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Noiseless Quantum Measurement and Squeezing of Microwave Fields
Utilizing Mechanical Vibrations

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A process which strongly amplifies both quadrature amplitudes of an oscillatory signal necessarily adds noise. Alternatively, if the information in one quadrature is lost in phase-sensitive amplification, it is possible to completely reconstruct the other quadrature. Here we demonstrate such a nearly perfect phase-sensitive measurement using a cavity optomechanical scheme, characterized by an extremely small noise less than 0.2 quanta. The device also strongly squeezes microwave radiation by 8 dB below vacuum. A source of bright squeezed microwaves opens up applications in manipulations of quantum systems, and noiseless amplification can be used even at modest cryogenic temperatures.

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The sensitive measurement of electromagnetic waves is instrumental in science and technology. A sinusoidally oscillating field \( X(t) = X_1 \cos(\omega t) + X_2 \sin(\omega t) \) at the frequency \( \omega \) is characterized by the quadrature amplitudes \( X_1 \) and \( X_2 \). In quantum mechanics, the quadratures are noncommuting observables which cannot be measured simultaneously. In a usual measurement which responds equally to both quadratures, noise must hence increase by at least half the zero-point fluctuations [1, 2].

In a phase-sensitive measurement, the two quadratures are amplified at different gain factors \( \mathcal{G}_1 \) and \( \mathcal{G}_2 \), such that the output quadratures are \( Y_i = \mathcal{G}_i X_i \). If either of the gains becomes very small and thus the information in this quadrature is discarded, the other quadrature can be perfectly measured. At the same time, the fluctuations in the discarded quadrature can become squeezed below the zero-point fluctuation level. Here we demonstrate such a nearly perfect measurement, proposed very recently [3], of microwave light using a cavity optomechanical setup. Along with the practical device that shows promise for applications, we realize the phase-mixing amplifier [3], and evolve the concept further.

The most sensitive measurements of microwave fields have taken advantage of nonlinearities of Josephson junctions [4–8]. Since late 1980s [9], Josephson junction parametric amplifiers have reached the impressive system noise performance of 0.62 added quanta of noise in the phase-insensitive mode, close to the fundamental limit, and 0.14 quanta in the phase-sensitive mode [10]. Therefore, these amplifiers are currently actively used in quantum science. Also electromechanical systems have been investigated to this end [11–15]. In recent work, Refs. [14, 15] demonstrate a phase-insensitive amplifier with a noise relatively low but not quite yet at the quantum limit.

Our realization of a practically noiseless amplifier can be pictured as a generic cavity optomechanical setup. It consists of a superconducting microwave resonator, the cavity, with frequency \( \omega_c \), coupled to a 15 \( \mu \)m wide membrane [16] vibrating at the frequency \( \omega_m \), as seen in Fig. 1(b). The two systems are coupled via the radiation-pressure coupling \( H_{\text{int}} = g_0 n_c (b^\dagger + b) \), where \( n_c = a^\dagger a \) is the number of microwave cavity photons, \( x = b^\dagger + b \) is the (dimensionless) position operator of the mechanical oscillator, and \( g_0 \) is a coupling constant. The cavity and the oscillator have the respective decay rates \( \kappa \) and \( \gamma \). The cavity is driven by two strong microwave tones of frequencies \( \omega_+ = \omega_c + \omega_m + \delta \) and \( \omega_- = \omega_c - \omega_m - \delta \) [Fig. 1(a)]. The pumps induce respective cavity fields of amplitudes \( \alpha_+ \) and \( \alpha_- \). Here, \( \delta \) describes the detuning from the blue or red sideband coresonance condition [17]. The pumping results in an enhanced linear coupling of strength \( G_\pm = g_0 \alpha_\pm \).

This pump scheme is related to backaction evading measurements [18] and squeezing [19–21] of the mechanical oscillator, and to the dissipative squeezing recently proposed in Ref. [22]. However, introducing a detuning \( \delta \gtrsim (G_+^2 - G_-^2)/\kappa \) drastically changes the resulting physics. In the following, we suppose the resolved-sideband regime, where \( \omega_m \gg \kappa \). The Hamiltonian describing this system is [23]

\[
H = \delta b^\dagger b + G_+ (a^\dagger b^\dagger + ba) + G_- (a^\dagger b + b^\dagger a).
\] (1)

We make a Bogoliubov transformation of the cavity to a set of new operators \( a \) so that \( a = ua - v a^\dagger \). We choose \( u = \cosh(\xi) \), \( v = \sinh(\xi) \) with the real parameter \( \xi \) satisfying \( \tanh(\xi) = G_+/G_- \). The resulting cavity-oscillator Hamiltonian is that of a beam splitter with coupling strength \( G_{BG} = \sqrt{G_+^2 - G_-^2} \), known to lead to the cooling of the mechanical oscillator [16]. If the cavity is over-coupled, signals sent to it are completely reflected; i.e., the reflected signal at a given frequency \( \omega \) experiences only a
phase shift, or \( \sigma_{\text{out}}(\omega) = e^{i\phi(\omega)}\sigma_{\text{in}}(\omega) \). In a large range of frequencies determining the amplifier bandwidth, different phase shifts of the Bogoliubov wave at positive and negative frequencies translate into phase-sensitive or phase-mixing amplification in the original cavity frame [23]. Even though the mechanical oscillator has a high thermal occupation number \( n_{\text{m}}^T = k_B T/\hbar\omega_m \gg 1 \), the added noise referred to the input is of the order of \( (\kappa/G_{\text{BG}}^2)n_{\text{m}}^T \) and can be almost neglected in our setup. One can thus reach very nearly quantum limited operation even when the mechanical oscillator is far from its quantum ground state.

At this point, let us discuss a phase-sensitive amplifier [11], referring to Fig. 1(c). At the input on top of a coherent signal, there is quantum noise, which usually does not show a phase preference. Hence the possible values of the quadrature amplitudes \( X_1 \) and \( X_2 \) of the input signal \( X \) fall uniformly inside the gray circle representing the variance. Following phase-sensitive amplification, the input noise gets squeezed into an ellipse owing to unequal gains \( G_1 \) and \( G_2 \) for the input quadratures. The principal axes in the amplified input noise define the preferred (output) quadratures which obey \( Y_1 = G_1 X_1 \), \( Y_2 = G_2 X_2 \), and the average amplified signal is \( Y^2 = 1/2(Y_1^2 + Y_2^2) = G^2 X^2 \), with the total gain \( G \).

Phase-sensitive amplification requires specifying a carrier frequency around which the quadrature operators are defined [1]. In our setup, the carrier frequency is the center frequency of the pumps, \( \omega_c \equiv (\omega_+ + \omega_-)/2 \) that here also roughly equals \( \omega_c \). The carrier frequency not only defines the output quadratures, but the input (preferred) quadratures as well. Therefore, unless the input signal lies exactly at \( \omega_0 \), a rigorous definition of the input quadratures requires the presence of two fields symmetrically centered around \( \omega_0 \). The latter case means that one has to consider a field also at the idler (or, image) frequency \( \omega_{\text{id}} \), satisfying \( 2\omega_0 = \omega_{\text{in}} + \omega_{\text{id}} \), as illustrated in Fig. 1(a). In a homodyne detection with a mixer (see below) the information in the idler is retained, which allows for an improved signal compared to phase-insensitive (heterodyne) detection, where the idler is discarded.

A phase-sensitive amplifier can turn into a phase-mixing amplifier when \( \omega_{\text{in}} \neq \omega_0 \) [3]. It differs from the phase-sensitive amplifier because the input-output relations for the quadratures cannot be transformed to the preferred form; i.e., each output quadrature depends through (implicit) gains \( G_{ij} \) on both input quadratures: \( Y_1 = G_{11} X_1 + G_{12} X_2 \) and \( Y_2 = G_{21} X_1 + G_{22} X_2 \). As a result, the ellipse representing the added noise [Fig. 1(c)] is rotated with respect to the input noise ellipse.

A local oscillator (LO) phase \( \theta \) defines a detection frame for the quadratures. Typically, the detection is in the preferred basis [4,6,10]. In phase-mixing amplifiers, the added noise can have a nontrivial dependence on \( \theta \), and the signal-to-noise ratio can potentially be improved by tuning away from the basis defined by the gains. In Fig. 1(c), the detection is indicated by the projections on the \( Y_\theta^2 \) axis. We can hence define a \( \theta \) dependent gain \( G_\theta \) whose precise form depends on the phase of the input signal. The added noise is referred to the input, that is, the spectral density is \( S_\theta^N = S_\theta^R/G_\theta^2 \), where \( S_\theta^R \) represents the output noise when no input signal is present. Expressed in units of quanta at the signal frequency, the phase-dependent added noise is \( N_\theta^N = S_\theta^N/\hbar\omega_m \).

We perform the experiments in a dilution refrigerator. The basic signal scheme is shown in Fig. 1(b). The two
microwave pump tones and a weak signal tone are applied to the coupler port of the device. The amplification is measured with a network analyzer as the $S_{11}$ reflection parameter. In Figs. 2(a)–2(c), we demonstrate phase-insensitive amplification of microwave signals achievable with the scheme. The double-peak structure corresponds to the positions of the resonances of the signal and idler, that is, where a phonon in the mechanical oscillator is emitted or absorbed by a pump tone. As shown in Fig. 2(b), we observe high amplification up to 60 dB, or alternatively a broad 3 dB bandwidth ($\approx 430$ kHz). The data shown in Fig. 2(c) correspond to the noise measurements in Fig. 2(d), discussed below. The theoretical predictions [23], overlaid on the experimental data, show a good agreement. In order to quantitatively explain the gain profiles, we include a parametric modulation term to the mechanical oscillator

$$\omega_{p} \approx \omega_{m} \pm \omega_{c} \mp \pi / 2.$$  

Notice that in Figs. 2(a)–2(c), we used slightly varying $\omega_{0}$ (but $\approx \omega_{c}$) shifting the peaks.

For noise measurements, we use a 50 $\Omega$ resistor as a tunable known noise source. It is attached to a heater and a separate thermometer, and connected to the sample via a short superconducting coaxial cable. At the known calibration temperature $T_{R}$, the quantum-noise power from the resistor is $N_{in} = \coth[\hbar \omega_{c} / (2 k_{B} T_{R})] / 2$. This calibrated input noise gives rise to an output noise power of $P_{RT} = N_{in} G^{2} F + (N_{add} + N_{F} / G^{2}) G^{2} F$ at room temperature. Here, $F$ and $N_{F} = 18 \pm 2$ quanta [23] are, respectively, the gain and the technical noise due to all amplifiers and attenuation following the sample. We use the system noise $N_{eff} = N_{add} + N_{F} / G^{2}$ and $G^{2} F$ as adjustable parameters when fitting data to the expression for $P_{RT}$ at varying values of $T_{R}$. In Fig. 2(e) we display an example of the measured noise power, showing a good agreement with the expected quantum noise.

The total (averaged over quadratures) noise corresponds to a phase-insensitive measurement, with the measurement frequency different from $\omega_{0}$. As shown in Fig. 2(d), we observe a total system noise well below the single quantum level, and the added noise $N_{add}$ is consistent with the quantum limit of 0.5 quanta. The theory curves include dielectric heating of the baths by the pumps up to $n_{Tm} = 80$, $n_{T} = 1.1$. Here, $n_{T}$ is photon occupation of the internal bath of the cavity mode. In a previous cooldown, we made a rough calibration of the bath heating by using sideband cooling, observing a sharp onset of heating around the powers discussed here [23]. The low noise appears clearly

\[ \text{FIG. 2. Phase-insensitive amplification and noise. (a) Gain at a fixed pump power, and at varying pump detunings. Black curves are theory predictions with } G_{-}/2\pi = 580 \text{ kHz, } G_{+}/2\pi = 496 \text{ kHz. (b) Amplification at a high gain (red) or over a broad bandwidth (blue). Inset: Improvement of the signal-to-noise ratio of a coherent input signal (sharp peak). The original (black) noise floor is limited by the commercial cryogenic amplifier. When the pump tones are switched on (green) the signal-to-noise ratio in phase-insensitive amplification is improved by 12 dB. (c) Gain for } G_{-}/2\pi = 308 \text{ kHz, } G_{+}/2\pi = 304 \text{ kHz, and } \delta / 2\pi = 20 \text{ kHz. The black line is a theory prediction. (d) Added noise corresponding to panel (c): Blue circles are the total system noise, and red squares represent the added noise due to the parametric modulation. The solid black line is a theory curve, while the blue line shows the quantum limit. (e) Noise calibration by varying the power emitted by a known noise source. The noise at device output is shown. The data sets correspond to } \omega_{m}/2\pi = 6.914 \text{ GHz.}} \]
as an improvement of signal-to-noise ratio of a weak signal when the amplification is switched on, as displayed in Fig. 2(b) (inset) where the observed noise floor is dominated by amplified vacuum noise. Figure 2(d) shows that the added noise can be very close to 0.5 quanta even when the mechanical oscillator responsible for the amplification resides far from its ground state.

For phase-sensitive homodyne measurements, the output signal is digitally mixed to the center frequency $\omega_{LO} = \omega_0$ in the scheme of Fig. 3(a). By changing the phase $\theta$ of the LO, we obtain the quadratures $Y_1(t)$ and $Y_2(t) = Y_{1+e^{i\pi/2}}(t)$ oscillating at the center frequency. The noise measurements are made as described above, but individually for each $\theta$ [23]. We discover that the noise falls well below the quantum limit in one quadrature [Fig. 3(b)] and we estimate $N_{\text{add}} \lesssim 0.2$ quanta. The uncertainty is dominated by the statistical errors from fitting to the quantum noise. The theoretical added noise [3,23] (black) in Fig. 3(b) is evaluated using $n_{\text{add}}^2 = 300$, $n_I = 1.5$, capturing the main features involving the optimum noise offset from the preferred quadrature.

A fundamental property of a phase-sensitive amplifier is the possibility to generate squeezed propagating states as shown in many experiments in optics [26,27], and with Josephson devices, see, e.g., [6,9]. Quantum squeezing of the light emitted from optomechanical cavities has also recently been observed at optical frequencies [28–30]. Next, we show that our approach provides a way to generate squeezed radiation. This demonstrates a new mechanism over the previously utilized ponderomotive squeezing [28–30]. In the plane of the input of the cryogenic amplifier, we measure strong squeezing within a bandwidth of 700 kHz, with the maximum of $\sim 3.5$ dB below vacuum, as shown in Fig. 4. The calibration procedure is described in the Supplemental Material [23]. The theory predictions in Figs. 4(b)–4(d) are generated using $n_m^2 = 400$, $n_I = 1.6$.

Intense squeezed coherent states are a valuable resource [31–34]. When injected with a sinusoidal signal, we estimate the setup of Fig. 4 to produce a bright squeezed coherent state of up to $\sim 10^{14}$ photons/sec, or $\sim 65$ dBm. If realized in optics, our approach can provide luminous squeezed laser beams to overcome the quantum-noise limitation in gravitational wave observations. Our amplification scheme compares favorably over JPA because it does not require superconductivity, and is able to handle 4 orders of magnitude more input power than a corresponding JPA [6], or 2 orders of magnitude more than in Ref. [8]. Moreover, in contrast to cavity-based parametric amplifiers, the gain-bandwidth product is unlimited [23]. The bandwidth is smaller than in JPA, but it can be increased by stronger coupling, or by implementing an electromechanical metamaterial. With slight improvements, the device can operate below the

![FIG. 3. Phase-mixing amplification. (a) In homodyne detection, a mixer [dashed box in Fig. 1(b)] extracts the quadrature amplitudes oscillating at the local oscillator (LO) frequency $\omega_{LO}$. The quadrature axes $Y_1$ and $Y_2$ are determined by the digitally tunable phase shift $\theta$. (b) Input noise (left scale) and gain (right scale) as a function of quadrature angle $\theta$. The blue circles are the system noise $N_{\text{eff}}$, and the red squares the added noise $N_{\text{add}}^2$. The black curve is the prediction for $N_{\text{add}}^2$. The regime below the quantum limit is colored in blue. The parameters are $G_-/2\pi = 320$ kHz, $G_+/2\pi = 315$ kHz, $\delta/2\pi = 460$ kHz, and $\omega_{m}/2\pi = 6.915$ 165 GHz.

![FIG. 4. Mechanical squeezing of microwave light. (a) Noise emitted from the device normalized to the vacuum level (in dB) as a function of the local oscillator phase $\theta$ and frequency. (b) Theoretical prediction corresponding to (a). (c) Crosscut along the horizontal lines $\theta = 2.93$ (blue) and $\theta = 1.38$ (red). The thin solid lines are the corresponding theoretical curves. (d) Squeezing at $\omega/2\pi = -0.369$ MHz as a function of $\theta$. In (c) and (d) the area under the zero-point fluctuation level is colored in blue. The pump amplitudes are $G_-/2\pi = 690$ kHz, $G_+/2\pi = 590$ kHz.}
quantum limit at modest cryogenic temperatures of a few kelvin, hence offering an attractive technology for narrow-band measurements in particle physics [35], with superconducting qubits [36–38], or finally, in microwave optomechanics.

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[17] Notice that generally the pumps need not be symmetrically detuned with respect to $\omega_c$. For simplicity, we focus on the (optimum) case of symmetric detuning.
[25] The quantum limit of added noise is below 0.5 if the gain is not very large.
I. EXPERIMENTAL DETAILS AND CALIBRATIONS

A. Bath temperatures and dynamical backaction

The mechanics is coupled to a bath corresponding to the phonon number $n_{Tm}$. When the mechanical resonator is thermalized to the cryostat temperature $T$, the temperature of the bath simply follows $\hbar \omega_m n_{Tm} = k_B T$ when $n_{Tm} \gg 1$. A single pump tone applied at the red sideband will transduce the mode temperature in the motional sideband peaks representing the thermal motion of the mechanics. We observe the regular motional sidebands as shown in Fig. S1a. With a weak pump tone, the bath temperature is expected to appear in the area in the motional sideband peaks, following a linear temperature dependence. The linear behavior is observed down to $\sim 30 \text{ mK}$ (Fig. S1b), below which the local bath, to which the mechanics is coupled, starts to saturate. The equilibrium temperature of the bath, holding at the lowest pump powers, is around 20 mK, corresponding to $n_{Tm} \sim 40$. This is a representative thermalization figure for samples produced by our fabrication process.

However, we usually observe a varying heating of the mechanics bath $n_{Tm}$ by the pump microwave tone(s), often posing the limiting factor for device performance. We estimate this heating as in main text Ref. [19]. In a previous cooldown on the same sample, we made a rough independent calibration of the bath heating by using sideband cooling. The results are summarized in Fig. S2. We observe an unchanged bath temperature up to $G_- \sim 200 \text{ kHz}$, followed by a fast increase. In the main text in Fig. 2d and in Fig. S7 (see below), we used as adjustable parameters the values $n_{Tm} = 80$ and $n_{Tm} = 220$, respectively, which are somewhat in line with Fig. S2. In Fig. S7, the heating matches with the sideband cooling, but in Fig. 2d, the heating is weaker. We attribute this to our typical observation that the heating varies between cooldowns. A modest heating of the bath representing the cavity internal losses $n_I$ is observed as well.

Under a red-sideband pumping inducing a photon number $n_-$ in the cavity, the radiation-pressure interaction can be linearized, resulting in an effective coupling:

$$G_- = g_0 \sqrt{n_-} \gg g_0.$$  \hspace{1cm} \text{(S1)}

Under these conditions, the dynamical backaction enhances the damping of the mechanical resonator by the amount

$$\gamma_{\text{opt}} = \frac{4 G_-^2}{\kappa},$$  \hspace{1cm} \text{(S2)}
so that the total ("effective") damping of the mechanics is

\[ \gamma_{\text{eff}} = \gamma_{\text{opt}} + \gamma. \] (S3)

As a consistency check, we verified the expected linear dependence on the added damping (Eq. (S2)) on the generator power, as shown in Fig. S3.

B. Noise calibrations

Let us briefly re-state the noise calibration procedure described in the main text. There is inevitably significant attenuation between the sample and the cryogenic hemt amplifier. The corresponding transmission coefficient is \( t < 1 \).

We denote by \( F_0 \) the total gain of all the post-amplification including the hemt, and by \( N_{F0} \) the corresponding noise temperature.

The noise at the output is

\[ P_{\text{RT}} = N_{\text{in}} G^2 t F_0 + N_{\text{add}} G^2 t F_0 + N_{F0} F_0 = N_{\text{in}} G^2 F + \left( N_{\text{add}} + N_F / G^2 \right) G^2 F \] (S4)

where we defined the effective post-amplification gain and noise as \( F = t F_0 \) and \( N_F = N_{F0} / t \).
When Eq. (S4) is plotted with a varying $N_{in}$, we obtain a straight line with a slope $\mathcal{G}^2 F$, and a horizontal axis intercept $N_{in,0}$ satisfying

$$N_{in,0} = N_{\text{add}} + \frac{N_F}{\mathcal{G}^2}$$

(S5)

which allows for deducing $N_{\text{add}}$, given that the last term is known or insignificant. Figure S4 shows the same data as plotted in the lowest curve in Fig. 2e in the main text, but now plotted as a function of $N_{in}$, instead of the noise source temperature, recovering the expected linear behavior. The linear fit provides the same result as the fits in Fig. 2e. Note that this method is independent of $F$, and that the effect of technical noise is negligible at large gain $\mathcal{G}$, here satisfied approximately when $\mathcal{G} \gtrsim 25$ dB.

![FIG. S4. Noise calibration on linear scale. Data from the lowest curve in Fig. 2e in the main text as a function of input noise quanta. The solid line is a linear fit.](image)

Equation (S4) holds also for the phase-sensitive mode of operation. In addition to what is discussed above, the measurement is repeated for all the desired quadrature angles. The data, complete with quantum noise fits similar to Fig. 2e in the main text are shown in Fig. S5. From the fits we obtain the noise data in Fig. 3b in the main text.

In Fig. S6 we show the measurement of the effective added noise $N_P$ of the entire microwave measurement system following the sample. This figure is likely dominated by the cryogenic amplifier because it has a high gain. This noise is measured the same way as the added noise of the optomechanical device, however, without any pump tones applied. This way, the sample acts as a slightly absorbing mirror. By changing the temperature of the noise calibration resistor,

![FIG. S5. Phase-sensitive noise. Noise calibration with the homodyne detection for Fig. 3b in the main text, at different LO phases as indicated.](image)
we observe a linearly increasing noise power when plotted as a function of the number of input noise quanta as seen in Fig. S6 obtaining $N_F \simeq 18 \pm 2$.

![Graph showing noise power as a function of input noise quanta](image)

FIG. S6. Noise of the cryogenic amplifier. Noise at the output measured without any pump tones applied.

In Fig. S7 we show additional data similar to Fig. 2c and d in the main text, but measured with a higher pump power. The gain profile is smoother because the two peaks start to overlap, but the noise is slightly worsened presumably because dielectric heating affects the mechanics bath temperature.

C. Dynamic range of the amplification

A figure of merit for an amplifier is how large powers can be applied to the input before saturation takes place. This is important because in many applications (such as microwave optomechanics), strong coherent pump tones are present, and they can saturate the amplifier. The power handling capability is often characterized with the 1 dB compression point, which is the input power where the gain has dropped by 1 dB. Its value depends on the gain in the linear regime. We measured the gain as a function of the input power as seen in Fig. S8. When the gain is 22 dB for instance, 1 dB compression occurs at approximately -80 dBm.

![Gain and noise graphs](image)

FIG. S7. Gain and added noise. Similar to Fig. 2c,d in the main text, but higher pump power $G_\omega / 2\pi \approx 413$ kHz. a, Gain. b, The corresponding added noise.
D. Calibration of squeezing

As in the main text, in the following the quantities written with a superscript $\theta$ refer to a specific quadrature, whereas those without the superscript are summed over the quadratures. We discuss the squeezing measured in the input of the hemt amplifier. The variables labeled with the letter $N$ are the spectral densities in units of quanta at the hemt input, and those at the output are labeled with $S$. The calibration of the amount of squeezing is done by using as a reference the noise level in one quadrature $S_{\theta \text{ off}}^\theta$ when the pumps are off, equaling half the true input noise $N_\text{hemt} = 2N_\theta^\theta$ of the hemt. When the pumps are on or off, respectively, the measured noise is

$$S_\theta^\theta = G^2(N_\theta^\theta + N_\text{hemt}^\theta),$$

$$S_{\theta \text{ off}}^\theta = G^2(N_{\theta \text{ off}}^\theta + N_\theta^\theta)$$

Here, $N_\theta^\theta$ is the quantity of interest, the (possibly) squeezed noise radiating into the hemt input. $N_{\theta \text{ off}}^\theta = N_\theta^\theta + N_0^\theta$ can be larger than the vacuum value $N_{zp}^\theta = 1/4 = N_{zp}/2$ because of a possibly present extra thermal noise $N_0^\theta$.

The amount of squeezing is conveniently expressed as the noise in one quadrature in units of $N_{\theta zp}^\theta$. The quantity plotted in Fig. 4 in the main text is

$$\text{squeezing} = \frac{N_\theta^\theta}{N_{\theta zp}^\theta}$$

and hence a value $< 1$ (or 0 dB) entails squeezing below vacuum. Using Eqs. (S6,S7), Eq. (S8) becomes

$$\text{squeezing} = \frac{S_\theta^\theta / G^2 - N_{\theta \text{ hemt}}^\theta}{N_{\theta zp}^\theta} \approx \frac{N_{\theta \text{ hemt}}^\theta (S_{\theta \text{ off}}^\theta - 1)}{N_{\theta zp}^\theta}$$

A caveat in using Eq. (S9) is that in the present setting we cannot directly measure $N_{\text{hemt}}$. What we can accurately measure is the effective input noise $N_F = N_{\text{hemt}}/t$, which includes the (power) transmission $t < 1$ between the sample and the hemt. In our system with a standard superconducting coaxial cabling, we estimate a typical $t \sim 1.5...2.5$ dB, which leads to relatively large error bars to the measured squeezing. We note that if for some reason the attenuation would be larger than we estimate, the squeezing would be stronger than claimed now. The attenuation is unlikely to be smaller than the stated values because even if all components are working ideally, the total attenuation amounts to approximately 1.5 dB.

At the high pump powers used in the squeezing demonstration, we observe that the noise calibration resistor heats up so that it stays at $\sim 160$ mK (Fig. S9b), hence emitting $N_0 \approx 0.14$ quanta of extra thermal noise, as included in Eq. (S7). However, $N_0 \ll N_{\text{hemt}}$ and hence can be neglected.

From the theory, we obtain the squeezing directly at the output of the sample. Losses between the sample and the hemt input, however, reduce the amount of squeezing by bringing the state towards a thermal state. In order to compare the data to theory, we make a rough estimation that the difference of the squeezing from vacuum is reduced by the attenuation $t$. The theoretical predictions in Fig. 4 in the main text are obtained this way. In Fig. S9, we include also the predicted squeezing at the sample plane.
As in Ref. 19, in phase-sensitive measurements, we obtain the best fit by introducing a direct parametric modulation (amplitude $\sim (2\pi) \cdot 320$ kHz) of the cavity beyond the ideal optomechanical model. Although the parametric modulation somewhat enhances the squeezing and is needed to quantitatively understand the data, the observed squeezing is essentially of optomechanical origin. This can be seen in Fig. S9b, which also shows theory curves plotted without the cavity parametric term.

The ideal strongly squeezed vacuum has the total phonon number $N \simeq G^2/4$ when the squeezed quadrature has $N^0 \simeq 1/(4G^2)$. With our maximum squeezing 8 dB right after the sample, we thus ideally have $N \simeq 1.5$. Figure S9b, however, implies $N \simeq 60$ which means the purity of the squeezing is poor, although it is of the same order as in Ref. [6]. The low purity is contributed by the finite temperature of all baths and by the internal cavity losses. At zero temperature, the emitted radiation closely approaches an ideal squeezed vacuum.

### II. THEORETICAL DETAILS

In this section we outline the theoretical model presented in detail in Ref. [3]. In section II B we introduce a new interpretation of the operation of the phase-sensitive optomechanical amplifier.

#### A. Equations of motion

The system Hamiltonian (Eq. (1) of the main text), expressed in terms of cavity Bogoliubov modes, can be derived from the conventional linearised optomechanical Hamiltonian,

$$H = \omega_c a^\dagger a + \omega_m b^\dagger b + \left( G_+ e^{-i\omega_+ t} + G_- e^{-i\omega_- t} \right) (a^\dagger b^\dagger + a^\dagger b) + \text{h.c.}$$  \hspace{1cm} (S10)

Neglecting fast oscillating terms (rotating-wave approximation) and moving to a frame rotating at $\omega_c$ and $\omega_m - \delta$ for the cavity and the mechanical field respectively, we can write Eq. (S10) as

$$H = \delta b^\dagger b + G_+ a^\dagger b^\dagger + G_- a^\dagger b + \text{h.c.}$$  \hspace{1cm} (S11)

Due to the symmetry induced by the linearisation scheme it is possible to recast Eq. (S11) in terms of Bogoliubov modes for either the mechanical field

$$H = \delta [\cosh^2 \xi + \sinh^2 \xi] \beta^\dagger \beta + \delta \cosh \xi \sinh \xi \left( \beta^{\dagger 2} + \beta^2 \right) + G_{BG} (a^\dagger \beta + a \beta^\dagger)$$  \hspace{1cm} (S12)

or for the cavity field

$$H = \delta b^\dagger b + G_{BG} (a^\dagger b + ab^\dagger),$$  \hspace{1cm} (S13)
where $\beta = \cosh \xi b + \sinh \xi b^\dagger$ and $\alpha = \cosh \xi a + \sinh \xi a^\dagger$, $\cosh \xi = G_- / \sqrt{G_-^2 - G_+^2}$, $\sinh \xi = G_+ / \sqrt{G_-^2 - G_+^2}$. In order to maintain stability of the system, we suppose $G_- > G_+$. With a view to the physics discussed in the main text, we focus on the form given by Eq. (S15), where the beam-splitter term $G_{BG} (\alpha^\dagger b + \alpha b^\dagger)$ points towards the cooling of the mechanical motion to the temperature of the Bogoliubov cavity mode, and entails squeezing of the original cavity mode $a$.

From Eq. (S13) we can determine the following quantum Langevin equations in the frequency domain \[ \text{II} \] for $\alpha$ and $b$

\[
-i\omega \alpha = -iG_B b - \frac{\kappa}{2} \alpha + \sqrt{\kappa} \alpha \omega_i \omega
\]

\[
-i\omega b = -i\delta b - iG_B \alpha - \frac{\kappa}{2} \omega + \sqrt{\kappa} b_i \omega_i \omega.
\]  

(S14)

The mechanical degrees of freedom can be eliminated from Eq. (S14), leading to the following equation for the Bogoliubov mode $\alpha$

\[
-i\omega \omega_i \omega = -G_B^2 \chi_m \alpha - \frac{\kappa}{2} \omega_i \omega_i + \sqrt{\kappa} \alpha \omega_i \omega
\]

(S15)

where $\chi_m = [\gamma / 2 - i (\omega - \delta)]^{-1}$.

Equation (S15) can be solved to give

\[
\alpha = \chi_\text{eff} \sqrt{\kappa} \alpha \omega_i \omega
\]

(S16)

where

\[
\chi_\text{eff} = \frac{1}{\kappa/2 - i\omega + G_B^2 \chi_m}
\]

\[
\sqrt{\kappa} \alpha \omega_i \omega = \sqrt{\kappa} \alpha \omega_i \omega - iG_B \chi_m \sqrt{\kappa} b_i \omega_i \omega
\]

(S17)

The output field $\alpha \omega_i \omega$ in terms of the input modes $\alpha \omega_i \omega$ is the obtained from

\[
\alpha \omega_i \omega = \sqrt{\kappa} \alpha \omega_i \omega - \alpha \omega_i \omega
\]

(S18)

The gains and added noise, discussed in section \[ \text{II C} \] are then obtained as detailed e.g. in Ref. [11].

**B. Phase-sensitive amplification via phase-sensitively reflected Bogoliubov wave**

As stated in the main text, Eq. (S13) represents the Hamiltonian of a beam splitter, and coincides with the Hamiltonian describing the cooling of a mechanical resonator by means of the coupling to a cavity Bogoliubov mode. Therefore, while it is natural to assume that the mechanics – albeit sub-optimally, due to the presence of $\delta$ – is cooled by the coupling with the Bogoliubov mode, the resulting phase-sensitive amplification for the output field is more surprising. The discussion of the system in terms of an ideal parametric amplifier can be understood in a somehow simplified picture in terms of phase shifts induced by the cavity coupled to the mechanical resonator. Since we focus here on the simplified analysis of the amplification process, we neglect all noise sources described above. We can write the I/O equations for the output field in Eq. (S18) as

\[
\alpha \omega_i \omega = (\kappa \chi_\text{eff} - 1) \alpha \omega_i \omega.
\]  

(S19)

Since the cavity is overcoupled ($\kappa_c \simeq \kappa$), we have that $|\kappa \chi_\text{eff}^\omega - 1| = |\kappa \chi_\text{eff}^\omega (-\omega) - 1| \simeq 1$. Defining the phases of the reflected cavity modes at frequencies $\pm \omega$, $\phi_\omega^+ \equiv \text{Arg} \left[ \kappa \chi_\text{eff}^\omega (\omega) - 1 \right]$ and $\phi_\omega^- \equiv \text{Arg} \left[ \kappa \chi_\text{eff}^\omega (-\omega) - 1 \right]$, we can write the I/O equation for the original output modes $a_{\alpha \omega}$ as

\[
a_{\alpha \omega} = \cosh \xi \alpha_{\alpha \omega} - \sinh \xi \alpha^\dagger_{\alpha \omega} = |\chi_\text{eff}^\omega - 1| \left( \cosh \xi e^{i\phi_\omega^+} \alpha_{\omega_i \omega} - \sinh \xi e^{i\phi_\omega^-} \alpha^\dagger_{\omega_i \omega} \right)
\]

\[
= \left( \cosh^2 \xi e^{i\phi_\omega^+} - \sinh^2 \xi e^{i\phi_\omega^-} \right) \alpha_{\omega_i \omega} - \cosh \xi \sinh \xi \left( e^{i\phi_\omega^+} - e^{i\phi_\omega^-} \right) \alpha^\dagger_{\omega_i \omega}.
\]  

(S20)
For $\delta \geq 4G_{BG}/\kappa$, to the first order in $G_{BG}/\kappa$ and $\delta/\kappa$, we have

$$\phi_{+\omega_{\text{max}}}^+ = \pi + \frac{2}{\kappa} \left( \frac{2G_{BG}^2}{\delta} - 2\delta \right)$$

$$\phi_{-\omega_{\text{max}}}^- = -\frac{\delta}{\kappa}$$

(S21)

and

$$\phi_{+\omega_{\text{max}}}^+ = \frac{4\delta}{\kappa}$$

$$\phi_{-\omega_{\text{max}}}^- = \pi - \frac{2}{\kappa} \left( \frac{2G_{BG}^2}{\delta} - 2\delta \right)$$

(S22)

Focusing on $\omega = \omega_{\text{max}}$ ($\omega_{\text{max}}$ is defined in Eq. (S27) below) and neglecting the small $1/\kappa$ terms, Eq. (S20) can be written as

$$a_{o\omega} = (\cosh^2 \xi + \sinh^2 \xi) a_{in\omega} - 2\cosh \xi \sinh \xi a_{in\omega}.$$  

(S23)

It is possible to recognize in this expression the relation for a phase-sensitive amplifier. In this case the gains in the preferred quadratures are given by

$$G_1 = (\cosh \xi + \sinh \xi)^2 = \frac{G_- + G_+}{G_- - G_+} = \exp(2\xi)$$

$$G_2 = (\cosh \xi - \sinh \xi)^2 = \frac{G_- - G_+}{G_- + G_+} = \exp(-2\xi).$$  

(S24)

Thus the amplifier in this limit behaves like an ideal parametric amplifier. We note also that the expression for the square of the gain $G_1^2$ can be recast as

$$G_1^2 \rightarrow G_-^2 = \frac{16G_+^2G_-^2}{(G_+^2 - G_-^2)^2}.$$  

(S25)

On the other hand, for $\delta \simeq 0$, $\phi_{+\omega_{\text{max}}}^+ = \phi_{-\omega_{\text{max}}}^- = \phi_{+\omega_{\text{max}}}^- = \phi_{-\omega_{\text{max}}}^+ = 0$, and Eq. (S20) leads to the (trivial) I/O relation

$$a_{o\omega} = a_{in\omega},$$

(S26)

describing the complete reflection without any amplification.

C. Gain and added noise

While the detailed analysis is found in Ref. [3], we quote here some important results. As seen in the main text, the gain peaks at the frequencies

$$\omega_{\text{max}} = \pm \delta \left[ 1 + \frac{G_{BG}^2}{\delta^2 + \kappa^2/4} \right] \simeq \pm \delta.$$

(S27)

The linewidth of each peak is

$$\gamma_{\text{eff}} = \frac{G_{BG}^2\kappa}{\delta^2 + \kappa^2/4}.$$  

(S28)

Equations (S27,S28) are valid in the limit $\kappa_e \simeq \kappa$, $\gamma \simeq 0$, $\omega_{\text{max}} \gg \gamma_{\text{eff}}$. The gain is given by Eq. (S25), and the gain-bandwidth product under the same approximations and $G_- \gg G_+$ as

$$G_1\gamma_{\text{eff}} = \frac{16G_+G_-}{\kappa}.$$

(S29)

Equation (S29) indicates that the gain-bandwidth product can be increased without limit by increasing the effective couplings while keeping $G_- \gg G_+$ in order to maintain stability. We have also verified numerically, without the stated approximations, that the gain-bandwidth product is unlimited.
At this point we mention the experimental values for the gain-bandwidth product. The values correspond to the phase-insensitive mode, and hence the prediction is that of Eq. (S29) divided by \sqrt{2}. In Fig. 2a in the main text, we have \( G_{\text{eff}} \approx (2\pi) \cdot 460 \text{ kHz} \), in a rough agreement with Eq. (S29). In the case of Fig. 2b (red, high gain, \( G_\gamma/2\pi \approx 341 \text{ kHz} \), \( G_{BG}/2\pi \approx 12.4 \text{ kHz} \), \( \delta/2\pi \approx 150 \text{ kHz} \)), the sharp peaks make determining the bandwidth difficult. For Fig. 2b (large bandwidth, blue, \( G_\gamma/2\pi \approx 767 \text{ kHz} \), \( G_{BG}/2\pi \approx 603 \text{ kHz} \), \( \delta/2\pi \approx 131 \text{ kHz} \)) we obtain experimentally \( G_{\text{eff}} \approx (2\pi) \cdot 2.2 \text{ MHz} \). This value is clearly larger than predicted by Eq. (S29) because the equation holds only when the gain peaks are clearly separate. The case of somewhat overlapping gain peaks as in Fig. 2b becomes the optimal operating conditions for the amplifier because this provides the largest bandwidth for given pumping powers, without degradation of noise.

In the pure phase-sensitive mode, taking place when \( \omega = \omega_{\text{max}} \), further supposing \( n_I \ll n_T m \) the added noise can be calculated as

\[
N_{\theta}^{\text{add}} \approx \frac{1}{2} \frac{1}{2 \gamma \kappa} \left( 2n_T^m + 1 \right) \left[ \left( G_+ + G_- \right)^2 \sin^2 \theta + \left( G_+ - G_- \right)^2 \cos^2 \theta \right] \left[ \left( G_+ + G_- \right)^4 \sin^2 \theta + \left( G_+ - G_- \right)^4 \cos^2 \theta \right] \]  

(S30)

In the phase mixing mode that can take place when \( \omega \neq \omega_{\text{max}} \), analytical results become too lengthy and we evaluate the \( \theta \) dependence of \( N_{\theta}^{\text{add}} \) according to Ref. [3].

In Fig. S10 we show the predicted phase-sensitive added noise as a function of cryostat temperature \( T \). Here we suppose that both baths will thermalize with the cryostat, i.e., we take \( n_T^m = \left[ \exp(\hbar \omega_m / k_B T) - 1 \right]^{-1} \), \( n_I = \left[ \exp(\hbar \omega_c / k_B T) - 1 \right]^{-1} \). In the figure, we show predictions for two different effective couplings. The input frequency and local oscillator phase have been optimized in each case.

![Predicted added noise.](image)

**FIG. S10.** Predicted added noise. The blue curve corresponds to the parameters in Fig. 3b in the main text. The red curve is otherwise the same, but with a higher coupling \( G_\gamma/2\pi = 1.5 \text{ MHz} \), \( G_{BG}/2\pi = 1.4 \text{ MHz} \). The other parameters are as in Fig. 3b.

It is interesting to spell out the parameter values where amplification is still just barely observable. We estimate numerically that amplification becomes possible if the cooperativity

\[
C = \frac{4G^2}{\gamma \kappa} 
\]  

(S31)

becomes at least of the order one. This is a very modest requirement, and is satisfied in a significant portion of the recently published cavity optomechanical experiments. In order to lower the noise below the mechanics bath temperature, nonetheless, it is required that the optical damping \( \gamma_{\text{opt}} \) competes with the mechanical decoherence \( \gamma n_T^m \), implying \( C \gtrsim n_T^m \).

**D. Cooling of the Bogoliubov modes**

Finally, let us comment on one more possible interpretation of the physics in terms of cooling or squeezing of the Bogoliubov modes. The squeezing of mechanics studied in Refs. [18-20] allows for a simple picture. Namely, at zero pump detuning \( \delta = 0 \), the Hamiltonian (S12) describes cooling of the mechanics Bogoliubov modes towards the ground state that corresponds to a squeezed state of the bare mechanics. One can work out a corresponding picture for the present case \( \delta \neq 0 \), although it is not as transparent.
Equation (S12) indicates that if $\delta \neq 0$, the mechanics Bogoliubov mode $\beta$ attains squeezing terms $\beta^2 + \text{h.c.}$ which are about as strong as the other terms. Thus, $\beta$ becomes squeezed, which means the bare mechanics $b$ is not much squeezed, although the beam splitter term is still competing. Then, based on (S13), the $b$ mode provides a strongly coupled bath, competing with $\kappa$ decay, for the $\alpha$ mode to thermalize with $b$. Hence $\alpha$ ends up in a roughly thermal state, which means that bare cavity $a$ is squeezed (although probably not squeezed below the vacuum level).

One can numerically check the above picture. In the case of Fig. 4 in the main text, we obtain that the mechanics is in a relatively low entropy state with a total energy $n_m \sim 3$ quanta, and showing a small thermal-noise squeezing by $\sim 20\%$. The cavity has a noise energy of $\sim 0.5$ quanta above the ground state. The cavity is not squeezed below vacuum, but one quadrature shows energy by about three times more than the orthogonal. Notice, however, that the radiation emitted from the cavity is squeezed below vacuum due to interference with the incoming vacuum noise.

[1] The subscript $\omega$ stands for the frequency, and we use the Fourier convention where $a^\dagger_{\omega}$ is the conjugate of $a_\omega$. 