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Microwave Admittance of Gold-Palladium Nanowires with Proximity-Induced Superconductivity

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Quantitative electrical admittance measurements of diffusive superconductor–normal-metal–superconductor (SNS) junctions at gigahertz frequencies and millikelvin temperatures are reported. The gold-palladium-based SNS junctions are arranged into a chain of superconducting quantum interference devices. The chain is coupled strongly to a multimode microwave resonator with a mode spacing of approximately 0.6 GHz. By measuring the resonance frequencies and quality factors of the resonator modes, the dissipative and reactive parts of the admittance of the chain are extracted. The phase and temperature dependence of the admittance near 1 GHz are compared with theory based on the time-dependent Usadel equations. This comparison allows the identification of important discrepancies between theory and experiment that are not resolved by including inelastic scattering or elastic spin-flip scattering in the theory.

1. Introduction

The transport of direct current (dc) between two superconductors (S) separated by a diffusive normal-metal (N) link is in general well understood both theoretically and experimentally.1–6 At low temperatures and currents, Andreev reflection7 leads to the formation of a gap in the density of quasiparticle states in N and allows a dissipationless supercurrent to flow. This gap has been directly observed in tunnel spectroscopy measurements.8,9 Shapiro steps, supercurrent enhancement, and other non-equilibrium effects under intense microwave irradiation have also been extensively studied.10–16

In contrast, the near-equilibrium response of superconductor–normal-metal–superconductor (SNS) junctions to weak microwave radiation has become an active area of investigation only recently.17–21 While the adiabatic contribution to the kinetic inductance can be calculated from the dc current–phase relation, at high frequencies both reactive and dissipative contributions arise also from other mechanisms, such as driven transitions between quasiparticle states and oscillation of the Andreev level populations.18 Surprisingly, however, little experimental data has been published on the topic thus far.22,23 The parameter regimes and materials studied in the published experiments are very sparse, hence limiting the extent to which theoretical predictions17–21 can be tested. In practice, data on the effective inductance and losses also expedites the process of designing high-frequency SNS-junction-based circuits, such as the SNS nanobolometer.24,25

Previous experimental studies22,23 have probed flux- and temperature-dependent changes in the linear microwave response of a superconducting ring with a gold normal-metal inclusion. The superconducting ring consisted of ion-beam-deposited tungsten in the first experiments,22 and sputter-deposited Nb with a thin Pd layer at the SN interface in the later experiments.23 A single SNS ring was biased with a dc magnetic flux and coupled weakly to a multi-mode microwave resonator. By measuring flux-dependent shifts in the quality factors and resonance frequencies, the authors determined how the complex-valued electrical susceptibility χ changes.22,23 The change in χ, as a function of flux and temperature, was reported to be in excellent agreement with theoretical predictions based on Usadel equations and numerical simulations. However, the implicit offsets in both the real and imaginary parts of the reported susceptibility prevent a comparison to the theoretically predicted absolute values of Re[χ] and Im[χ]. They also prevent the accurate prediction of the effective inductance (Re[χ]−1) and the loss tangent (Im[χ]/Re[χ]), which are the key quantities for practical high-frequency applications of SNS junctions.

In this article, we present measurements of the SNS junction admittance Z−1(ω) = χ/|ω| for gold-palladium based junctions at angular frequencies ω of order 2π × 1 GHz. We use a chain of
SNS superconducting quantum interference devices (SQUIDs) with a strong capacitive coupling to a multimode microwave resonator with a typical mode spacing of 0.6 GHz. Each chain consists of 20 SNS SQUIDs in series. The strong coupling and the large number of SQUIDs lead to significant changes in the frequencies and quality factors of the resonator modes, which allow determining $Z^{-1}$ without an offset. The absence of an offset enables us to show that the Usadel-equation-based theory we consider cannot simultaneously explain the observed real and imaginary parts of $Z^{-1}$.

2. Samples

We study the linear electrical response of SNS junctions at millikelvin temperatures in two samples: Sample 1 and 2. The gold-palladium nanowires used as the normal-metal are deposited simultaneously in the same fabrication steps as for our recent nanobolometer circuits.[25] We determine the Au:Pd atomic ratio of the alloy to be approximately 3:2 using energy-dispersive X-ray spectroscopy (see Experimental Section). The nominal junction length $l$ is 300 nm and the nominal cross-sectional area is $30 \text{ nm} \times 120 \text{ nm}$. The normal-state resistance of a single junction $R_0 = 15 \text{ Ω}$ is estimated based on four-wire dc measurements of reference samples. From the reference measurements, we also estimate the upper bound for the contact resistance $R_0$ to be approximately 1 Ω. We only give an upper bound for the contact resistance because of the uncertainties introduced by the crossed-wire geometry[26] of the reference samples. Reference samples are similar to those in Figure 1b in Reference [25].

The key parameter determining the strength of the proximity-induced superconductivity is the Thouless energy $E_T = \hbar D l^2$, where $D = 22 \text{ cm}^2 \text{ s}^{-1}$ is the diffusion constant. We obtain the diffusion constant by measuring the resistivity of the nanowires and scaling the result according to literature values for the proportionality $D \propto \rho^{-1}$.[27] The result we obtain is approximately 30% higher than that in Reference [27]. In our samples, the Thouless energy is $E_T = k_B \times 190 \text{ mK} \times h \times 3.9 \text{ GHz}$, where $h = 2\pi\hbar$ is the Planck constant and $k_B$ is the Boltzmann constant. The superconducting sections are 100-nm-thick aluminum, which implies that the energy gap $\Delta$ in the superconductors is much larger than the Thouless energy ($\Delta/E_T \approx 13$). Both S and N parts are fabricated using electron beam lithography and evaporation. Further fabrication details are reported in the Experimental Section.

The SNS junctions are arranged into a chain of SQUIDs, as shown in Figure 1. Each SQUID loop has a relatively small area of 20 μm² in order to minimize sensitivity to external magnetic field noise. In addition, we measured Sample 2 in a double-layer magnetic shield. The geometric inductance of each loop is small ($L_C < 10 \text{ pH}$) compared to the effective inductance of the SQUID, as verified by the results below. A dc magnetic flux bias is applied by an external coil that provides a uniform flux bias $\Phi$ for each SQUID in the 100-μm-long chain. Assuming identical junctions, the field biases each SNS junction at a phase difference $\Phi/2\Phi_0 = m/2\Phi_0$ at dc, where $\Phi_0$ is the magnetic flux quantum $h/2e$ and $m$ is an integer that minimizes $|\Phi - m\Phi_0|$. We note that the flux bias we label as $\Phi = 0$ may be offset from the actual zero flux condition by an integer multiple of $\Phi_0$.

The SQUID chains in the two samples are nominally identical, except for the addition of heat sinks to Sample 2 (see Figure 1c). The heat sinks are designed to reduce the hot-electron effect,[28] i.e., the increase of the quasiparticle temperature above the phonon bath temperature $T_B$ that we measure. For each SNS junction, the heatsink consists of two large (0.5 μm³) reservoirs of gold-palladium that are thermally strongly coupled to the junction.

In addition to the SQUID chain, the chip contains a transmission line resonator of length 10 cm (see Figure 1a). The last column emphasizes that Sample 2 includes additional large gold-palladium heat sink reservoirs for enhancing electron–phonon coupling.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$C_C$ [pF]</th>
<th>$C_0$ [pF]</th>
<th>$l_0$ [mm]</th>
<th>$f_0$ [MHz]</th>
<th>$Z_0$ [Ω]</th>
<th>Heat sinks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.15</td>
<td>14.5</td>
<td>97.2</td>
<td>637</td>
<td>39</td>
<td>No</td>
</tr>
<tr>
<td>2</td>
<td>0.44</td>
<td>15.2</td>
<td>96.5</td>
<td>633</td>
<td>39</td>
<td>Yes</td>
</tr>
</tbody>
</table>
termination, i.e., samples without the SQUID chain. From the control measurements, we extract the fundamental frequency of the transmission line resonator $f_0 = 635$ MHz, and confirm that the internal quality factor $Q_{in}$ is $10^3$ for the resonances we consider ($n = 3$). The latter implies that we can neglect the losses in the transmission line part of the resonator, and in the Al$_2$O$_3$ used as the dielectric material in the lumped element capacitors ($C_C$ and $C_J$). This is valid because introducing the SQUID chain lowers $Q_{in}$ to the order of $10^2$, as observed below. We also deduce the characteristic impedance $Z_0 = 39$ $\Omega$ of the transmission line from the measured $f_0$, the length of the resonator, and the design value for the inductance per unit length.

3. Measurement Scheme and Sample Characterization

We determine the admittance of the SQUID chain by embedding it as the termination of the transmission line microwave resonator, as illustrated in Figure 1a. We first determine the resonance frequency $f_n$ and the internal quality factor $Q_{in}$ of each mode $n$ by measuring the frequency-dependent transmission coefficient $S_{21}(\omega)$ through the feedline. By comparing $f_n$ and $Q_{in}$ to values measured in control samples with no SQUID chains, we can determine the admittance of the SQUID chain at multiple frequencies. Specifically, we use a circuit model (Figure 1a) that allows extracting the admittance of the SQUID chain $Y_{chain}$ from the response of the combined resonator/SQUID-chain system. The admittance of each individual SNS junction is then given by $10 Z_{chain}^i$, assuming that the junctions are identical and that geometric inductance is negligible.

Figure 2 shows the normalized transmission through the feedline at frequencies near 1.4 GHz, probing the third ($n = 3$) mode in Sample 2. The normalization (defined precisely in the Experimental Section) removes all spurious features in the transmission data that do not depend on flux. This procedure reveals the oscillatory flux dependence of the resonance frequency $f_n$ with a period we identify as $\Phi_0$. As the flux bias is increased away from integer multiples of $\Phi_0$, we measure a decrease in both the resonance frequency and the loaded quality factor $Q_{in}$. This behavior is more clearly visible in Figure 2b,c with individual slices of the transmission data for $\Phi/\Phi_0 = 0$ and $\Phi/\Phi_0 = 0.3$. These changes in the resonance indicate that both the inductance and the losses are maximized in the SQUID chain.

To extract $f_n$ and $Q_{in}$ quantitatively, we fit the measured normalized transmission for the $n$th mode to the model:

$$S_{21}(\omega) = 1 - \frac{Q_{in} - 2iQ_{in} \delta f_n}{Q_{in} + i2Q_{in} \delta f_n} \frac{f_n}{f_n - f_n}$$

where $\delta f_n$ is a fit parameter that characterizes asymmetry, and the external quality factor $Q_{out}$ is governed by the coupling ($C_C$) to the feedline. From the obtained fit parameters $Q_{in}$ and $Q_{out}$, we compute the contribution of losses due to the SQUID chain as $Q_{chain} = Q_{in} - Q_{out}$. Figure 3 shows the extracted values of $f_n$ and $Q_{in}$, for frequencies up to 12 GHz ($n = 20$).

In the low-frequency and low-temperature regime, the SQUID chain behaves like an inductor, i.e., most of the admittance is reactive $|\text{Im}[Z_{chain}]| >> |\text{Re}[Z_{chain}]|$, and $\text{Im}[\omega Z_{chain}]$ varies slowly as a function of the angular frequency $\omega = 2\pi f$. Consequently, we parametrize the admittance as $Z_{chain} = R + (i\omega L)^{-1}$. Figure 3 demonstrates that this is a good parametrization by showing qualitative agreement between the experimental data and predictions for the mode shifts and quality factors using a simplified model where $L$ and $R$ are constant.

Let us now discuss the extraction of the admittance $Z_{chain}^{-1}$ from the measured $f_n$ and $Q_{in}$ values. For an ideal transmission line resonator with open-circuit conditions at both ends ($C_C = 0$), the $n$th mode is located at frequency $f_n$. In contrast, for the samples with the SQUID chains, the frequency-dependent reactive, i.e., imaginary parts of the termination admittances $i\omega C_C$ and $[(i\omega C_C)^{-1} + Z_{chain}]$ lead to the non-zero modeshift of Figure 3, that we use to determine $\text{Im}[Z_{chain}]$. Similarly, the measured $Q_{in}$ gives information about the dissipative, i.e., real part of $Z_{chain}^{-1}$. 

![Figure 2](image-url)

**Figure 2.** a) Magnitude of the normalized transmission coefficient $|S_{21}|$ for the third resonator mode ($n = 3$) of Sample 2 at $T_b = 10$ mK as a function of frequency and flux-bias $\Phi$. b) Traces from a) along $\Phi/\Phi_0 = 0$ and $\Phi/\Phi_0 = 0.3$ with fits (solid lines) to the model in Equation (1). c) Same as (b) but for the phase arg($S_{21}$). The extracted resonance frequencies and internal quality factors are $f_3 = (1452.6 \pm 0.4)$ MHz; $Q_{in} = 127 \pm 8$ and $f_3 = (1411.1 \pm 0.4)$ MHz; $Q_{in} = 78 \pm 4$ for $\Phi/\Phi_0 = 0$ and $\Phi/\Phi_0 = 0.3$, respectively. The discontinuities in the background in a) are artifacts caused by the normalization procedure of $S_{21}$ (see Experimental Section).
The inductances near 1 GHz. The reported values provide an important reference for designing high-frequency devices based on gold-palladium SNS junctions. That is, they imply that an effective inductance of a few hundred picohenries per junction and a loss tangent of a few percent are observed around 1 GHz at millikelvin temperatures.

The accuracy of the admittance measurement is limited by uncertainty in the fit parameters \( f_0 \) and \( Q_n \), and the systematic device parameters measured in control samples (e.g. \( f_0, C_0 \)). The error bars of the \( R \) and \( L \) values throughout this article are propagated from the uncertainties in the fitted \( f_0, Q_n \) values at the 68% confidence level. There remains a 10% relative error in the reported \( R \) and \( L \) values from the systematic device parameter uncertainties at 1 GHz. This arises predominantly from chip-to-chip variations in the unterminated resonance frequency, \( \Delta f_0 \approx 3 \) MHz, based on measurements of control samples. As frequency increases, \( \Delta f_0 \) has a larger effect on the uncertainty in \( L \) in Equation (2). For example, in Sample 2, the uncertainty \( \Delta f_0 \approx 3 \) MHz leads to a relative error in \( L \) of 70% at 10 GHz. For this reason, we concentrate on resonances near 1 GHz in the discussion below.

4. Theory

In the next section, we compare the experimental results to theoretical predictions[18] based on the time-dependent Usadel equation.[19] The low-frequency and low-temperature regime \( \hbar \omega, k_B \approx E_F \) considered below, the imaginary part of the admittance of the junction is expected to be mostly determined by the adiabatic Josephson inductance associated with the supercurrent, i.e., the \( \Phi \) derivative of the dc supercurrent. The real part, on the other hand, mainly arises from driven quasiparticle transitions in the junction. The availability of such transitions is sensitive to the density of quasiparticle states. In particular, the presence of a proximity-induced energy gap \( E_g \approx E_T \) in the density of states should lead to an exponential increase in the resistance as \( k_B T \) decreases below \( E_g \).

However, the low-temperature values of \( L \) and \( R \) we measure (Table 2) are dramatically larger than those predicted using the parameters considered in Reference [18]. This is evident from a cursory comparison of Figure 1 in Reference [18] to our \( \omega \) and \( R \) values. The inductance per junction \( L/10 \approx 300 \) pH is also an order of magnitude higher than the expected adiabatic Josephson inductance \( L_J = 2 R_0 e \Phi / |\Phi| = 50 \) pF, where we approximate the dc supercurrent \( I_J(\Phi) = I_c \sin(\Theta/\Phi) \) and the critical current \( I_c \) as the ideal value 6.7 \( eR_0 \) for \( \Delta/|\Delta| = 13.\) Moreover—in the results below—we observe a weak temperature dependence of \( R(\Phi = 0) \) measured near 1 GHz, which is in stark contrast to the theoretically predicted exponential dependence.

The observed values of \( L \) and \( R \) imply that the proximity-induced superconductivity is significantly weaker than expected. We consider two distinct scattering mechanisms as potential explanations for this. First, we include dephasing due to inelastic scattering by choosing a phenomenological relaxation rate \( \Gamma_\text{ef} \).\[16\] Second, we include a spin-flip scattering rate \( \Gamma_\text{sf} \) which could arise from dilute magnetic impurities in the weak link.\[17\] Specifically, we include the spin-flip scattering as an additional self-energy \( \tilde{\Sigma} = -\frac{\hbar}{2} I_c g \Gamma_\text{sf} \) in the equations defined.
in Reference [18]. Considering spin-orbit impurity scattering in a similar approximation would not affect the superconducting proximity effect within the model considered here. Although quantitative details differ, both of the scattering mechanisms generally lead to increased dissipation and increased inductance. Increased dissipation occurs mainly due to the suppression of $E_{\text{p}}$, while increased inductance occurs mainly due to the increase in $L_T$.

Theoretical work on the microscopic origin of the scattering rates in disordered metals is reviewed in References [33] and [34]. Experiments have also been carried out with high-purity metal wires. However, we are not aware of measurements on the gold-palladium alloy used here, which prevents direct comparison to existing literature. Instead, our goal is to estimate the scattering rates required for a qualitative match to the experimental results. We find that in order to reproduce the experimentally observed $L$ or $R$, the phenomenological rates $\Gamma$ and $\Gamma_{\text{sf}}$ must be large, i.e., comparable to $E_{\text{F}}/h$ and $k_{\text{B}}T/h$.

Inelastic scattering and spin-flip scattering are not the only possible explanations for observing proximity-induced superconductivity that is weaker than what is predicted by the ideal Usadel-equation-based theory. Although we do not attempt to exhaustively cover all candidates, we note that the SN contact resistance in our samples is much smaller ($R_R < 1 \, \Omega$) than the normal-state resistance ($R_N = 15 \, \Omega$). While the smallness of the ratio $R_R/R_N$ does not conclusively exclude explanations based on imperfect interfaces, it limits them significantly.

5. Temperature and Flux Dependence near 1 GHz

Below, we compare the predicted and observed dependences of $Z_{\text{chain}}$ on the bath temperature and magnetic flux. We choose to analyze two low-$n$ resonances near 1 GHz, mainly because the $L$ values we extract for them suffer the least from the uncertainty in $f_0$.

Figure 4 shows the measured flux dependence of $R$ and $L$ for the third ($n = 3$) resonance in Sample 2. The bath temperature is $T_b = 195$ mK, which should be high enough for neglecting the hot-electron effect, i.e., for assuming that $T = T_b$. As expected, we observe that $R$ and $L$ are periodic in flux, and that the inductance $L$ and the loss tangent $\tan \delta L/R$ are minimized (maximized) at integer (half-integer) values of $\Phi/\Phi_0$.

Figure 4 also includes theoretical predictions for two different rates of inelastic scattering. The weaker of the two rates ($\Gamma = 2.3 \, k_{\text{B}}T_b/h$) reproduces $R(\Phi) = 0$ well and gives a reasonable prediction for its flux-dependent oscillations. Furthermore, if we could only measure changes in $L$, we might conclude that the predicted flux modulation of $L$ is in fair agreement with the experimental data for this moderate value of $\Gamma$. However, the absolute value of the prediction for $L(\Phi)$ is several times smaller than the observed value at nearly all flux values. This highlights the importance of measuring $L$ and $R$ without offsets if theories are to be rigorously tested. Note that we can improve the agreement between the predicted and measured $L(\Phi)$, especially around integer values of $\Phi/\Phi_0$, by using a very strong inelastic scattering rate of $\Gamma = 8k_{\text{B}}T_b/h$ in the theoretical calculation. However, this value of $\Gamma$ leads to a clear disagreement in the amplitude of the oscillations in $R(\Phi)$ as shown in Figure 4b.

Figure 4 also shows the theoretical predictions that include strong spin-flip scattering. By choosing $\Gamma_{\text{sf}}$ appropriately, the predictions become nearly identical to the case of strong inelastic scattering. Therefore, the conclusions of the previous paragraph also apply to predictions where scattering is spin-flip dominated. Furthermore, the similarity of the predictions shows that, in this parameter regime, the source of additional dephasing is unimportant.

To gain further insight, we study the temperature dependence of $R(\Phi = 0)$ and $L(\Phi = 0)$ for one resonance from each sample near 1 GHz (see Figure 5). In addition to the measured data points, Figure 5 shows theoretical predictions with scattering parameters that—at 195 mK—are identical to those in Figure 4. However, we note that considerable freedom remains in choosing the temperature dependence of the scattering rates. Rigorously justifying a particular temperature scaling would require knowledge of the specific microscopic mechanism responsible for the scattering. However, as the theoretical predictions already disagree with the measured results at the phenomenological level at a fixed temperature (Figure 4), identifying any specific microscopic mechanism seems implausible. As instructive examples, we choose $\Gamma \propto T$ and a constant $\Gamma_{\text{sf}}$ in Figure 5. Unsurprisingly, none of the predictions simultaneously matches the observed temperature dependence of $L$ and $R$. Nevertheless, the experimental data in Figure 5 may serve an important role in testing alternative theories in the future.
with a comparison to quasiclassical theory for diffusive SNS previously. Thus, our experimental technique provides a more elements of the electrical admittance without arbitrary offsets. In on the Usadel equation. This disagreement manifests itself values that exceed theoretical pre-

We note that the discrepancies shown here are not in direct contradiction with the previous experiments that both the SNS junctions and the measurement scheme presented here are very different from these preceding studies. Firstly, the weak link material is different than in the previous experiments. We cannot rule out the possibility of effects specific to gold-palladium that reduce coherence in the weak link. Secondly, we measure both the reactive and dissipative components of the electrical admittance without arbitrary offsets. In contrast, only changes in the admittance have been reported previously. Thus, our experimental technique provides a more stringent test of the accuracy of the theory and reveals quantitative disagreements more easily.

In conclusion, we reported measurements of microwave frequency admittance for gold-palladium SNS junctions, together with a comparison to quasiclassical theory for diffusive SNS weak links. These discrepancies between measurement results and theoretical predictions suggest that dephasing caused by inelastic scattering, or elastic spin-flip scattering, is probably not the correct mechanism for explaining why the proximity-induced superconductivity is weaker than expected in our gold-palladium SNS junctions. Further theoretical work is required for reaching simultaneous agreement for the magnitude, temperature dependence, and flux dependence of both the dissipative and reactive parts of the admittance. Mechanisms that may need to be taken into account include imperfect interfaces, electron–electron and fluctuation effects in low-dimensional superconducting structures, and para-magnon interaction. Magnetic effects could be particularly important in SNS junctions that include palladium, which is paramagnetic in bulk and can even become ferromagnetic in nanoscale particles. In general, the relationship between microscopic materials properties and coherence at microwave frequencies in normal-metal Josephson junctions should be clarified, both experimentally and theoretically. A productive experimental approach may be to first investigate systems such as Nb/Cu weak links that, based on previous experiments, are expected to behave in an ideal fashion at dc.

7. Experimental Section

Resonator fabrication: The substrates were 4” (0.5-mm-thick) high-resistivity (>10 Ohm cm) Si wafers with 300 nm of thermal oxide. First, a niobium thin film (thickness 200 nm) was sputter-deposited on the entire wafer. Next, the coplanar waveguide (CPW) structures were defined with AZ5214E positive photoresist that was reflowed in air at 150 °C for 1 min to ensure a positive etch profile of the resulting Nb features. Then CPWs were etched with an rf-generated plasma under a constant flow of SF6 /O2 gases at constant power. The remaining resist was removed with solvents and an additional O2-plasma cleaning step. The 4” wafer was then coated with a protective layer of resist and pre-diced with partial cuts along device pixel outlines on the back of the wafer.

Capacitor dielectric fabrication: The Al2O3 dielectric for the on-chip Nb-Al2O3-Al capacitors was formed by atomic layer deposition with 455 cycles in a H2 O/TMA process at 200 °C, resulting in a thickness of 42 nm. The thickness was verified in ellipsometry using an index of refraction nAl2O3 = 1.64. Measurements of reference Nb/Al2O3/Al capacitors yielded a capacitance per unit area of 1.4 fF μm-2.

Nanostructure fabrication: The gold-palladium nanowires and aluminum superconducting leads were fabricated by electron beam lithography in two separate evaporation and liftoff steps. In the first step, gold and palladium pellets were evaporated from the same crucible with an electron beam heater. Afterward, unwanted AuPd was lifted off with an electron beam. Finally, after liftoff of the Al film, individual resonator pixels were snapped along the pre-diced lines and packaged for measurement.

The chemical composition of the gold-palladium material was determined with energy-dispersive X-ray spectroscopy for incident electron beam energies 5 keV, 10 keV, and 20 keV (Figure 6). The average Au:Pd atomic ratio (weight ratio) was approximately 3.2 (3:1).

Cryogenic Measurements: Measurements were carried out in a commercial cryostat with a base temperature of 10 mK. The transmission coefficient was probed with a vector network analyzer. The device input line had >100 dB fixed attenuation. For all measurements, the output signal was amplified by a broadband low-noise cryogenic amplifier and by additional room temperature amplifiers. For some measurements (e.g. Sample 2, n = 3) two cryogenic isolators were placed on the base cooling stage between the low-noise cryogenic...
amplifier and the sample. Each sample was placed in a custom-printed circuit board and sealed within a metal enclosure. The external flux coil consisted of a superconducting solenoid with 100 turns that was fixed outside the metal enclosure. One \( \Phi_0 \) period in Figures 4 and 5 corresponds to a current change of \( \Delta I_{\text{mag}} = 7 \text{ mA} \) through the coil. Magnetic shielding surrounded both the enclosure and the flux coil in the case of Sample 2.

The measurement power incident at the transmission line input was approximately \(-128 \text{ dBm} \) for the data shown in Figures 4 and 5. This drove a current of roughly 5 nA through the SQUID chain at \( \Phi = 0 \) for \( n = 3 \) of Sample 2. This is far below the estimated critical current of the SQUID chain. Furthermore, experimentally we ensured that we measured the linear response by making sure that the measured \( S_{21} \) was not sensitive to factor-of-two changes in the measurement power.

**Normalized transmission coefficient:** We defined the normalized transmission coefficient \( S_{21}(\Phi) = S_{21}(\Phi)/S_{21}(\Phi_{\text{ref}})/S_{21}(\Phi_{\text{ref}}) \), where \( S_{21}(\Phi) \) is the raw transmission coefficient, including contributions from the cabling and other external circuitry and \( S_{21}(\Phi_{\text{ref}}) \) is the prediction of Equation (1) at a reference flux value \( \Phi_{\text{ref}} \). First, dividing \( S_{21}(\Phi) \) by \( S_{21}(\Phi_{\text{ref}}) \) removes all flux-independent features introduced by the external circuitry and unintentional reflections. The ratio \( S_{21}(\Phi)/S_{21}(\Phi_{\text{ref}}) \) only includes information about the response of the flux-varying SQUID chain since the external circuit leads to identical contributions in both scans. Second, we multiply \( S_{21}(\Phi)/S_{21}(\Phi_{\text{ref}}) \) by \( S_{21, \text{ref}}(\Phi_{\text{ref}}) \) in order to remove the systematic contribution of the reference in the final \( S_{21} \) result, which would otherwise appear at all values of \( \Phi \) as a static vertically inverted mirror image \((1/S_{21, \text{ref}}(\Phi_{\text{ref}})) \) of the reference resonance. We emphasize that we also determine \( S_{21, \text{ref}}(\Phi_{\text{ref}}) \) itself by fitting a ratio of two scans. That is, we fit \( S_{21}(\Phi)/S_{21}(\Phi_{\text{ref}}) \) to the quotient of two instances of Equation (1), each with a different set of fit parameters. These fit parameters determine the \( S_{21, \text{ref}}(\Phi_{\text{ref}}) \) used in the normalization process described above.

In practice, the scan used as the reference alternates between \( \Phi_{\text{ref}} = \Phi_0/2 \) and \( \Phi_{\text{ref}} = 0 \), changing from one to the other whenever \( \Phi \) crossed \( \Phi_0(1/4 + k/2) \), where \( k \in \mathbb{Z} \). This kept the resonance in the reference far from the resonance frequency at the \( \Phi \) value being analyzed. These changes in \( \Phi_{\text{ref}} \) caused the apparent discontinuities in the background color in Figure 2.

As shown in Figure 2b,c the normalization procedure revealed clear Lorentzian-like lineshapes for \( S_{21} \), as one would expect for a resonator of this type. Raw \( S_{21}(\Phi = 0) \) and \( S_{21}(\Phi = 0.3) \) data are shown in Figure S3 of the Supporting Information for reference. Disentangling the SQUID chain response from the rest of the circuit is also aided by the fact that the SQUID chain is the only circuit element that responds to flux periodically, with a period that is identical for all of the identified resonances. In general, the external circuit and background noise do not have such a response as we confirmed in measurements of control samples with no SQUIDs.

**Supporting Information**

Supporting Information is available from the Wiley Online Library or from the author.

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