Wang, Xu Chen; Díaz-Rubio, Ana; Tretyakov, Sergei

**Microwave and millimeter-wave characterization of conductive ink film in rectangular waveguide**

*Published in:* European Microwave Week 2017

*DOI:* 10.23919/EuMC.2017.8230976

Published: 19/12/2017

*Document Version*  
Peer reviewed version

*Please cite the original version:*  
https://doi.org/10.23919/EuMC.2017.8230976

This material is protected by copyright and other intellectual property rights, and duplication or sale of all or part of any of the repository collections is not permitted, except that material may be duplicated by you for your research use or educational purposes in electronic or print form. You must obtain permission for any other use. Electronic or print copies may not be offered, whether for sale or otherwise to anyone who is not an authorised user.
Microwave and Millimeter-wave Characterization of Conductive Ink Film in Rectangular Waveguide

Xu-chen Wang 1, Ana Díaz-Rubio 2, Sergei Tretyakov 3
Department of Electronics and Nanoengineering, Aalto University
P. O. Box 15500, FI-00076 Aalto, Finland

1xuchen.wang@aalto.fi

Abstract—An accurate approach to characterize the sheet impedance of thin conductive ink in rectangular waveguide is reported. This method allows retrieval of the sheet impedance without prior characterization of the supporting dielectric substrate. The ink sample is positioned between two waveguide flanges, introducing a discontinuous gap. The effect of the physical discontinuity can be removed by an additional measurement of the sample without ink layer. We have examined the extraction accuracy of the proposed method within the operating band of the waveguide, and also for various substrate thicknesses. Experimental results demonstrate the applicability of the proposed approach for sheet impedance measurements of ink films on both thin or thick (up to about quarter wavelength) substrates.

I. INTRODUCTION

Ink-printing technologies are quickly developing and it is expected that they will be widely used in the fabrication of millimeter and terahertz circuits as a pathway to low-cost, high efficiency and large-area electronics and photonics [1]–[3]. In last years, various printing technologies such as inkjet printing, screen printing, offset, and reverse offset printing (RO) have been reported [4]–[7]. The sheet impedance of ink (conductivity of the printed ink as well as the sheet reactance) depends largely on the printing method and sintering environment (curing time and temperature), varying several orders of magnitude [8]. Therefore, it is important to accurately measure the electrical properties of printed ink films at the operational frequencies.

Many works have been devoted to measurements of sheet impedance of thin film. Among them, measurements in waveguides have been repeatedly reported for broadband characterization [9]–[12]. For sheet impedance measurements in rectangular waveguide setups, a crucial issue is the placement of the sample. If the sample is cut to the size of the waveguide cross section and then inserted into the waveguide opening, it is hardly possible to ensure accurate orthogonal positioning of the sample and good ohmic contact between the conductive film and the walls of the waveguide. Practically, there always exist small gaps with non-negligible parasitic capacitance between the sample and the waveguide walls [12]. In order to ensure good contact of the thin conductive film with the waveguide walls, printed samples are placed between two flanges of a waveguide junction [11]. However, in this case the two metal flanges are not connected galvanically [see Fig. 1]. Energy leakage along the flange plates and parasitic junction reactances need to be carefully considered to avoid significant errors in the extracted values of the sheet impedance.

II. METHODOLOGY

In the proposed measurement method, a rectangular waveguide with the cross-section dimensions \( a \times b \) operating in its fundamental TE\(_{10}\) mode is employed for wave confinement. A dielectric substrate with an ink layer deposited on its one side, is fixed between two waveguide flanges, as shown in Fig. 1. The measurement comprises two steps: (i) characterization of the discontinuity between the two flanges (including the holders) after the insertion of a naked substrate sample; (ii) characterization of the conductive ink printed on the substrate.

In this set-up, the gap between two flanges consists of four parts [see Fig. 1]: dielectric substrate, two metal flanges, and metallic screws. Theoretically, the complex environment in the gap can be represented by an equivalent \( \Pi \)-circuit. In the most general case, the circuit is defined by three independent components (called gap parameters): a series impedance, \( X_g \) and two parallel admittances, \( Y_{g1} \) and \( Y_{g2} \). Since the waveguide gap with only naked substrate is a symmetric system, in this case the two parallel admittances are equal \( Y_{g1} = Y_{g2} = Y_g \).
expressions in terms of the S side of the substrate. The thin layer is modelled by a parallel circuit can be described in terms of its ABCD matrix for solving Y_21.

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = M_1 M_2 M_3 = \begin{bmatrix} 1 + X_g Y_g & X_g \\
X_g Y_g + 2Y_g & 1 + X_g Y_g \end{bmatrix}, \quad (1)
\]

where \(M_1 = M_3 = \begin{bmatrix} 1 & 0 \\
y_g & 1 \end{bmatrix}\) and \(M_2 = \begin{bmatrix} 1 & X_g \\
0 & 1 \end{bmatrix}\).

Each element in (1) can be expressed in terms of the S-parameters of the set-up [13]. \(X_g\) and \(Y_g\) can be solved by equating the matrix element B and C, respectively, to their expressions in terms of the S-parameters:

\[
X_g = B = Z_0 \frac{(1 + S_{11})^2 - S_{21}^2}{2 S_{21}}, \quad (2)
\]

\[
X_g Y_g^2 + 2Y_g = \frac{(1 - S_{11})^2 - S_{21}^2}{2 S_{21} Z_0}. \quad (3)
\]

Here \(Z_0 = \omega \varepsilon_0 / \sqrt{\omega^2 \varepsilon_0 - \left(\frac{1}{Z_0}\right)^2}\) is the characteristic impedance of the TE_{10} mode in the waveguides.

Then, we characterize the conductive ink deposited on one side of the substrate. The thin layer is modelled by a parallel admittance \(Y_{s'}\), as depicted in Fig. 2. Following the same approach as in the previous step, the ABCD matrix of the gap with the impedance sheet is expressed as

\[
\begin{bmatrix} A' & B' \\
C' & D' \end{bmatrix} = M_1 M_1' M_2 M_3'.
\]

\[
\begin{bmatrix} A' & B' \\
C' & D' \end{bmatrix} = M_1 M_1' M_2 M_3' = \begin{bmatrix} a_{11} & a_{12} \\
a_{21} & a_{22} \end{bmatrix}, \quad (4)
\]

where \(M_1 = \begin{bmatrix} 1 & 0 \\
y_{s'} & 1 \end{bmatrix}\), \(M_1' = \begin{bmatrix} 1 & Y_{s'} \\
0 & 1 \end{bmatrix}\), \(M_2 = \begin{bmatrix} 1 & X_s \\
0 & 1 \end{bmatrix}\), and \(M_3' = \begin{bmatrix} 1 & Y_{s'} \\
0 & 1 \end{bmatrix}\).

For solving \(Y_{s'}\) from measured S-parameters, we should carefully use the S-parameters. As is demonstrated in [11], extracting the ink parameters from the measured reflection coefficient \(S_{11}'\) leads to significant errors if the ink layer is highly conductive (\(|S_{11}'| \approx 1\). It is important to note that due to leakage of energy between the flanges the reflection coefficient \(S_{22}'\) from the side of the substrate which is not covered by conducting ink is much smaller than \(S_{11}'\). This allows us to improve extraction accuracy by using both transmission and reflection coefficients \((S_{21}'\) and \(S_{22}'\), yet avoiding the inevitable uncertainties related to high reflections from highly conducting inks.

\[
S_{21}' = \frac{2}{A' + B'/Z_0 + C' Z_0 + D'}, \quad (5)
\]

\[
S_{22}' = \frac{-A' + B'/Z_0 - C' Z_0 + D'}{A' + B'/Z_0 + C' Z_0 + D'}, \quad (6)
\]

where \(A', B', C',\) and \(D'\) can be expressed in terms of the gap parameters (4). Assuming that \(Y_{s1}'\) and \(Y_{s2}'\) are equal to \(Y_g\) obtained in the characterization of the bare substrate in the gap, the sheet impedance of the conductive layer \(Z_s = 1/Y_s\) can be easily solved:

\[
Z_s = \frac{S_{21}' Z_0}{1 - S_{22}' - S_{21}' - Z_0 Y_g (1 + S_{22}' + S_{21}')}. \quad (7)
\]

In this work, the proposed approach based on formula (7) is compared with the simple extraction method used in [11], where the effects of the gap are not considered. The extraction formula [11] reads

\[
Z_s = -\frac{S_{21}' Z_0 (Z_0 \cos(k_d d) + j Z_d \sin(k_d d))}{D}, \quad (8)
\]

where \(D = 2Z_0 Z_d (S_{21}' \cos(k_d d) - 1) + j (Z_0^2 + Z_d^2) S_{21}' \sin(k_d d),\)

\(k_z = \sqrt{\omega^2 \varepsilon_0 - \left(\frac{1}{Z_0}\right)^2}\) is the propagation constant of the TE_{10} mode in the substrate-filled waveguide, \(Z_d = \omega \varepsilon_d / k_z\) is the characteristic impedance of this waveguide section filled with the substrate material, \(d\) and \(\varepsilon_d\) denote the thickness and permittivity of the dielectric substrate, respectively.

III. NUMERICAL RESULTS

Using Ansoft HFSS, a rectangular waveguide with the dimensions 2.54 mm × 1.27 mm is modeled, including two metallic holders for fixing the samples [see Fig. 1]. The dimensions of the metallic holders are 22 mm × 20 mm. The range of frequencies under study is from 75 GHz to 110 GHz, ensuring that only TE_{10} mode is propagating in the waveguides. In this analysis, a flexible polyimide substrate with the thickness of \(d = 125 \mu m (\sim 0.07 \lambda_d = 0.0725 \lambda_{110})\) is used as a dielectric substrate. The complex relative permittivity of the substrate is 3.2(1 - j0.045). The substrate size is arbitrary selected as 5 mm × 5 mm so that it can fully cover the rectangular waveguide openings of the holders.

We start with the extraction of the sheet impedance defined as \(Z_s = 100 - j50 \Omega/\text{sq}\) in the frequency range of 75–110 GHz. Fig. 3 shows a comparison of the results given by (7) and (8). It is evident that our proposed method gives much smaller errors in the extraction of both resistive and reactive parts of the sheet impedance, while the extraction inaccuracy using (8) is significant and the corresponding extracted curve is not flat in the considered frequency window. The impact
of the gap thickness on the retrieval accuracy of the two approaches is shown in Fig. 4. For a given value of the sheet impedance $Z_s = 100 \, \Omega/sq$ at a single frequency point $f = 9.5 \, \text{GHz}$ we have extracted the impedance values for different thicknesses of the set-up gap (in the range $0.05 \lambda_d$–$0.8 \lambda_d$). It is expected that close to the Fabry-Perot resonant thickness $d = \lambda_d/2$, the methods based on measurements of the transmission coefficient fail, and we indeed observe parasitic peaks in the extracted values. On the other hand, for $d < \lambda_d/2$, the proposed approach exhibits small errors. As a comparison, for the method proposed in [11], the extraction uncertainty deteriorates in the investigated thickness range as the thickness $d$ increases.

Our extraction method is based on the use of $S_{22}$. Therefore, we must investigate its accuracy when $|S_{22}|$ is large due to extremely thin substrate. We numerically introduce perturbations of $\pm 5\%$ in magnitude and $\pm 10\%$ in phase for $S_{22}$, and calculate the average extraction errors of the resistivity in the frequency band (see Fig. 5). The results show that the reduced gap thickness does not degrade the accuracy of this extraction method.

The accuracy of the proposed method is mainly affected by the uncertainty of measured S-parameters. We have investigated the extraction uncertainty contributed by two separate measurements (substrate and ink sample), using differential analysis. The results in [14] tell that the measurement error of this method is below 10% for sheet resistivity from 1Ω/sq to 1000 Ω/sq.

IV. EXPERIMENTAL VALIDATION

The thin film used in our experiment is silver nanoparticles (Ag NP)-based ink printed on polyethylene naphthalate (PEN) substrate. The measurement steps comprise:

1) Adhere a bare PEN substrate to one of the metallic holders and ensure that the rectangular waveguide openings are completely covered by the substrate.
2) Connect the two waveguide sections and measure both the magnitude and phase of the scattering parameters. Then, substitute the $S$-parameters to Equations (2) and (3) and calculate the gap parameters $X_g$ and $Y_g$.
3) Cut a sample piece with printed ink and attach it to the holder. The size and thickness of the substrate should be the same as that used in Step 1, to create an identical gap structure.
4) Connect the waveguides and measure the scattering parameters. Using (7), the ink impedance is finally extracted.

Figure 6 displays the extracted sheet impedance of a sample with $Z_s = 4.5 \, \Omega/sq$ measured under DC condition. The ink layer is printed using reverse offset (RO) technology and sintered for 60 minutes under 180°C. The substrate of the ink sheet is a single-layer PEN with $d = 125 \, \mu\text{m}$. As expected, it is evident that extraction using (7) is more accurate than (8) which suffers more severe oscillations in both real and imaginary parts of the extracted value. The predicted resistivity by the method proposed in [11] is always lower than the actual value of the resistivity. This is because it assumed that the measured $S_{21}$ excludes energy leakage through the flange gap, which creates an illusion that the measured sample is more reflective than it really is.
In order to verify the extraction capability of the proposed method for thick substrates, three additional PEN layers are adhered to the sample creating a thick substrate with $d = 500 \, \mu m$ (about $0.28 \lambda_g$). The measured results using (7) and (8) are compared in Fig. 7. Obviously, for a thick substrate, the proposed method still offers high accuracy and stability within the whole frequency window, while the extraction method in [11] totally fails and the extracted curves deviate from the expected value with severe perturbations. The inaccurate results using (8) are caused by multiple resonances triggered by reflections of the TEM wave travelling inside the gap between the two flanges.

Fig. 6. Extracted sheet impedance of the measured sample using (7) and (8). $Z_0 = 4.5 \, \Omega/\text{sq}$ via DC method.

Fig. 7. The measured results for the sample with four-layer substrates using (7) and the method in [11]. The ink impedance is $Z_0 = 4.5 \, \Omega/\text{sq}$ predicted under DC condition.

V. CONCLUSION

We have proposed an accurate method for sheet impedance measurements in a non-consecutive waveguide set-ups, most suitable for measuring thin ink layers supported by dielectric substrates. By using an equivalent circuit of the gap formed by two waveguide flanges and the substrate, we can successfully remove the effect of the waveguide gap without the necessity of substrate characterization. Numerical studies have been conducted to verify the validity for broadband extraction and also for thick substrates. Experimental results of ink sheet impedance measurements with thin and thick substrates further confirm the validity and accuracy of the method. This method is not limited to the measurements of ink impedance, it can be used in the characterization of many thin conductive materials such as silver nanowire composites and carbon-based layers.

ACKNOWLEDGMENTS

This work was supported in part by the Academy of Finland through the LATERA project. The authors would like to thank Subash Khanal, Irina Nefedova and Juha Ala-Laurinaho for their help in measurements, and Viktar Asadchy for his technical discussion. Ink samples have been provided by Sneck Asko, Alastalo Ari and Mäkelä Tapio from VTT Technical Research Centre of Finland Ltd.

REFERENCES