should always lie between 0 and 1.

C. Determining discount rate α

In this section we propose a method for determining the discount rate α_i in the presence of NU. The discount rates are measured individually for every SU depending on the NU interval associated with it and as such each SU will have its own unique discount rate α_i . In this regard, the first piece of information required for calculating α_i is the NU parameters σ_{ni}^2 and ρ_i of each SU.

Now, considering a single SU performing spectrum sensing in the presence of NU, the objective in NP criterion based energy detection is to maximize P_{di} (probability of detection at the i^{th} SU) for a given value of P_{fi} (probability of false alarm at the i^{th} SU) $\leq \beta_i$, where β_i is the false alarm constraint at the i^{th} SU. Therefore we have,

$$\beta_i = \max_{\sigma_i^2 \in [\sigma_{li}^2, \sigma_{ui}^2]} Q\left(\frac{\eta_i - \mu_{0i}}{\sigma_{0i}}\right) = Q\left(\frac{\eta_i - N\sigma_{ui}^2}{\sqrt{N}\sigma_{ui}^2}\right).$$

The threshold η_i for a single SU ($U=1$) under NU is then given by

$$\eta_i = \sqrt{N}\sigma_{ui}^2 Q^{-1}(\beta_i) + N\sigma_{ui}^2. \quad (36)$$

Thus from (36) we observe that the threshold η_i at the i^{th} SU is a function of β_i and upper limit of noise variance σ_{ui}^2 . Under this condition, the maximum probability of detection P_{di} is achieved, when $\sigma_i^2 = \sigma_{ui}^2$ and minimum P_{di} for $\sigma_i^2 = \sigma_{li}^2$. Now based on threshold values evaluated from (36), receiver operating characteristic (ROC) curves for a single user are obtained for the best case ($\sigma_i^2 = \sigma_{ui}^2$) and worst case ($\sigma_i^2 = \sigma_{li}^2$). Once this is estimated, we calculate α_i as the difference between the best and the worst case P_{di} values.

$$\alpha_i(\beta_i) = P_{di}(\beta_i)_{\sigma_{ui}^2} - P_{di}(\beta_i)_{\sigma_{li}^2}. \quad (37)$$

This technique helps in ensuring $0 \leq \alpha_i \leq 1$ and also makes sure that the α_i value increases (or decreases) with increase (or decrease) in the NU interval as shown later in this section. Note that with the change in nominal SNR value, i.e., SNR_{ni} at the i^{th} SU and sample size N , ROC curves will also change for the same NU interval $[\sigma_{li}^2, \sigma_{ui}^2]$. Therefore, we can say that α_i is a function of three parameters viz. β_i , SNR_{ni} and N . However, considering N to be fixed for a SU, α_i becomes a function of β_i and SNR_{ni} as long as the NU interval of a SU remains constant. Thus (37) can be modified as

$$\alpha_i(\beta_i, \text{SNR}_{ni}) = P_{di}(\beta_i, \text{SNR}_{ni})_{\sigma_{ui}^2} - P_{di}(\beta_i, \text{SNR}_{ni})_{\sigma_{li}^2} \quad (38)$$

Analytically we can express $P_{di}(\beta_i, \text{SNR}_{ni})_{\sigma_{ui}^2}$ and $P_{di}(\beta_i, \text{SNR}_{ni})_{\sigma_{li}^2}$ in equation (38) as

$$\begin{aligned} P_{di}(\beta_i, \text{SNR}_{ni})_{\sigma_{ui}^2} &= \max_{\sigma_i^2 \in [\sigma_{li}^2, \sigma_{ui}^2]} Q\left(\frac{\eta_i - \mu_{1i}}{\sigma_{1i}}\right) \\ &= Q\left(\frac{\eta_i - N\sigma_{ni}^2(\rho_i + \text{SNR}_{ni})}{\sqrt{N}\sigma_{ni}^2(\rho_i + \text{SNR}_{ni})}\right) \end{aligned} \quad (39)$$

and

$$\begin{aligned} P_{di}(\beta_i, \text{SNR}_{ni})_{\sigma_{li}^2} &= \min_{\sigma_i^2 \in [\sigma_{li}^2, \sigma_{ui}^2]} Q\left(\frac{\eta_i - \mu_{1i}}{\sigma_{1i}}\right) \\ &= Q\left(\frac{\eta_i - N\sigma_{ni}^2\left(\frac{1}{\rho_i} + \text{SNR}_{ni}\right)}{\sqrt{N}\sigma_{ni}^2\left(\frac{1}{\rho_i} + \text{SNR}_{ni}\right)}\right). \end{aligned} \quad (40)$$

Using (39) and (40) in (38), α_i can be expressed in closed form as

$$\begin{aligned} \alpha_i(\beta_i, \text{SNR}_{ni}) &= Q\left(\frac{\eta_i - N\sigma_{ni}^2(\rho_i + \text{SNR}_{ni})}{\sqrt{N}\sigma_{ni}^2(\rho_i + \text{SNR}_{ni})}\right) - \\ &Q\left(\frac{\eta_i - N\sigma_{ni}^2\left(\frac{1}{\rho_i} + \text{SNR}_{ni}\right)}{\sqrt{N}\sigma_{ni}^2\left(\frac{1}{\rho_i} + \text{SNR}_{ni}\right)}\right), \end{aligned} \quad (41)$$

where the value of η_i is obtained from (36). Fig. 3 shows the plot of α_i as a function of β_i for different NU intervals and $\text{SNR}_n(\text{dB}) = -5$ dB. It can be clearly seen that with increase in NU interval, the α_i or discount rate of a SU also increases. For false alarm rate $\beta_i = 0.1$, the α_i values intersecting the black dotted line denotes the discount rate to be used depending on the NU interval associated with the SU.

Figure 3: Plot of α_i for different NU intervals as a function of β_i and $\text{SNR}_n(\text{dB}) = -5$ dB.

In order to calculate the discount rate α_i for each SU, we have assumed for convenience that the β_i is the same as the false alarm value used at the FC. Therefore, we can write $\beta_i = \beta$. However, note that the SUs do not make any local decisions and the value $\beta_i = \beta$ is only used for α_i calculation.

D. Data fusion at the FC

The BMA adjustment is performed locally at the SU with the corresponding discount rate α_i . For identical sensors, we can assume $\alpha_1 = \alpha_2 = \dots = \alpha_U = \alpha$. But if the NU interval is different for each SU_i the discount rates will also differ accordingly. The discounted BMA values are then sent to the FC via the reporting channel. In the FC, Dempster combination

rule is used to fuse the BMA values from all the U SUs, which gives us the combined basic mass $M(H_0)$ and $M(H_1)$ for hypothesis H_0 and H_1 respectively,

$$M(H_0) = \frac{1}{K} \sum_{A_1 \cap A_2 \cap \dots \cap A_U = H_0} \prod_{i=1}^U \hat{m}_i(A_i), \quad (42)$$

$$M(H_1) = \frac{1}{K} \sum_{A_1 \cap A_2 \cap \dots \cap A_U = H_1} \prod_{i=1}^U \hat{m}_i(A_i).$$

Finally, the test statistic at the FC is taken as the ratio of $M(H_1)$ and $M(H_0)$

$$T_{ds} = \frac{M(H_1)}{M(H_0)} \underset{H_0}{\overset{H_1}{\geq}} \eta_{ds}, \quad (43)$$

where η_{ds} is the threshold under DST scheme at FC. In this context, the threshold η_{ds} is a function of β and SNR_{ni} value at the i^{th} SU. For determining threshold η_{ds} , we take $\alpha_i = 0$ to ensure that the constraint $P_{fa} \leq \beta$ is maintained for all values of α_i .

E. Optimality under no NU

In this subsection, we show that in the absence of NU, i.e., $\Delta = 0$, the proposed DST based fusion rule reduces to the optimal fusion rule of LR. Note for $\Delta_i = 0$ dB, we have $\sigma_{li}^2 = \sigma_{ui}^2$, which along with (38) means that $\alpha_i = 0$. Therefore, for this case $\hat{m}_i(\omega) = 0$, $\hat{m}_i(H_0) = m_i(H_0)$ and $\hat{m}_i(H_1) = m_i(H_1)$ for $i = 1, \dots, U$ so that the test statistic T_{ds} in (43) becomes

$$\begin{aligned} T_{ds} &= \frac{M(H_1)}{M(H_0)} = \frac{\frac{1}{K} \prod_{i=1}^U m_i(H_1)}{\frac{1}{K} \prod_{i=1}^U m_i(H_0)} \\ &= \prod_{i=1}^U \frac{\frac{p(E_i; H_1, \sigma_i^2)}{p(E_i; H_0, \sigma_i^2) + p(E_i; H_1, \sigma_i^2)}}{\frac{p(E_i; H_0, \sigma_i^2)}{p(E_i; H_0, \sigma_i^2) + p(E_i; H_1, \sigma_i^2)}} \\ &= \prod_{i=1}^U \frac{p(E_i; H_1, \sigma_i^2)}{p(E_i; H_0, \sigma_i^2)}, \end{aligned} \quad (44)$$

which is the optimal LR test statistic for the binary hypothesis testing problem in (6) [36]. Therefore the detection performance of tests with T_{sum} and T_{ds} will have the same performance in the absence of NU.

VI. SIMULATION RESULTS

The simulation results are divided into three parts. In the first part, performance analysis of the proposed DST scheme is done in terms of P_d vs SNR_n (dB) plots considering homogeneous sensors for different channel conditions and different number of SUs. In the second part, performance comparison of the proposed scheme is carried out with the traditional sum and MRC fusion rules. In the third part, analytical SP wall results are validated in simulation for the sum fusion rule in different scenarios followed by comparison with DST based CED.

For our simulations, we have assumed that the PU signal is a zero-mean complex and circularly symmetric Gaussian signal.

Figure 4: P_d vs SNR_n (dB) plots of DST based CED scheme for different number of homogeneous SUs in the presence ($\Delta = 0.5$ dB) and absence ($\Delta = 0$ dB) of NU. Here, $\beta_i = \beta = 0.1$, $\sigma_n^2 = 1$, and $P_i = P = \text{SNR}_n \cdot \sigma_n^2$.

NP detector is assumed with the constraint on the false alarm probability β . Generally in a practical scenario, σ_i^2 of each SU can take any value within the interval $[\sigma_{li}^2, \sigma_{ui}^2]$. However, for NP criterion, the key objective under NU is to ensure that for low SNR values $P_{fa} \leq \beta$ is maintained at all times. In order to satisfy this constraint, the threshold at the FC for all the schemes (i.e. sum, MRC and DST) is determined by setting the true value of all SUs to the upper limit of noise variance, i.e., $\sigma_i^2 = \sigma_{ui}^2, \forall i$, for a fixed value of β . On the contrary, for performance analysis and comparison under NU, the true noise variance used for the experiments is the lower noise variance limit, i.e., $\sigma_i^2 = \sigma_{li}^2$ for each SU. This corresponds to the worst case scenario where the detector has been designed to maintain the false alarm constraint even for the highest allowed noise variance while the actual noise variance is the lowest allowed value of noise variance.

Unless stated otherwise, the number of cooperating SUs is $U = 3$. In this paper, simulation results are presented for two channel conditions: AWGN and fading. For the fading channel, we have considered a fast-fading channel such that $h_i[n]$ changes after every 10 transmitted PU samples with $\delta_i = 1$. For AWGN channel, $h_i[n] = 1$. The number of observations used for evaluating received signal energy is $N = 300$ while the number of realizations used for estimating the probability of detection is 10,000. For the proposed DST scheme, deriving the distribution of the test statistic T_{ds} under both the hypotheses is a non-trivial and tedious task. As such, the threshold η_{ds} is evaluated empirically. Note that the empirical evaluation of threshold can be done off-line as it depends on all the known parameters such as noise variance $\sigma_i^2 = \sigma_{ui}^2$, β , and nominal SNR (dB).

A. Performance analysis of proposed DST scheme

Fig. 4 shows the P_d vs SNR_n (dB) plots of DST scheme for $\beta = 0.1$ assuming homogeneous SUs and same received power at all SUs. First observation from the figure is that the presence of NU leads to performance degradation for

Figure 5: Comparison of P_d vs $\text{SNR}_n(\text{dB})$ between the proposed DST and the sum fusion rule for CED with homogeneous SUs. Here $U = 3$, $\beta = 0.1$, $\sigma_n^2 = 1$, and $P_i = P = \text{SNR}_n \cdot \sigma_n^2$.

both single-user ($U = 1$) and cooperative sensing ($U = 3$) scenarios. Second observation is that the performance of DST scheme improves significantly with increase in the number of SUs, which validates the cooperation gain of the proposed scheme in the presence as well as in the absence of NU. Third observation is that the performance of the proposed DST scheme under fading is close to AWGN. Although this is also true for the sum fusion rule, the results are not shown here for conciseness. This is expected for the considered fast-fading channel with $\delta_i = 1$ for following reasons: the effects of channel fading averages out during the sensing time yielding results similar to that of AWGN while for $\delta_i = 1$, the definitions of SNR are same for the two channels. As the sensing results are same for both AWGN and fading channel considered in this paper, we only present sensing results for AWGN channel in the following sections for conciseness.

B. Comparison of the sum, MRC and DST fusion rules

In this part, performance comparison of the proposed scheme is carried out with the traditional sum and MRC fusion rules for two scenarios: (1) homogeneous sensors having same NU parameters and $P_i = P \forall i$ and (2) heterogeneous sensors having different NU parameters and $P_i \neq P \forall i$. For MRC, the test statistic is the weighted sum given in [31] by

$$T_{mrc} = \sum_{i=1}^U w_i E_i, \quad (45)$$

where the weight corresponding to the i^{th} SU is chosen as

$$w_i = \frac{P_i / \sigma_{ni}^2}{\sum_{i=1}^U P_i / \sigma_{ni}^2}. \quad (46)$$

These weights can be calculated at the FC, which is assumed to have P_i and σ_{ni}^2 values for all the sensors.

1) *Homogeneous and equal power (EP)*: The SUs are assumed to have identical NU parameters and equal received signal power. Fig. 5 shows the performance comparison of the proposed DST fusion rule to that of the sum fusion rule in

Figure 6: P_d vs P_{avg} (log scale) comparison of proposed scheme with sum and MRC rule for SUs with heterogeneous NU parameters. Here EP and UP denote the scenarios where different SUs have equal and unequal powers, respectively.

terms of P_d vs $\text{SNR}_n(\text{dB})$ plots for different NU intervals of $\Delta = 0, 0.5$ and 1 . Since in this case $P_i = P$ and $\sigma_{ni}^2 = \sigma_n^2 \forall i$, the performance of MRC will be same as the sum rule. Hence, the plots for MRC have not been included in this figure for conciseness. Note that the NU parameters and received power at all SUs are considered same so that the nominal SNRs are also same, i.e. $\text{SNR}_{ni} = \text{SNR}_n$. The nominal variance at each SU for this plot is $\sigma_n^2 = 1$. First observation from the figure is that under no NU ($\Delta = 0$), the performances of both fusion rules overlap. This results from the fact that both the test statistics T_{sum} and T_{ds} are equivalent to the optimal LR test statistic under no NU as was shown in Sec. V-E. Second observation from the figure is that for NU of $\Delta = 0.5$ dB and $\Delta = 1$ dB, performances of both the fusion rules degrade. However, proposed DST based approach significantly outperforms the traditional sum fusion rule in the presence of NU.

2) *Heterogeneous and unequal power (UP)*: All SUs have different NU parameters and the received signal power are also different for all SUs, i.e., $P_i \neq P \forall i$. The number of SUs are taken as $U = 3$. The signal power at the three SUs are chosen as $1.5P$, P and $0.5P$ such that the average power P_{avg} is P . For simulation purpose, the NU intervals for the three SUs are chosen as $\Delta_1 = 0.25$ dB, $\Delta_2 = 0.5$ dB, and $\Delta_3 = 1$ dB while the nominal noise variances are taken as $\sigma_{n1}^2 = 0.9$, $\sigma_{n2}^2 = 1$ and $\sigma_{n3}^2 = 1.1$. Fig. 6 shows the performance of DST, sum and MRC fusion rules in the form of P_d vs P_{avg} (log scale) plot. First observation from the plot is that there is slight improvement in the performance of the MRC as compared to the sum rule for the EP case. However, for the UP case, the MRC gives significant gain as compared to the simple sum rule. Finally, for both cases of EP and UP, the DST gives the best performance as compared to the sum rule as well as the MRC.

C. SP wall analysis and comparison

In order to verify the formation of SP wall in CED under NU, five different cases are taken into account based on

different combinations of nominal noise variance σ_{ni}^2 and uncertainty factor ρ_i . They are as follows:

- Case I : All SUs have identical nominal noise variance and uncertainty parameter (homogeneous).
- Case II : All SUs have different nominal noise variances but identical uncertainty factor.
- Case III : All SUs have identical nominal noise variance but different uncertainty factors.
- Case IV : All SUs have different nominal noise variances and uncertainty factors.
- Case V : NU parameters same as case IV but $P_i \neq P \forall i$.

Table II shows the NU parameters for all the 5 considered scenarios. The received signal power $P_i = P \forall i$ for cases I-IV. For generalized SNR wall, the constraint on the probabilities of detection and false alarm are $P_d \geq 0.9$ and $P_{fa} \leq \beta = 0.1$, respectively.

Table II: SP wall for the sum fusion rule in AWGN. Theory and simulations are well in par in characterizing the SP wall.

Case No.	NU parameters ($U = 3$)	SP _{wall} (Theory)	SP _{wall} (Sim.)
I	$\sigma_{n1}^2 = \sigma_{n2}^2 = \sigma_{n3}^2 = 1$ $\Delta_1 = \Delta_2 = \Delta_3 = 0.75$ dB $\rho_1 = \rho_2 = \rho_3 = 1.189$	0.349	0.3520
II	$\sigma_{n1}^2 = 0.9, \sigma_{n2}^2 = 1, \sigma_{n3}^2 = 1.1$ $\Delta_1 = \Delta_2 = \Delta_3 = 1$ dB $\rho_1 = \rho_2 = \rho_3 = 1.259$	0.4646	0.4670
III	$\sigma_{n1}^2 = \sigma_{n2}^2 = \sigma_{n3}^2 = 1$ $\Delta_1 = 0.5, \Delta_2 = 0.75, \Delta_3 = 1$ dB $\rho_1 = 1.122, \rho_2 = 1.188, \rho_3 = 1.259$	0.3475	0.3510
IV	$\sigma_{n1}^2 = 0.9, \sigma_{n2}^2 = 1, \sigma_{n3}^2 = 1.1$ $\Delta_1 = 0.25, \Delta_2 = 0.5, \Delta_3 = 1$ dB $\rho_1 = 1.0593, \rho_2 = 1.122, \rho_3 = 1.259$	0.2818	0.2846
V	Same as case IV but with $P_i \neq P$ $P_1 = 1.5P, P_2 = P, P_3 = 0.5P$ $P_{avg} = \sum_{i=1}^U P_i = P$	0.2818	0.2870

Figure 7: Sample size N (in log scale) vs average signal power, P_{avg} for the considered cases in AWGN channel. The figure clearly shows that the SP wall varies for different values of uncertainty parameters associated with the sensors.

1) *SP wall for the sum fusion rule:* Fig. 7 shows the plots of sample size N (in log scale) vs average signal power P_{avg} for all the five cases. The figure clearly demonstrates the

Figure 8: Comparison of the DST and sum fusion rules in terms of sample size N as a function of P and $\text{SNR}_n(\text{dB})$ with NU parameters corresponding to cases I and III in Table II with $\sigma_{ni}^2 = \sigma_n^2$. Here, $P_i = P \forall i$.

existence of SP wall for all the scenarios, homogeneous (case I) and heterogeneous (case II, III, IV and V). Table II shows a comparison of theoretical and simulated SP wall values for the sum fusion rule, where the theoretical values of SP wall for all the five cases are calculated using (16). For cases I-IV, we choose $P_i = P \forall i$. In case V, the NU parameters are same as case IV but the power levels at each SU are different and chosen as shown in Table II. However, irrespective of different P_i , the SP wall for case V is same as case IV. This is because $P_{avg} = \frac{1}{U} \sum_{i=1}^U P_i = P$ in this case which proves that SP wall value depends on the average signal power across all the SUs. Furthermore, it can be seen that both the theoretical and simulated values of SP wall for the all the considered cases are very close, which validates the theoretical analysis of SP wall as discussed in Section III.

2) *Comparison between DST and sum fusion rule:* Figs. 8 and 9 show the comparison between the sum and DST based CED schemes in terms of location of SP wall. The detection

Figure 9: Comparison of DST and sum fusion rules in terms of N as a function of P_{avg} for scenarios corresponding to cases II and IV in Table II.

performance at the FC is chosen as $P_{fa} \leq 0.1$ and $P_d \geq 0.9$. Fig. 8 shows the SP wall plots for cases I and III of Table II, where $\sigma_{ni}^2 = \sigma_n^2$ and $P_i = P \forall i$. This makes it possible to use $\text{SNR}_n(\text{dB})$ in addition to P as visible from the figure. First observation from the figure is that the SP wall plots for both the schemes for the fast-fading scenario almost overlaps with AWGN. Secondly, the SP wall value for the proposed scheme is significantly lower than that of sum rule. On the other hand, Fig. 9 shows the SP wall plots for cases II, IV, and V of Table II, where $\sigma_{ni}^2 \neq \sigma_n^2$, making it impossible to use $\text{SNR}_n(\text{dB})$ as visible from the figure. It can be clearly observed from this figure that even for cases II, IV and V the proposed scheme is able to significantly lower the sample size to achieve the same detection performance at the FC. Moreover, the value of SP wall for the proposed scheme is much lower than that of traditional sum rule in all the considered scenarios.

VII. CONCLUSION

In this paper, we have derived the expression for generalized SNR wall for the sum fusion rule based CED with heterogeneous SUs, i.e., when the nominal noise variances and the uncertainty intervals at different SUs are different and also the signal powers are unequal for all SUs. We have termed the generalized SNR wall as SP wall as it is possible to represent the performance limitation in terms of SP and not in SNR in more general scenario as shown in the paper. It has been also shown that when the SUs are homogeneous, i.e., have same nominal noise variances and same NU intervals, SP wall simplifies to the traditional SNR wall for local as well as for CED.

We have also proposed a DST based approach for CED. A new BMA method has been introduced based on the energy of the received signal. In the absence of NU, it has been shown that the proposed DST based CED test statistic is the same as the LR test statistic. In the presence of NU, the proposed DST based CED approach incorporates the uncertainty present in the noise variance by discounting the BMA from each SU by a rate proportional to the amount of NU associated with that SU. The detection performance of the proposed DST based CED is better than the sum and the MRC fusion rules in the presence of NU. Moreover, the proposed DST based scheme significantly lowers the SP wall values as compared to the sum fusion rule.

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