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Interference and Background Noise Effects in Wireless Networks with Poisson Fields of Transmitters

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Abstract—In this work, we analyze joint effects of background Gaussian noise and interference in wireless random networks where transmitting nodes form Poisson point processes (PPPs). We analyze different spatial models of interferers such as PPPs and Poisson cluster processes. In our analysis, Rayleigh fading and path-loss effects are taken into account.

Index Terms—Fading, interference, outage probability, Poisson cluster processes, Poisson point processes, wireless networks.

I. INTRODUCTION

Wireless network operating is characterized by interference coming from both licensed and unlicensed sources. Analysis and design of wireless networks under fading, path-loss, interference, and background noise effects, are challenging tasks where interference and transmitting nodes spatial distributions play important roles. This is due to path-loss effects that cannot be considered negligible in many wireless applications. Therefore, spatial models of transmitting and interfering nodes are of special interest since their statistical distributions affect the statistics of signal-to-joint interference ratio (SINR), which is a crucial network factor specifying its performance metrics.

Regular grids and Poisson point processes (PPPs) are frequently used spatial models of transmitting nodes. While regular grids were primarily used for modeling of cellular networks, the PPPs were the mainly applied to modeling of ad hoc networks. Recent studies [1]–[3] showed, however, that PPPs match well to modeling of cellular networks too. It was proven that PPPs model real cellular networks as accurately as regular grids do, but in contrast to the regular grids, PPPs provide tractable estimates of network performance metrics.

In many practical scenarios, statistical models of real interference diverge significantly from normal distributions, and Gaussian interference models often become non-acceptable [3], [4]. Frequent presumptions about the interferer spatial distribution are also based on PPPs [1]–[6]. Recently, Poisson cluster processes (PCPs) attracted much interest in network modeling because often the nodes in wireless networks tend to cluster [7]–[8]. The PCPs are more sophisticated Poisson models applied to modeling of transmitting and interfering nodes.

This work concentrates on analyzing of the outage probability (OP) in wireless networks where the transmitting nodes form PPPs. We analyze two spatial models of interferers,

PPPs and PCPs, and additionally take into account Gaussian background noise, Rayleigh fading and path-loss effects. The OP is a very important characteristic of a wireless network affecting many its functionalities. The OP provides statistical characterization of node connectivity, and thus it plays an important role in design of wireless networks, for instance, in cell planning.

Wireless networks with different types of interferers and under different operational scenarios were analyzed in many works that is impossible to refer to in this paper. We can refer, *e.g.*, to [9]–[11] where random node distributions were taken into account along with fading effects. Considerations for Poisson fields of interferers were restricted the mainly by interference-limited scenarios.

The paper is organized as follows. In Section II, we present statistical models of wireless propagation. The OP analysis is given in Section III, Section IV presents numerical estimates, and Section V concludes the work.

II. SYSTEM MODEL

We consider wireless networks operating in two-dimensional Euclidean space $\mathbb{R}^2$. Below, we present models of wireless propagation.

A. Path-loss Model

We apply a conventional distance-dependent path-loss model [12]. The path-loss signal power attenuation γ_{PL} is specified by the transmitter (Tx)-receiver (Rx) distance R as

$$\gamma_{PL} = K_0 (R)^{-\eta} \quad (1)$$

where η is the path-loss exponent, and K_0 denotes the value of γ_{PL} at $R = 1$. In the typical environment, $\eta \in [2, 6]$ [12].

B. Poisson Point Processes and Poisson Cluster Processes

The homogeneous PPP is a widely applied model of node spatial distribution characterized by the node density λ . A PPP feature is that the number of points inside any compact set $\mathcal{B} \in \mathbb{R}^2$ is a Poisson random variable with the mean $\lambda|\mathcal{B}|$ (where $|\mathcal{B}|$ is the area of $\mathcal{B}$), and the numbers of points in non-overlapping areas are independent. There exists a large number of good tutorials on PPPs such as [3], [13].

PCPs are less studied and more sophisticated point processes than PPPs. In a PCP, parent points (cluster centers) form a PPP with the density λ_p . Offspring points (cluster members) conditioned on a parent point are independent, and identically distributed around the cluster center according to a symmetric normal distribution with a given variance Σ^2 , see Fig. 1. The number of cluster members may be fixed or random. If the number of cluster points follows the Poisson distribution, then we deal with the Thomas cluster process.

III. OUTAGE PROBABILITY ANALYSIS

Without loss of generality, based on Slivnyak's theorem [14], we assume that the receiver of interest is located at the origin. We consider a Poisson field of transmitters Ξ_T . A set of interferers Ξ_I is specified below in subsections III.A-III.B. Then taking into account background Gaussian noise with the variance σ^2 , we can specify the SINR for a Tx belonging to Ξ_T and located at x as

$$\text{SINR} = \frac{P_T K_0 \|x\|^{-\eta} |h_x|^2}{\underbrace{\sum_{I \in \Xi_I} P_I K_0 \|y_I\|^{-\eta} |h_I|^2 + \sigma^2}_{I_{\text{aggr}}}} = \frac{P_T K_0 \|x\|^{-\eta} |h_x|^2}{I_{\text{aggr}}} \quad (2)$$

where P_I is the interferer transmit power, $\|\cdot\|$ denotes the norm, $|h_x|^2$ and $|h_I|^2$ are the channel power gains representing the fading effects. In this work, we focus on Rayleigh fading leaving more sophisticated distributions for a future work. We assume that both $|h_x|^2$, $|h_I|^2 \sim \exp(1)$.

Then the OP, $P_{\text{out}}(\gamma_0)$ can be specified as the probability that the SINR defined by (2) for all $x \in \Xi_T$ is less or equal to a predetermined level γ_0 , that is

$$P_{\text{out}}(\gamma_0) = Pr \left\{ \max_{x \in \Phi_T} P_T L_0 |h_x|^2 \|x\|^{-\eta} \leq \gamma_0 I_{\text{aggr}} \right\} \quad (3)$$

where P_T is the Tx power.

Then taking into account the cumulative distribution function (CDF) of exponential distribution, we find that

$$P_{\text{out}}(\gamma_0) = E_{I_{\text{aggr}}, x} \left\{ \prod_{x \in \Phi_T} \left(1 - \exp \left[-\gamma_0 (L_0 P_T)^{-1} I_{\text{aggr}} \|x\|^\eta \right] \right) \right\} = E_{I_{\text{aggr}}} \left\{ \exp \left[-\lambda_T \int_{x \in \mathbb{R}^2} \exp \left(-\gamma_0 (L_0 P_T)^{-1} I_{\text{aggr}} \|x\|^\eta \right) dx \right] \right\} \quad (4)$$

where we used the PPP probability generating functional [3]. Expressing the integral in (4) in terms of gamma functions, we find that the OP can be expressed via the moment generating

function (MGF) of $I_{\text{aggr}}^{-\frac{2}{\eta}} M_{I_{\text{aggr}}^{-\frac{2}{\eta}}}(-s) \triangleq E \left\{ \exp(-s I_{\text{aggr}}^{-\frac{2}{\eta}}) \right\}$ (where E means the expectation) as

$$P_{\text{out}}(\gamma_0) = E_{I_{\text{aggr}}} \left\{ \exp \left[-\pi \lambda_T \underbrace{\left(\frac{P_T L_0}{\gamma_0} \right)^{\frac{2}{\eta}} \Gamma \left(1 + \frac{2}{\eta} \right)}_{\Omega} I_{\text{aggr}}^{-\frac{2}{\eta}} \right] \right\} = M_{I_{\text{aggr}}^{-\frac{2}{\eta}}}(-\Omega). \quad (5)$$

We note that the parameter Ω in (5) is real and $\Omega > 0$.

Further OP evaluation depends on the concrete interference type since it specifies $M_{I_{\text{aggr}}^{-\frac{2}{\eta}}}(-s)$. Using a series expansion of $\exp(-s I_{\text{aggr}}^{-\frac{2}{\eta}})$, representing $I_{\text{aggr}}^{-\frac{2m}{\eta}}$ in terms of gamma function as $I_{\text{aggr}}^{-\frac{2m}{\eta}} = \frac{1}{\Gamma(\frac{2m}{\eta})} \int_0^\infty \exp(-tI) t^{\frac{2m}{\eta}-1} dt$ [15, vol. 1, eq. (2.3.18.2)], and applying Fubini's theorem [16], we can find that

$$M_{I_{\text{aggr}}^{-\frac{2}{\eta}}}(-s) = 1 + \sum_{m=1}^{\infty} \frac{(-s)^m}{m! \Gamma(\frac{2m}{\eta})} \times \int_0^\infty t^{\frac{2m}{\eta}-1} M_{I_{\text{aggr}}}(-t) dt. \quad (6)$$

Thus, (6) relates $M_{I_{\text{aggr}}^{-\frac{2}{\eta}}}(-s)$ and $M_{I_{\text{aggr}}}(-s)$ and in view of (5), it provides an opportunity of OP evaluation on the basis of known $M_{I_{\text{aggr}}}(-s)$. From (2), we can find that

$$M_{I_{\text{aggr}}}(-t) = M_I(-t) \exp(-t\sigma^2) \quad (7)$$

where we separate interference coming from interferers belonging to Ξ_I and background Gaussian noise.

Below, we consider two different spatial distributions of interfering nodes, which are Poisson and Poisson cluster fields of interferers.

A. Interfering Nodes Form Poisson Point Process

In [1], [4]- [6], MGF expressions were derived for different operational scenarios with Poisson fields of interferers. The MGF expressions have a similar structure that can be represented as

$$\mathcal{M}_I(-t) = \exp \left(-K t^{\frac{2}{\eta}} \right) \quad (8)$$

where K is defined by network and fading parameters with explicit expressions given in [1], [4]- [6].

Then plugging (8) into (6) and taking into account (5), we find that

$$P_{\text{out}}(\gamma_0) = 1 + \sum_{m=1}^{\infty} \frac{(-\Omega)^m}{m! \Gamma(\frac{2m}{\eta})} \times \underbrace{\int_0^\infty t^{\frac{2m}{\eta}-1} \exp \left(-K t^{\frac{2}{\eta}} \right) \exp(-t\sigma^2) dt}_{\text{Int}_m}. \quad (9)$$

Next we note that the integral in (9) can be solved in a closed form with the help of [15, vol. 4, eq. (2.2.1.22)] if $2/\eta = l/k$ where l and k are integers. In this case,

$$\text{Int}_m = \frac{k^{1/2} l^{\nu+1/2} (\sigma^2)^{-\nu-1}}{(2\pi)^{(k+l)2-1}} \times G_{l,k}^{k,l} \left[\frac{K^k l^l}{k^k \sigma^{2l}} \middle| \begin{matrix} \Delta(l, -\nu) \\ \Delta(k, 0) \end{matrix} \right] \quad (10)$$

where $G_{l,k}^{k,l}[\cdot]$ is the Meijer G function, $\Delta(k, a) = \frac{a}{k}, \dots, \frac{a+k-1}{k}$, and $\nu = \frac{2m}{\eta} - 1$. It is worth mentioning that the Meijer G function is implemented in many standard software packages.

Using (9)-(10) for practical purposes requires truncation of infinite series in (9) by $N_{\max}$ terms. Thus, the residual $r_{N_{\max}}$ must be assessed. Taking into account that the parameter Ω in (5) is real, and $\Omega > 0$, we note that the series in (9) is alternating, and thus $r_{N_{\max}}$ can be assessed with the help of Leibnitz rule as [15, vol. 3, I.3.3.5]

$$\left| \sum_{m=N_{\max}+1}^{\infty} \frac{(-\Omega)^m \text{Int}_m}{m! \Gamma\left(\frac{2m}{\eta}\right)} \right| < \frac{(\Omega)^{N_{\max}+1} \text{Int}_{N_{\max}+1}}{(N_{\max}+1)! \Gamma\left(\frac{2(N_{\max}+1)}{\eta}\right)}. \quad (11)$$

B. Interfering Nodes Form Poisson Cluster Process

In this subsection, we consider the PCPs with a fixed point number M in a cluster. MGF expressions for this scenario were obtained in [7]. In this case, interference at the Rx of interest depends on whether the Rx belongs to a cluster or does not belong. Under the former scenarios, both the intra- and inter-cluster interference must be taken into account, while under the latter scenarios, only inter-cluster interference is of interest.

In this sub-section, we analyze OP lower bounds by using lower bounds on MGFs of intra- and inter-cluster interference derived in [7, eq.(16) and eq. (19)]. Both bounds are represented by (8) with K equal to $K = K_1$ and $K = K_2$ for intra- and inter-cluster interference, respectively. Then taking into account independence of two interference types, we note that for real $s > 0$, the MGF of joint inter-plus- intra-interference, can be lower-bounded as

$$\mathcal{M}_I(-s) \geq \exp \left[-(K_1 + K_2) s^{\frac{2}{\eta}} \right] \quad (12)$$

where $K_1 = \frac{P_1^{\frac{2}{\eta}} 2\pi/\eta}{\sin(2\pi/\eta)} \frac{(\tilde{m}-1)}{4\tilde{\Sigma}^2}$, $K_2 = P_1^{\frac{2}{\eta}} \pi \lambda_p \tilde{m} \frac{2\pi/\eta}{\sin(2\pi/\eta)}$, and $\tilde{m} < M$ is the average number of active nodes (following a Poisson distribution) within each cluster.

Then using the integral comparison theorem, we find from (9)-(10) an approximate OP lower bound as

$$P_{\text{out}}(\gamma_0) \geq 1 + \sum_{m=1}^{N_{\max}} \frac{(-\Omega)^m}{m! \Gamma\left(\frac{2m}{\eta}\right)} \text{Int}_m |_{K=K_{\text{appr}}} \quad (13)$$

where K_{appr} depends on whether the Rx of interest belongs to a cluster or does not belong. For the former scenarios, $K_{\text{appr}} = K_1 + K_2$, and for the latter scenario, $K_{\text{appr}} = K_1$.

The difference δ between an exact OP lower bound and approximate bound given by (13) is defined by the residual of infinite series in (9), and δ is always upper bounded by (11).

From (5), it can be seen that the parameter Ω in (9), (11), and (13) depends on γ_0 thus specifying dependence of $P_{\text{out}}(\gamma_0)$ on γ_0 .

IV. NUMERICAL RESULTS

In this section, we present numerical results for a few operational scenarios based on analytical estimates of Section III. Taking into account expressions for K given in [1], [4]-[6], as well the expressions for K_1 and K_2 in (12), one can note that all OP estimates given in Section III can directly be expressed in terms of signal-to-noise power ratio $\text{SNR} = \frac{P_T}{\sigma^2}$ and signal-to-interference power ratio $\text{SIR} = \frac{P_T}{P_I}$. In all numerical examples, the value of path-loss exponent $\eta = 7/2$, and $\gamma_0 = 0$ dB.

In Fig. 2-3, we show OP numerical estimates for scenarios where both TxS and interferers form PPPs. In Fig. 2, OP estimates are shown versus the SNR for a few values of $\text{SIR} = P_T/P_I$ and λ_T/λ_I . In all considered cases, irreducible error floors are observed due the presence of interferers.

In Fig. 3, OP numerical estimates are shown versus the SIR for a few values of SNR and λ_T/λ_I . As in Fig. 2, error floors are observed due to background noise. For SIR values considered in our evaluations, the error floor is evident for $\text{SNR}=0$ dB. The error floors can conveniently be evaluated based on results of Section III.

Finally, in Fig. 4, we show numerical results for scenarios where TxS form PPPs, and interferers form PCPs. We consider scenarios where the Rx of interest belongs to a cluster and evaluate OP lower bounds versus the SIR (13) for a few values of SNR, λ_T/λ_I , and $\tilde{m}$. For comparison, we also present OP simulation results, which show that the OP lower bound are rather tight.

V. CONCLUSION

The outage probability is an important performance metric of wireless networks, which characterizes statistically many network functions. In this work, we present a technique providing OP analytical evaluation under interference and background Gaussian noise. Rayleigh fading and path-loss effects are taken into account. We analyze scenarios where the transmitters form Poisson point processes, and interferers form either Poisson point processes or Poisson cluster processes. For former scenarios, we derive OP analytical expressions, while for latter scenarios, we obtain OP lower bounds.

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Fig. 1. Poisson point process (squares) and Poisson cluster processes (dots).

Fig. 2. Outage probability versus SNR for different values of SIR and λ_T/λ_I ; transmitters and interferers form PPPs.

Fig. 3. Outage probability versus SIR for different values of SNR and λ_T/λ_I ; transmitters and interferers form PPPs.

Fig. 4. Outage probability and OP lower bounds versus SIR for different values of SNR and λ_T/λ_I ; transmitters form PPP, and receivers form PCPs.

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