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AN APPROXIMATE ESTIMATION OF VELOCITY PROFILES AND TURBULENCE FACTOR MODELS FOR AIR-FLOWS ALONG THE EXTERIOR OF TEFC INDUCTION MOTORS

by

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Compared to a number of other existing correlations for heat transfer, the empirical correlations for forced convection from a short horizontal cylinder in axial air-flows usually do not involve the effects of changes in air-flow velocity and/or air-flow turbulence. Therefore, a common analysis of the heat transfer by using only one energy balance equation for entire outer surface of a solid is considered insufficient for induction motor applications because it fails to include aforementioned effects. This paper presents a novel, empirically-based methodology to estimate approximately the values of air-flow velocities and turbulence factors, that is, velocity profiles and turbulence factor models for stationary horizontal cylinders with and without fins (frame and two end-shields) in axial air-flows. These velocity profiles and turbulence factor models can then be used in analytical modelling of steady-state heat transfer from the exterior of totally enclosed fan-cooled induction motors.

Key words: air-flow velocity profile, empirical correlation, steady-state heat transfer, TEFC induction motor, turbulence factor model

Introduction

In order to precisely apply an analytical model for steady-state heat transfer to any totally enclosed fan-cooled (TEFC) induction motor it is necessary to know the following: (1) two different air-flow velocities, one for the beginning and another for the end of the cooling channels; and (2) three different turbulence factor models, two particular models for the flat and cylindrical outer surfaces of the end-shields and one common for the cooling fins and the inter-fin surfaces. The introduction of these profiles and models facilitates estimation of the air-flow velocities and the turbulence factors for various TEFC induction motors.

The air-flow velocities at the beginning and at the end of the cooling channels of a TEFC induction motor are necessary to calculate the appropriate values of the Reynolds number. The Reynolds numbers are then used to calculate the average Nusselt numbers (by means of empirical correlations) and corresponding heat transfer coefficients due to forced convection (from end-shields, cooling fins, and inter-fin surfaces). However, the dependence of heat transfer coefficients for the cooling fins and the inter-fin surfaces on longitudinal distance from the fan cowl and corresponding temperature distribution (along the frame) can not be determined

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solely on the basis of the air-flow velocity profile [1]. This can be solved by introducing the dependence of the turbulence factor, \( K_{\xi,y} \), on the longitudinal distance, \( y \), in the form \( K_{\xi,y} = K_{\xi,y}(y) \), which will be known as the turbulence factor model.

According to the Heiles [2], the turbulence factor, \( K_{\xi,y} \), was introduced in the thermal analysis of the electrical machines based on the studies conducted by Rietschel in 1914 and Schutte in 1937. Moreover, according to them, the turbulence factor, \( K_{\xi,y} \), is constant, does not depend on the air-flow velocity and typically has a value between 1.7 and 1.9. In 1965 Kovalev et al. [1] reported that for the frame of the motor with a shaft height of 180 mm, the degree of air-flow turbulence, \( \xi \), decreases with the increase of the longitudinal distance from the fan cowl. The turbulence factor, \( K_{\xi,y} \), should be distinguished from the degree of air-flow turbulence, \( \xi \). Practically, the turbulence factor, \( K_{\xi,y} \), represents the ratio between two values of the same heat transfer coefficient due to forced convection, \( h_{c,i}(\xi_1) \) and \( h_{c,i}(\xi_2) \), where the first corresponds to the degree of air-flow turbulence greater than zero (\( \xi_1 > 0 \)) and the second to the degree of air-flow turbulence equal or close to zero (\( \xi_2 = 0 \) or \( \xi_2 \approx 0 \)).

This paper proposes a set of new correlations for the air-flow velocity profiles and the turbulence factor models. The velocity profile is assumed to be a biquadratic polynomial function of the longitudinal distance from the fan cowl similar to the one from [3], but with no terms of odd-degree. The coefficients of this polynomial will depend on air-flow velocity at the beginning of the cooling channels, number of poles, height of the shaft, rated power and angular velocity of the rotor. Air-flow velocity at the beginning of the cooling channels and peripheral velocity of the fan wheel have been correlated by the one-n-th-power law. The effect of the fan wheel shape on the air-flow velocities at the beginning and at the end of the cooling channels of induction motors is neglected. Velocity profiles have been obtained on the basis of the published experimental data on more than eighteen different TEFC induction motors and finned frames. The turbulence factor models, based on the existing data on five different TEFC induction motors, are assumed to be reduction functions similar to the one from [4].

### Air-flow velocity profiles

According to [3], Pohlhausen in 1921 solved the momentum integral equation for a velocity distribution/profile within a laminar boundary layer along a flat plate. The velocity profile satisfies a polynomial of fourth degree [3]. Substituting the boundary layer thickness by the frame/fin length \( L_5 \) [m] and ignoring the terms of odd-degree in the polynomial, it is reasonable to assume that the profile of air-flow velocity along the cooling channels \( V_y = f(y) \) [ms\(^{-1}\)] can be expressed as:

\[
V_y = C_1 V_0 \left[ 1 - \Theta_0 \left( \frac{y}{L_5} \right)^2 + 0.5 \Theta_0 \left( \frac{y}{L_5} \right)^4 \right]
\]

where \( C_1 \) and \( \Theta_0 \) are unknown dimensionless coefficients, \( V_0 \) [ms\(^{-1}\)] – the air-flow velocity at the beginning of the cooling channels, and \( y \) [m] – the longitudinal distance from the fan cowl. The coefficients \( C_1 \) and \( \Theta_0 \) are determined based on the published experimental data with relevance to this problem.

The length, \( L_5 \), and the longitudinal distance, \( y \), are defined in fig. 1, where the exterior of a standard TEFC induction motor is also presented. The exterior of a standard TEFC induction motor in the direction of fan wheel from the drive side consists of the following elements: (A) drive pulley, (B) metal key between pulley and shaft, (C) drive shaft extension, (D) drive end-shield, (E) finned frame, (F) terminal box, (G) metal nameplate, (H) eyebolt, (I) motor
mounting feet, (J) non-drive end-shield, (K) non-drive shaft extension, (L) metal key between fan wheel and shaft, (M) fan wheel, and (N) fan cowl. Element labels A-N correspond to the labelling within fig. 1. Furthermore, according to fig. 1, \( r_1 \) [m] is the frame radius under the fins, \( r_{ci} \) [m] is the inner radius of the fan cowl, and \( r_{fo} \) [m] is the outer radius of the fan wheel.

It is established that the coefficient \( C_1 \) depends only on the air-flow rate conditions. Each cooling channel along an actual frame may have a different air-flow rate. The difference in air-flow rate between different cooling channels is explained by means of the motor elements (bolt lugs and terminal box) which block the inlet and outlet zones of some cooling channels [5]. The value of the coefficient \( C_1 \) amounts to 1 or 2/3, which means that there is a negligible or significant effect of mechanical obstructions on the air-flow rate, respectively. As can be seen from fig. 4, the value \( C_1 = 2/3 \) corresponds approximately to the average air-flow rate along the frame cooling channels [5].

Moreover, it is assumed that the coefficient, \( \Theta_0 \), depends on the ratio between the air-flow velocity at the beginning of the cooling channels \( V_0 \) [m/s] and the peripheral velocity of the fan wheel \( V_p \) [m/s], the shaft height \( H_{sh} \) [m], the angular velocity of the rotor \( \Omega_m \) [rpm], the number of poles \( p \), and the rated power \( P_n \) [kW]. This theory-based and empirically established dependence is described by the equation:

\[
\Theta_0 = \left( \frac{V_0}{V_p} - \frac{H_{sh}}{C_2} \right) \left( \frac{\Omega_m}{C_2} \right)^{2 - \frac{C_3}{8}}
\]

(2)

where \( C_2 \) [m·rpm] is a constant, and \( C_3 \) is an unknown dimensionless coefficient which depends on the number of poles, \( p \), and the rated power, \( P_n \). Fitting the available experimental data with the function (1) alone will give the values of \( C_2 \) and \( C_3 \).

Approximate equations for the ratio \( V_0/V_p \) for steady, fully developed, laminar, and turbulent flows of a Newtonian fluid through an annular gap between an equivalent non-drive end-shield and the fan cowl can be written as [6-9]:

\[
\frac{V_0}{V_p} = \left( 1 - \frac{r_{ci}^2}{r_{fo}^2} \right) \quad \text{for} \quad \text{Re} < 2320
\]

(3)

and

\[
\frac{V_0}{V_p} = \left( 1 - \frac{r_{ci}}{r_{fo}} \right)^{\frac{1}{2}} \quad \text{for} \quad \text{Re} \geq 2320
\]

(4)
respectively, where

\[ n = \left[ 0.2525 - 0.0229 \cdot \log(\text{Re}) \right]^{-1} \]  

is the exponent dependent on the Reynolds number \( \text{Re} = 2u_{\text{r}}/\nu \), \( u = 0.5V_p \) is the mean velocity in the case of laminar flow, and \( u = 49V_p/60 \) is the mean velocity in the case of turbulent flow. Equation (4) is an empirical equation known as the one-nth-power law/equation or the Nikuradse’s equation for turbulent velocity profile in a pipe. According to the theory, the flow in a circular pipe is laminar if the Reynolds number is less than 2320 and turbulent, if it is greater than 4000. Between these two Reynolds numbers is a transitional zone where the flow can be laminar or turbulent or in the process of continuous switching between the two regimes. In the particular case of TEFC induction motors, it is also assumed that the eq. (4) can be a good approximation to the transition velocity profile. It is fairly safe because of the fact that the fan wheel and its cowl create the high turbulences at the beginning of the cooling channels [10-14].

The maximum of the velocity profile is at the outer radius of the fan wheel, \( r_{f,o} \), and it amounts to the peripheral velocity of the fan wheel, \( V_p \).

A number of studies have been published dealing with the profiles of air-flow velocity along the cooling channels of different TEFC induction motors, and a summary of them is presented in tab. 1. The values of the coefficients \( C_1 \), \( C_2 \), and \( C_3 \) are estimated based on these experimental data. Some of the \( V_0/V_p \) ratios in references cited in tab. 1 were reported as assumed, measured or averaged, but some of them were unspecified. In cases where it was possible the values of the ratio \( V_0/V_p \) are estimated by means of the one-7th-power equation and the definition of the peripheral velocity of the fan wheel:

\[ V_p = \frac{2\pi r_{f,o} \Omega}{60} \]  

where \( V_p \) is in [ms\(^{-1}\)] for \( r_{f,o} \) in [m], and \( \Omega \) in [rpm]. The one-7th-power equation is specifically used to avoid the calculation of the exponent \( n \) as well as the Reynolds number in the cases of absence of adequate geometric and heat transfer data in the literature.

Furthermore, in accordance with [4] for TEFC induction motors it can be stated that: (1) when the number of poles, \( p \), increases the air-flow velocity along the cooling channels becomes smaller and smaller and (2) for motors having more than 4 poles it is acceptable to use only one value of the air-flow velocity along the cooling channels. Both these facts are fairly well implemented into the eq. (1).

### Table 1. Summary of some published results relevant to the profiles of air-flow velocity along the cooling channels of TEFC induction motors

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Data for the test TEFC induction motors</th>
<th>Results(^{*})</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>[11]</td>
<td>( H_{\theta} ) [mm], ( p ) [kW], ( P_n ) [rpm], ( \Omega_m ) [rpm], ( r_{f,o} ) [mm], ( r_c ) [mm], ( r_{f,o} - r_c ) [mm], ( L_{s,\eta} ) [mm] or [p. u.], ( V_0/V_p ) [ms(^{-1})] or [p. u.], ( V_p = f(y) )</td>
<td>( V_0 \approx 0.75V_p ), ( h_c = f(V_0) ), and ( h_c = f(V_p) )</td>
<td>It was assumed that ( V_0 \approx 0.75V_p ), ( h_c = f(V_0) ), and ( h_c = f(V_p) )</td>
</tr>
<tr>
<td>[15]</td>
<td>( H_{\theta} ) [mm], ( p ) [kW], ( P_n ) [rpm], ( \Omega_m ) [rpm], ( r_{f,o} ) [mm], ( r_c ) [mm], ( r_{f,o} - r_c ) [mm], ( L_{s,\eta} ) [mm] or [p. u.], ( V_0/V_p ) [ms(^{-1})] or [p. u.], ( V_p = f(y) )</td>
<td>( V_0 \approx 0.7V_p ), ( h_c = f(V_0) ), and ( h_c = f(V_p) )</td>
<td>It was assumed that ( V_0 \approx 0.7V_p ), ( h_c = f(V_0) ), and ( h_c = f(V_p) )</td>
</tr>
</tbody>
</table>
Table 1. (continuation)

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Data for the test TEFC induction motors</th>
<th>Results$^x$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H_a$</td>
<td>$p$</td>
<td>$P_n$</td>
</tr>
<tr>
<td></td>
<td>[mm]</td>
<td>[–]</td>
<td>[kW]</td>
</tr>
<tr>
<td>[4]</td>
<td>100</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>[4]</td>
<td>100</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>[16-18]</td>
<td>112</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>[10]</td>
<td>112</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>[19]</td>
<td>112</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>[5]</td>
<td>112</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>[20]</td>
<td>112</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>[14]</td>
<td>132</td>
<td>4</td>
<td>7.5</td>
</tr>
<tr>
<td>[10]</td>
<td>132</td>
<td>4</td>
<td>7.5</td>
</tr>
<tr>
<td>[21]</td>
<td>132</td>
<td>4</td>
<td>7.5</td>
</tr>
<tr>
<td>[22]</td>
<td>132</td>
<td>4</td>
<td>7.5</td>
</tr>
</tbody>
</table>

$^x$ $V_0$ and $V_y$ were expressed as a percentage of their respective rated values.
## Table 1. (continuation)

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Data for the test TEFC induction motors</th>
<th>Results&lt;sup&gt;ab&lt;/sup&gt;</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H_a$, $p$, $P_n$, $\Omega_m$, $r_{al}$, $r_{al}^<em>$, $r_{bf}^</em>$, $L_{ai}$, $V_0/V_p$</td>
<td>$V_c = f(y)$&lt;sup&gt;*&lt;/sup&gt;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[mm] [–] [kW] [rpm] [mm] [mm] [mm] [mm] or [p. u.]</td>
<td>[-] [ms&lt;sup&gt;-1&lt;/sup&gt;] or [p. u.]</td>
<td></td>
</tr>
<tr>
<td>[12]</td>
<td>134/142 n/a 5.5 1800 111 134 67.5 250</td>
<td>0.739 fig. 4(g)-A</td>
<td>2 non-standard frames identical to the frames from [13] were analyzed under artificial ventilation. The frames were with: (1) $N_{fe} = 36$ and $H_{fe} = 23$ mm and (2) $N_{fe} = 48$ and $H_{fe} = 23$ mm. Small differences were found. The tests corresponded approximately to $p = 6$.&lt;sup&gt;ab&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>134/142 n/a 5.5 1500 111 134 67.5 250</td>
<td>0.745 fig. 4(g)-C</td>
<td>Four non-standard frames were analyzed under artificial ventilation. The frames were with: (1) $N_{fe} = 48$ and $H_{fe} = 31$ mm, (2) $N_{fe} = 48$ and $H_{fe} = 23$ mm, (3) $N_{fe} = 36$ and $H_{fe} = 31$ mm, and (4) $N_{fe} = 36$ and $H_{fe} = 23$ mm. Small differences were found. The tests corresponded approx. to $p = 6$.&lt;sup&gt;ab&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>134/142 n/a 5.5 1200 111 134 67.5 250</td>
<td>0.719 fig. 4(g)-E</td>
<td></td>
</tr>
<tr>
<td></td>
<td>134/142 n/a 5.5 900 111 134 76 250</td>
<td>0.768 fig. 4(g)-G</td>
<td></td>
</tr>
<tr>
<td></td>
<td>134/142 n/a 5.5 600 111 134 76 250</td>
<td>0.733 fig. 4(g)-I</td>
<td></td>
</tr>
<tr>
<td></td>
<td>134/142 n/a 5.5 300 111 134 76 250</td>
<td>0.775 fig. 4(g)-K</td>
<td></td>
</tr>
</tbody>
</table>

### Remarks

- **Air-flow velocity along the frame was averaged.**
- **Air-flow velocity along one of the unblocked channels was recorded.**
- **Changes in air-flow velocities along the channels were averaged.**
- **Average air-flow velocity along the frame was modelled.**


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All values for the radius $r_3$ except those corresponding to [1, 5, 12, 13, 15] are estimated according to [24].

All values for the radii $r_{ci}$ and $r_{fo}$ except those corresponding to [1, 5, 15] are taken from [24], and they correspond to the TEFC induction motors which were analyzed in the aforementioned references.

In most aforementioned references, the longitudinal distance $y$ was referred to the frame/fin length $L_5$, that is, $y$ was expressed in [p. u.].

In order to estimate the unspecified values of the ratio $V_0/V_F$ as well as the velocities $V_F$, $V_o$ and $V_y$ in [ms$^{-1}$], the eqs. (4) and (6) for $n = 7$ and the radius $r_c$ are used.

Here $h_{c5}$ [Wm$^{-2}$K$^{-1}$] is the heat transfer coefficient due to forced convection from the surfaces of the cooling fins, and $h_{c6}$ [Wm$^{-2}$K$^{-1}$] is the heat transfer coefficient due to forced convection from the surfaces amongst the cooling fins.

Here $N_{FE}$ is the number of the equivalent cooling fins, and $H_{FE}$ [m] is the height of the equivalent cooling fins.

Values for $p$, $P_n$ and $\Omega_m$ are estimated according to [25].

Turbulence factor models

Based on the fact that the heat transfer coefficients due to forced convection from the cooling fins and the inter-fin surfaces simultaneously depend on the air-flow velocity profile and the degree of air-flow turbulence [1], the authors proposed the following reduction function for the turbulence factor model:

$$K_{y} = K_1 + (K_2 - K_1)e^{-K_3 y}$$  \hspace{1cm} (7)

where $K_1$ is a dimensionless coefficient representing the minimum value of the turbulence factor at very great distance from the fan cowl ($y \to \infty$), $K_2$ – the dimensionless coefficient representing the maximum value of the turbulence factor at zero distance from the fan cowl ($y = 0$), and $K_3$ [m$^{-1}$] – the coefficient which describes the reduction of the turbulence factor with the increase of the longitudinal distance from the fan cowl.

By means of an identical reduction function Di Gerlando and Vistoli [4] modelled the air-flow velocity profile along the same frames of two motors having the shaft height of 100 mm and different pole numbers. The experimental data on the degree of air-flow turbulence from [1] have been correlated with the reduction function (7) by setting $K_1 = 0.122$ and $K_2 = 0.4$ in accordance with [2] as well as $K_3 = 4.62$ in accordance with [4]. As can be seen from fig. 2, the generated solid curve coincides well with the experimental data represented by the cross markers. It is therefore evident that the reduction of the turbulence factor can be modelled by the eq. (7).

For the purposes of this study, the values of coefficients $K_1$ and $K_2$ are calculated by means of two auxiliary analytical algorithms. The algorithms are developed by assuming that a TEFC induction motor can be treated as a solid whose exterior can be described by a single energy balance equation. The flowchart, presented in fig. 3, describes simultaneously both algorithms. In this flowchart, all the assignments and conditions are common. The inputs, statements, functions and outputs which are outside of the round brackets are common for both algorithms or belong to the first one. The inputs, statements, functions and outputs inside the brackets, which are also preceded by the disjunctive or, only relate to the second algorithm.

The first/second algorithm comprises the energy balance equation in which the amount of heat dissipated (by forced convection and radiation) through both end-shields is expressed as
a multiple of the amount of heat dissipated through the drive/non-drive end-shield. According to fig. 3, the multiples associated with the first and second algorithms are designated with \( \Delta q_D \) and \( \Delta q_{ND} \), respectively, where the subscripts \( D \) and \( ND \) refer to the drive and non-drive end-shield. Therefore, once the amounts of heat dissipated through the two end-shields are known, the energy balance equation can be solved without a number of empirical correlations. In the case of the first algorithm, for instance, correlations for forced convection from the flat and cylindrical outer surfaces of the non-drive end-shield [26-28] are not included in the iterative loop (in the 2nd set of statements). After the exit from the iterative loop, these correlations [26-28] are used for the calculation of the turbulence factors \( K_{1F} \) and \( K_{1C} \) corresponding to the flat and cylindrical outer surfaces of the non-drive end-shield, respectively. The subscripts \( F \) and \( C \), respectively, denote the flat and cylindrical outer surfaces of the end-shields.

The parameters displayed in fig. 3 have the following meanings: \( Q_{tot} \) [W] is the total power loss, \( T_a \) [K] – the ambient air temperature, \( h \) [Wm\(^{-2}\)K\(^{-1}\)] – an initial value for all heat transfer coefficients due to forced convection and radiation, \( K_{1F} \) [-] and \( K_{1C} \) [-] – are the turbulence factors corresponding to the flat and cylindrical outer surfaces of the drive end-shield, respectively, \( V_1 \) [ms\(^{-1}\)] – is the air-flow velocity at the end of the cooling channels, \( E \) [K] – the specified accuracy, \( J \) – the index of the current iteration, \( T_s \) [K] – the average temperature of the motor exterior, \( T'_s \) [K] and \( T''_s \) [K] – are the programme variables reserved for values of the temperature \( T_s \), \( Pr \) is the Prandtl number, \( Re_{NDF} \), \( Re_{NDC} \), \( Re_{DF} \), and \( Re_{DC} \) – corresponding Reynolds numbers, and \( Nu_{NDF} \), \( Nu_{NDC} \), \( Nu_{DF} \), and \( Nu_{DC} \) – corresponding Nusselt numbers.

Moreover, \( S_1, S_2, \ldots, S_9 \) and \( S_{10} \) [m\(^2\)] are portions of the outer surface area of the motor. The coefficients \( h_{1F}, h_{2F}, \ldots, h_{9F} \) and \( h_{10F} \) [Wm\(^{-2}\)K\(^{-1}\)] and \( h_{1C}, h_{2C}, \ldots, h_{9C} \) and \( h_{10C} \) [Wm\(^{-2}\)K\(^{-1}\)] are the programme variables reserved for values of the heat transfer coefficients due to forced convection. The coefficients \( h_{1F}, h_{2F}, \ldots, h_{9F} \) and \( h_{10F} \) [Wm\(^{-2}\)K\(^{-1}\)] and \( h_{1C}, h_{2C}, \ldots, h_{9C} \) and \( h_{10C} \) are

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**Figure 3. The flowchart illustrating the algorithms used for the calculation of \( K_1 \) and \( K_2 \)**
The programme variables reserved for values of the heat transfer coefficients due to radiation. The subscripts $i = 3$ and $i = 8$ relate to the flat outer surfaces of the drive and non-drive end-shields, respectively. Also, $i = 4$ and $i = 7$ relate to the cylindrical outer surfaces of the drive and non-drive end-shields, respectively.

The values of the coefficient $K_i$ describing the reduction of the turbulence factor models are obtained by fitting curves through the values of $K_1$ and $K_2$, which were previously determined.

**Results and discussions**

**Air-flow velocity profiles**

Figure 4 illustrates the fitting procedure for determining the coefficients $C_2$ and $C_3$. The markers indicate measured air-flow velocities, expressed in [ms$^{-1}$] or [p. u.], and the curves except the solid curve with point markers corresponding to fig. 4(b)-E indicate calculated air-flow velocities.

Based on the comparisons, it is found: (1) that the coefficient $C_2$ amounts to 1500 m·rpm, and (2) that the values of the coefficient $C_3$ are: $C_3 = 3$ for $p \leq 4$ and $P_n \leq 15$ kW, $C_3 = 2$ for $p \leq 4$ and $P_n > 15$ kW, and $C_3 = 8/3$ for $p > 4$ and $\forall P_n$. This is in satisfactory agreement with most of the experimental data. The quantity of $P_n = 15$ kW that corresponds with the change in value of the coefficient $C_3$ also coincide with the fact that the TEFC induction motors of the rated powers up to 15 kW have an aluminum frame, while the motors of the rated powers above 15 kW have a cast iron one [16].

It is also important to note the large spread of values obtained in fig. 4. The cause for the large spread in data can be explained by the fact that the researchers utilized significantly different TEFC induction motors and frames, different loading conditions as well as different assumptions in their studies. Excluding the results reported in [10, 16-18], all other data are reliable and comparable. The best manner to show that is to discuss the results related to the four similar four-pole, 4 kW, 50 Hz TEFC induction motors from tab. 1, which are presented in fig. 4(b). Moreover, this motor is amongst the most commonly studied TEFC induction motors in the literature [5, 10, 16-20].

Amongst other details, it can be noticed from fig. 4(b) that the solid curve, fig. 4(b)-B, and the dashed curve, fig. 4(b)-D, fit well within the measured data which are labelled with the cross markers, fig. 4(b)-A. The solid curve passes over the maximum measured values of the air-flow velocity along the frame, while the dashed curve intersects almost ideally with the solid curve with point markers, fig. 4(b)-E, representing the average air-flow velocity along the frame. Compared to a curve that would pass through the square markers, fig. 4(b)-C, the dashed curve provided far better agreement with the cross markers, that is, with the measured data from [10, 16-18]. According to these references, the value of the average air-flow velocity along the frame is significantly lower than 5 m/s and amounts to approximately 4 m/s. Moreover, the dotted curve, fig. 4(b)-G, does not fit so well within the measured data from [5] which are labelled with the plus sign markers, fig. 4(b)-F. However, substituting the estimated value of the ratio $V_r/V_p = 0.717$ (which is calculated by using the eqs. (4) and (6) for $n = 7$) by the tabulated value $V_r/V_p = 0.672$ into the eq. (2), it would be possible to provide better agreement between the dotted curve and the plus sign markers.

From the same authors, such the differences were also obtained for four-pole TEFC induction motors of the rated powers of 7.5, 15, 30, and 55 kW [10, 16-18]. It can also be noticed in fig. 4(e). The reasons for this are the same as those previously described. The fact that there is an underestimation or overestimation of the average air-flow velocities along the frames of these motors can also be supported by data from the motor guides [29, 30]. By comparing
the values of the air-flow velocity at the beginning of the cooling channels from [10, 16-18] and the corresponding minimum values from [29], it can be concluded that there are significant differences between these values. These differences amount to $-12.8$, $-32.2$, $-9.7$, $-4$, and $+13.3\%$ for motors of the rated powers of 4, 7.5, 15, 30, and 55 kW, respectively. If one excludes the underestimation (or overestimation) of the average air-flow velocities along the frames, then the following conclusion can be drawn: these motors were not designed in accordance with the standards. However, that was not the case with these motors. Therefore, the experimental results reported in [10, 16-18] could not be used in a better manner for this study.

**Turbulence factor models**

If it is assumed that the turbulence factors at zero distances from the fan cowls are the same for all the motors and that their values depend only on the longitudinal distance; then,
based on the experimental data related to the 4 and 15 kW motors from [5] and by means of two auxiliary analytical algorithms, which were previously presented in the section Turbulence factor models, the following equations are obtained:

\[
K_{\xi,y} = 1 + (1.4163 - 1)e^{-4.62y} \quad \text{– for flat outer surfaces of end-shields} \quad (8)
\]

\[
K_{\xi,y} = 1 + (1.6776 - 1)e^{-13.087y} \quad \text{– for cylindrical outer surfaces of end-shields} \quad (9)
\]

\[
K_{\xi,y} = 1 + (1.8 - 1)e^{-13.087y} \quad \text{– for fins and inter-fin surfaces} \quad (10)
\]

The values of coefficients \(K_1\), \(K_2\), and \(K_3\) defined in the aforementioned three equations are obtained in the following manner:

– For the motors having longer axial lengths, based on the results reported in [1], it is obvious that the degree of air-flow turbulence at the drive end approximately equals to zero. That is why the effect of turbulence on the heat transfer at the drive end of motors with longer axial lengths can be ignored and why the coefficient \(K_1\) equals to 1 in all three equations.

– In [5] it is reported that the amounts of heat dissipated through the drive end-shield and the non-drive end-shield of the 15 kW motor at rated operation equal to 6 and 12 % of \(Q_{tot}\) respectively. By introducing this experimental data into associated energy balance equation for the 15 kW motor exterior and assuming that \(K_1 = K_{1F} = 1\) at a distance \(y = L_5 = 0.37\) m, the authors found that \(K_2\) amounts to \(K_{2F} = 1.4163\) and \(K_{2C} = 1.6776\) for the flat and cylindrical outer surfaces of the end-shields, respectively.

– According to [5], the portions of heat dissipated through the drive end-shield and the non-drive end-shield of the 4 kW motor at rated operation equal to 9 and 19 % of \(Q_{tot}\) respectively. By substituting this data into associated energy balance equation for the 4 kW motor exterior and assuming that \(K_2 = K_{2F} = 1.4163\) and \(K_2 = K_{2C} = 1.6776\) (for the flat and cylindrical outer surfaces of the end-shields), the authors found that \(K_1\) amounts to \(K_{1F} = 1.2385\) and \(K_{1C} = 1.0686\) for the flat and cylindrical outer surfaces of the end-shields, respectively.

– It is estimated that the models which fit well within the previously-calculated values of the turbulence factors for the flat and cylindrical outer surfaces of the end-shields have the exponents \(K_3 = K_{3F} = 4.62\) and \(K_3 = K_{3C} = 13.087\), respectively. Figure 5 depicts the models (8) and (9), that is, the dashed and dotted curves, together with corresponding calculated values of the turbulence factor (represented by the square and plus sign markers).

– The turbulence factor model (10) is derived from the model (9) by substitution of the calculated value of the coefficient \(K_2 = 1.6776\) by the arithmetic mean of the two bounds of the range 1.7-1.9, which is reported in [2]. The model (9) is derived by using the correlations for turbulent forced convection from a cylinder in axial air-flows, which correspond to the degree of air-flow turbulence \(\zeta = 0.1\%\) [26-28]. Therefore, if the degree of air-
flow turbulence is equal to zero ($\xi = 0\%$), the model (10) can be used instead of (9) for the cylindrical outer surfaces of the end-shields, but in combination with the appropriate correlations for laminar forced convection.

**Conclusion**

The authors developed an empirically-based methodology for estimating air-flow velocity profiles and turbulence factor models along the frames of various TEFC induction motors. By using the velocity profiles and the turbulence factor models, it is possible now to express the energy balance on the exterior of any TEFC induction motor with just a single equation and to determine accurately the steady-state axial temperature distribution (along the cooling channels). Therefore, the methodology establishes a general framework for developing novel analytical models for steady-state thermal analysis of induction motors. The velocity profiles and turbulence factor models have been obtained on the basis of a representative sample of TEFC induction motors.

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**References**
