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Position Estimation for Synchronous Motor Drives: Unified Framework for Design and Analysis

Marko Hinkkanen*, Seppo E. Saarakkala*, Hafiz Asad Ali Awan*, Eemeli Mölsä*, and Toni Tuovinen†

*Aalto University School of Electrical Engineering, Espoo, Finland

†ABB Oy Drives, Helsinki, Finland

Abstract—This paper deals with the model-based speed and position estimation for synchronous reluctance motors (SyRMs) and interior permanent-magnet synchronous motors (IPMs). A unified design and analysis framework for a class of observers is developed and the links between apparently different observer structures are brought out. Adaptive state observers equipped with a speed-adaptation law are shown to equal voltage-model-based flux observers equipped with a position-tracking phase-locked loop (PLL). The error signal driving the adaptation law or the PLL is presented in a generalized form. The developed framework enables revealing potential instability issues of existing observer designs and exploiting already existing stability results. A stabilizing gain design is reviewed and design guidelines are given. The observer design is experimentally evaluated using a 6.7-kW SyRM drive.

Index Terms—Observer, stability conditions, speed sensorless.

I. INTRODUCTION

In the future, energy-efficient SyRMs, with or without permanent magnets (PMs), and IPMs could replace induction motors in many applications, such as pumps, fans, and conveyors. These kind of industrial drives should obviously be sensorless for cost savings. A fundamental-wave rotor-position observer, based on a mathematical motor model, is a core algorithm in sensorless control [1]–[12]. The observer should provide (locally) stable and sufficiently fast estimation-error dynamics at all speeds and loads (except in the zero-speed condition, which is unobservable). Of course, the observer should also be robust against the measurement noise and parameter errors. Crossing the zero speed even with a large load torque as well as smooth starting and stopping in a no-load condition should be possible without additional algorithms. However, if the application requires sustained operation in the vicinity of the zero speed under a load torque, an observer should be combined with a signal-injection scheme [1]–[3], [9]. These requirements are not trivial to fulfil, since the estimation-error dynamics unavoidably become nonlinear.

Various position observers may look quite different at first glance, but there are only a few basic structures. A classical approach is to combine a voltage-model-based flux observer with a phase-locked loop (PLL) for rotor-position tracking [1], [2]. As will be shown, this classical structure is mathematically equivalent to the standard state observer augmented with a speed-adaptation loop [3], [4]. Despite the apparent simplicity of these structures, they provide enough degrees of freedom to meet the above-mentioned demands, if the observer gain

and the speed-adaptation loop are properly designed. The observers proposed in [9], [10] are similar to [1], [2], but since an additional integral action is used in their state observer structures, their order is higher and they have more design parameters to be tuned. The observer structure used in [5] is similar to [3], [4], except that the PM-flux magnitude is also estimated, which also increases the order of the observer.

The magnetic saliency of SyRMs and IPMs generally complicates the observer design. For salient machines, the estimation-error dynamics have been rigorously analyzed only in a few papers [3]–[5], [8], [12]. A concept of the fictitious flux has been used in order to turn the salient machine model into a more favourable nonsalient form [5]. However, the d-axis current component was assumed to be constant in this analysis. In [3], a linearized model was developed for the stability analysis and a potential problem of the low-speed instability in the regenerating mode was revealed. Later, a stabilizing gain design was developed based on the same model [4]. This gain design has a unique feature of decoupling the flux observer from the speed-adaptation law (or from the PLL).

The main contributions of the paper are:

- 1) A unified design framework for the observers similar to [1]–[4] is developed. Links between these observers are brought out. Here, the classical structure [1], [2] is adopted for the framework, since it leads to the simplest form of the equations. The developed framework enables exploiting existing results, such as the linearized model [3] and stabilizing gains [4].
- 2) Instability issues of sensorless observers are analyzed. A risk of an unstable region, appearing in the field-weakening range at high torque values, in the classical gain design [1], [2] is discovered.

Furthermore, the design procedure of speed-adaptive observers is clarified. The observer design is experimentally evaluated using a 6.7-kW SyRM drive.

II. MOTOR MODEL AND SENSORLESS CONTROL SYSTEM

Real space vectors will be used. For example, the stator-current vector is $i_s = [i_d, i_q]^T$, where i_d and i_q are the components of the vector. The identity matrix is $\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and the orthogonal rotation matrix is $\mathbf{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Space vectors in stator coordinates are marked with the superscript s.

The electrical rotor angle is ϑ_m and the electrical angular rotor speed is $\omega_m = d\vartheta_m/dt$. The electrical radians are used

Fig. 1. (a) Sensorless control system. The observer is implemented in estimated rotor coordinates, but it could be equivalently implemented in stator coordinates instead. Any other control scheme could be used instead of the current controller. (b) Internal structure of the speed-adaptive observer.

throughout the paper. In rotor coordinates, the inductance matrix and the PM-flux vector, respectively, are denoted by

$$\mathbf{L} = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \quad \boldsymbol{\psi}_f = \begin{bmatrix} \psi_f \\ 0 \end{bmatrix} \quad (1)$$

where L_d is the direct-axis inductance, L_q is the quadrature-axis inductance, and ψ_f is the PM flux.

The machine model will be expressed in *estimated* rotor coordinates, whose d-axis is aligned at $\hat{\vartheta}_m$ with respect to the stator coordinates. The stator flux linkage is

$$\boldsymbol{\psi}_s = \mathbf{L}' \mathbf{i}_s + \boldsymbol{\psi}_f' \quad (2)$$

where the inductance matrix and PM-flux vector, respectively,

$$\mathbf{L}' = e^{(\vartheta_m - \hat{\vartheta}_m) \mathbf{J}} \mathbf{L} e^{-(\vartheta_m - \hat{\vartheta}_m) \mathbf{J}} \quad \boldsymbol{\psi}_f' = e^{(\vartheta_m - \hat{\vartheta}_m) \mathbf{J}} \boldsymbol{\psi}_f \quad (3)$$

depend on the rotor-position estimation error $\vartheta_m - \hat{\vartheta}_m$. The stator voltage is

$$\mathbf{u}_s = R_s \mathbf{i}_s + \frac{d\boldsymbol{\psi}_s}{dt} + \hat{\omega}_m \mathbf{J} \boldsymbol{\psi}_s \quad (4)$$

where R_s is the stator resistance and $\hat{\omega}_m = d\hat{\vartheta}_m/dt$ is the angular speed of the coordinate system. The electromagnetic torque is

$$T_e = \frac{3p}{2} \mathbf{i}_s^T \mathbf{J} \boldsymbol{\psi}_s \quad (5)$$

where p is the number of pole pairs and the superscript T marks the transpose. As special cases, this motor model represents the surface-mounted PM motor if $L_d = L_q$ and the SyRM if $\psi_f = 0$.

Fig. 1(a) shows the block diagram of a typical sensorless control system, part of which is a speed-adaptive observer.

A speed controller is not shown in the figure for simplicity, but it uses the estimated speed $\hat{\omega}_m$ as its feedback signal. The pulse-width modulator (PWM) calculates the duty ratios based on the voltage reference $u_{s,ref}^s$ and the DC-bus voltage u_{dc} . In this example system, the observer is implemented in estimated rotor coordinates, but it could be equivalently implemented in stator coordinates instead.

III. SPEED-ADAPTIVE OBSERVER

A. Structure

1) *State Observer*: Without loss of generality, the speed-adaptive observer is considered to be implemented in estimated rotor coordinates, cf. Fig. 1(b). A voltage-model-based state observer is defined by

$$\frac{d\hat{\boldsymbol{\psi}}_s}{dt} = \mathbf{u}_s - R_s \mathbf{i}_s - \hat{\omega}_m \mathbf{J} \hat{\boldsymbol{\psi}}_s + \mathbf{K} (\mathbf{L} \mathbf{i}_s + \boldsymbol{\psi}_f - \hat{\boldsymbol{\psi}}_s) \quad (6)$$

where $\mathbf{K}$ is a 2×2 observer gain matrix and estimates are marked with a hat. The correction vector is simply the difference between the measured current and the estimated current, scaled by the inductance matrix: $\mathbf{L} \mathbf{i}_s + \boldsymbol{\psi}_f - \hat{\boldsymbol{\psi}}_s = \mathbf{L} (\mathbf{i}_s - \hat{\mathbf{i}}_s)$, where $\hat{\mathbf{i}}_s = \mathbf{L}^{-1} (\hat{\boldsymbol{\psi}}_s - \boldsymbol{\psi}_f)$. In sensorless drives, the correction vector generally differs from the real flux estimation error $\boldsymbol{\psi}_s - \hat{\boldsymbol{\psi}}_s$ during transients, even if accurate model parameter estimates are assumed, as can be seen from (2) and (3).

2) *Speed Adaptation*: The proportional-integral (PI) speed-adaptation law is used to drive the error signal ε to zero

$$\hat{\omega}_m = k_p \varepsilon + \int k_i \varepsilon dt \quad \hat{\vartheta}_m = \int \hat{\omega}_m dt \quad (7)$$

where k_p and k_i are the gains. The error signal is defined as the scalar product of the correction vector and the vector $\mathbf{J} \boldsymbol{\lambda}$,

$$\varepsilon = - (\mathbf{L} \mathbf{i}_s + \boldsymbol{\psi}_f - \hat{\boldsymbol{\psi}}_s)^T \mathbf{J} \boldsymbol{\lambda} \quad (8)$$

where the projection vector $\boldsymbol{\lambda}$ can be a constant vector or it may depend on $\hat{\boldsymbol{\psi}}_s$ and $\mathbf{i}_s$. The selection of the projection vector will be considered in Section IV-B. The orthogonal rotation matrix $\mathbf{J}$ has been included in (8) in order to simplify the expressions in Section IV.

It is worth noticing that the scalar product in (8) is similar to the torque expression (5). For example, if $\boldsymbol{\lambda} = \hat{\boldsymbol{\psi}}_s$ were chosen, the estimation error of the torque-producing current component would be driven to zero. If $\boldsymbol{\lambda} = \boldsymbol{\psi}_f$ were chosen, the q-axis error $\hat{\psi}_q - L_q i_q$ would be driven to zero.

B. Alternative Equivalent Structures

1) *Stator Coordinates*: The observer (6) can be transformed into stator coordinates [1], [2]

$$\frac{d\hat{\boldsymbol{\psi}}_s^s}{dt} = \mathbf{u}_s^s - R_s \mathbf{i}_s^s + \mathbf{K}^s (\mathbf{L}^s \mathbf{i}_s^s + \boldsymbol{\psi}_f^s - \hat{\boldsymbol{\psi}}_s^s) \quad (9)$$

where

$$\mathbf{L}^s = e^{\hat{\vartheta}_m \mathbf{J}} \mathbf{L} e^{-\hat{\vartheta}_m \mathbf{J}} \quad \boldsymbol{\psi}_f^s = e^{\hat{\vartheta}_m \mathbf{J}} \boldsymbol{\psi}_f \\ \mathbf{K}^s = e^{\hat{\vartheta}_m \mathbf{J}} \mathbf{K} e^{-\hat{\vartheta}_m \mathbf{J}} \quad (10)$$

Fig. 2. Linearized model of the estimation-error dynamics for analysis purposes.

The expression (8) for the error signal ε is still valid if all the vectors are expressed in stator coordinates.

In [1], [2], a simple observer gain $\mathbf{K}^s = \mathbf{K} = g\mathbf{I}$ was chosen. In this special case, (9) can be represented as

$$\hat{\psi}_s^s = \frac{s}{s+g} \frac{\mathbf{u}_s^s - R_s \mathbf{i}_s^s}{s} + \frac{g}{s+g} (\mathbf{L}^s \mathbf{i}_s^s + \psi_f^s) \quad (11)$$

where $s/(s+g)$ and $g/(s+g)$ are the first-order high-pass and low-pass filters, respectively, and s is used as the derivative operator. This form clearly shows that the state observer can be parametrized to behave as the voltage model at higher speeds and as the flux model at low speeds, the parameter g defining the corner frequency (typically $g = 2\pi \cdot 15 \dots 30$ rad/s). However, when the effect of the speed-adaptation loop is taken into account, the gain g results in unstable operating regions, as will be discussed later.

2) *Standard State Observer Form*: It is easy to show that the voltage-model-based state observer (6) is mathematically equivalent to another commonly used structure [3], [4]

$$\begin{aligned} \frac{d\hat{\psi}_s}{dt} = & -(\mathbf{R}_s \mathbf{L}^{-1} + \hat{\omega}_m \mathbf{J}) \hat{\psi}_s + \mathbf{R}_s \mathbf{L}^{-1} \psi_f + \mathbf{u}_s \\ & + \mathbf{K}' (\mathbf{L} \mathbf{i}_s + \psi_f - \hat{\psi}_s) \end{aligned} \quad (12)$$

if the observer gain is chosen as¹

$$\mathbf{K}' = \mathbf{K} - \mathbf{R}_s \mathbf{L}^{-1} \quad (13)$$

This structure corresponds to a standard state observer form, where an open-loop observer is augmented with an output-error-based correction term. When discretizing the observer, the form (12) is more flexible than the form (6). The error signal $\varepsilon = \hat{\psi}_q/L_q - i_q$ used in [3], [4] is obtained from (8) by selecting $\lambda = [1]_0/L_q$.

C. Linearized Model for Analyzing the Estimation-Error Dynamics

The nonlinear estimation-error dynamics consisting of (2)–(4) and (6)–(8) can be linearized for analysis purposes [3], [4], [12]. The operating-point quantities are marked with the subscript 0. The accurate model parameters are assumed, making the operating-point estimation errors zero, e.g., $\tilde{\vartheta}_{m0} = \vartheta_{m0}$, further leading to $\mathbf{L}'_0 = \mathbf{L}$ and $\psi'_{f0} = \psi_f$. The standard linearization procedure gives [3]

$$\frac{d\tilde{\psi}_s}{dt} = -(\mathbf{K}_0 + \omega_{m0} \mathbf{J}) \tilde{\psi}_s + \mathbf{K}_0 \mathbf{J} \psi_{a0} \tilde{\vartheta}_m \quad (14a)$$

$$\varepsilon = \lambda_0^T \mathbf{J} \tilde{\psi}_s + \lambda_0^T \psi_{a0} \tilde{\vartheta}_m \quad (14b)$$

¹The gain was scaled by the inductance matrix in [3], [4].

where $\tilde{\psi}_s = \psi_s - \hat{\psi}_s$ is the flux estimation error and other errors are marked similarly. To simplify the notation, an auxiliary flux linkage vector is defined as

$$\begin{aligned} \psi_{a0} &= (\mathbf{L} + \mathbf{J} \mathbf{L} \mathbf{J}) \mathbf{i}_{s0} + \psi_f \\ &= \begin{bmatrix} (L_d - L_q) i_{d0} + \psi_f \\ -(L_d - L_q) i_{q0} \end{bmatrix} = \begin{bmatrix} \psi_{ad0} \\ \psi_{aq0} \end{bmatrix} \end{aligned} \quad (15)$$

The linear system (14) can be represented by the transfer function

$$G(s) = \lambda_0^T \mathbf{J} (s\mathbf{I} + \mathbf{K}_0 + \omega_{m0} \mathbf{J})^{-1} \mathbf{K}_0 \mathbf{J} \psi_{a0} + \lambda_0^T \psi_{a0} \quad (16)$$

from $\tilde{\vartheta}_m(s)$ to $\varepsilon(s)$. Fig. 2 shows the block diagram of the resulting linearized model, where also the speed-adaptation law is included. According to the figure, the closed-loop transfer function from the actual speed to the estimated speed is

$$\frac{\hat{\omega}_m(s)}{\omega_m(s)} = \frac{(sk_p + k_i)G(s)}{s^2 + (sk_p + k_i)G(s)} = \frac{B(s)}{A(s)} \quad (17)$$

where $A(s)$ is the fourth-order characteristic polynomial and $B(s)$ is the third-order numerator polynomial. The closed-form expressions for these polynomials can be easily calculated using, e.g., any symbolic mathematics package.

The linearized model can be used to analyze the existing observer designs, which can be presented in the framework shown in Fig. 1(b). In a general case, the characteristics of (17) depend on the observer gain $\mathbf{K}_0$, the projection vector λ_0 , and the speed-adaptation gains k_p and k_i . It is to be noted that the linearized model is valid also for analyzing the observer (9) implemented in stator coordinates.

IV. STABILIZING GAIN DESIGN

A. Pole Placement

The fourth-order closed-loop system (17) is complicated and the gains can be difficult to tune. In order to simplify the tuning procedure, the flux-estimation dynamics can be decoupled from the speed-estimation dynamics. From (16), it can be seen that this decoupling is achieved if and only if $\mathbf{K}_0 \mathbf{J} \psi_{a0} = \mathbf{0}$ holds or

$$\mathbf{K}_0 = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \frac{\psi_{a0}^T}{\|\psi_{a0}\|^2} = \frac{1}{\psi_{ad0}^2 + \psi_{aq0}^2} \begin{bmatrix} k_1 \psi_{ad0} & k_1 \psi_{aq0} \\ k_2 \psi_{ad0} & k_2 \psi_{aq0} \end{bmatrix} \quad (18)$$

where k_1 and k_2 are the two remaining free gains in the observer gain matrix and the matrix is normalized by dividing it by $\|\psi_{a0}\|^2$ in order to simplify the following equations. The condition (18) makes the transfer function (16) to reduce to the static gain $G(s) = \lambda_0^T \psi_{a0}$.

The two remaining free gains of $\mathbf{K}_0$ determine the two flux-estimation poles, which can be placed as

$$\det(s\mathbf{I} + \mathbf{K}_0 + \omega_{m0} \mathbf{J}) = s^2 + bs + c \quad (19)$$

Solving (19) under the condition (18) yields

$$k_1 = b\psi_{ad0} + (\omega_{m0} - c/\omega_{m0})\psi_{aq0} \quad (20a)$$

$$k_2 = b\psi_{aq0} - (\omega_{m0} - c/\omega_{m0})\psi_{ad0} \quad (20b)$$

These gain elements can be inserted into (18), leading to [4]

$$\mathbf{K}_0 = \left[b\mathbf{I} + \left(\frac{c}{\omega_{m0}} - \omega_{m0} \right) \mathbf{J} \right] \frac{\psi_{a0}\psi_{a0}^T}{\|\psi_{a0}\|^2} \quad (21)$$

where the design parameters $b > 0$ and $c > 0$ determine the flux-estimation error dynamics. The factor $\psi_{a0}\psi_{a0}^T/\|\psi_{a0}\|^2$ can be recognized as an orthogonal projection matrix, which takes the vector projection of the correction vector in the direction of the vector ψ_{a0} . This matrix can be expressed in different forms

$$\begin{aligned} \frac{\psi_{a0}\psi_{a0}^T}{\|\psi_{a0}\|^2} &= \frac{1}{\psi_{ad0}^2 + \psi_{aq0}^2} \begin{bmatrix} \psi_{ad0}^2 & \psi_{ad0}\psi_{aq0} \\ \psi_{ad0}\psi_{aq0} & \psi_{aq0}^2 \end{bmatrix} \\ &= \frac{1}{1 + \beta^2} \begin{bmatrix} 1 & -\beta \\ -\beta & \beta^2 \end{bmatrix} \end{aligned} \quad (22)$$

where $\beta = -\psi_{aq0}/\psi_{ad0}$ is an auxiliary variable [4], [12]. As special cases, $\beta = 0$ holds for nonsalient machines and $\beta = i_{q0}/i_{d0}$ for SyRMs.

B. Speed-Adaptation Law

If the gains of the PI mechanism in (7) are positive, the stability condition for the speed-adaptation law is

$$G(s) = \lambda_0^T \psi_{a0} > 0 \quad (23)$$

The tuning of the speed-adaptation loop becomes very simple, if $G(s) = 1$ holds. This goal is achieved by choosing, for example, a projection vector

$$\lambda_0 = \frac{\psi_{a0}}{\|\psi_{a0}\|^2} \quad (24)$$

Another example is

$$\lambda_0 = \frac{1}{\psi_{ad0}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (25)$$

which corresponds to the speed-adaptation law used in [4]. Both these projection vectors cause (17) to reduce to

$$\begin{aligned} \frac{\hat{\omega}_m(s)}{\omega_m(s)} &= \frac{(s^2 + bs + c)(sk_p + k_i)}{(s^2 + bs + c)(s^2 + sk_p + k_i)} \\ &= \frac{sk_p + k_i}{s^2 + sk_p + k_i} \end{aligned} \quad (26)$$

where the speed-adaptation gains $k_p > 0$ and $k_i > 0$ are now directly the coefficients of the characteristic polynomial.

Even if the flux-observer dynamics cancel out from (26), they still are a part of a whole system and the flux-observer poles should be properly placed. With accurate model parameters, the estimation-error dynamics are locally stable in every operating point (marginally stable at zero speed). This observer design is a subset of all possible stable designs. However, it is easier to tune two second-order systems than one fourth-order system, which is a clear advantage of this gain selection.

The choice of the projection vector affects the sensitivity to parameter errors. The vector (24) yields a slightly better signal-to-noise ratio, while, based on the time-domain simulations, the vector (25) results in the better robustness against model parameter errors at lower speeds. It can also be shown that the choice $\lambda_0 = \psi_{s0}$ does not generally fulfil the stability condition (23) and, therefore, is not recommended.

C. Example: Selection of Design Parameters

In order to limit the observer gain and to reduce sensitivity to model parameter errors, the design parameters b and c should be selected such that the closed-loop flux-observer poles remain in the vicinity of the open-loop poles. Fig. 3(a) shows open-loop poles of a motor for $\omega_{m0} = 0 \dots 2$ p.u. It can be seen that the damping of the open-loop poles decreases as the speed increases.

It is favorable to increase the damping of the closed-loop poles at higher speeds. Since the poles can be arbitrarily placed using (21), a constant damping ratio at all speeds could be easily achieved. However, this choice would locate the both poles at the origin at $\omega_{m0} = 0$, causing the pure voltage-model behavior, which is undesirable and would complicate the starting of the motor. This starting problem can be remedied, e.g., using the design parameters

$$b = b' + \left(2\zeta - \frac{b'}{\omega_\zeta} \right) |\omega_{m0}| \quad c = \frac{b}{2\zeta} |\omega_{m0}| \quad (27)$$

where the constant ζ is the desired damping ratio at a given angular speed ω_ζ (e.g., the rated speed). The constant b' is recommended to be chosen larger than R_s/L_d and R_s/L_q . At standstill, the poles are placed at $s = 0$ and $s = -b'$.

The two remaining poles of (26) can be placed at $s = -\omega_o$, leading to the critically damped speed-estimation dynamics. This choice corresponds to the speed-adaptation gains

$$k_p = 2\omega_o \quad k_i = \omega_o^2 \quad (28)$$

where ω_o can be considered as an approximate speed-estimation bandwidth. In the implementation, the operating-point quantities in (15), (21), (25), and (27) are replaced with the estimated quantities

$$\omega_{m0} \leftarrow \hat{\omega}_m \quad \mathbf{i}_{s0} \leftarrow \mathbf{L}^{-1}(\hat{\psi}_s - \psi_f) \quad (29)$$

Under the assumption of accurate model parameters, the linearized model is still valid after these replacements. Alternatively, the measured current could be used, i.e. $\mathbf{i}_{s0} \leftarrow \mathbf{i}_s$.

V. RESULTS

A four-pole 6.7-kW SyRM is considered. The rated values are: speed 3175 r/min; frequency 105.8 Hz; line-to-line rms voltage 370 V; rms current 15.5 A; and torque 20.1 Nm. Two different designs are evaluated:

- 1) The observer gain is $\mathbf{K} = g\mathbf{I}$ with $g = 2\pi \cdot 20$ rad/s.
- 2) The observer gain $\mathbf{K}$ is defined by (21) and (27) with $b' = 2\pi \cdot 20$ rad/s, $\zeta = 0.4$, and $\omega_\zeta = 2\pi \cdot 105.8$ rad/s.

In both designs, the speed-adaptation projection vector (25) is used and the adaptation gains are given by (28) with $\omega_o = 2\pi \cdot 100$ rad/s.

A. Stability Analysis

The local stability of the two observer designs is analyzed by calculating the poles of (17) in the speed range of $\omega_{m0} = 0 \dots 2$ p.u. at the maximum positive torque, when the current magnitude is limited to 1.5 p.u. At each speed, the

optimal current components i_{d0} and i_{q0} corresponding to the maximum-torque-per-ampere (MTPA) locus, field weakening, and maximum-torque-per-volt (MTPV) limit are used.

Fig. 3(b) shows the pole locations of *Design 1*. It can be seen that the system is unstable at higher speeds. The unstable region could be decreased by limiting the current (and the torque) at higher speeds. Since the flux-observer dynamics are coupled with the speed-adaptation dynamics, also the speed-adaptation gains strongly affect the stability. Under the same conditions, Fig. 3(c) shows the pole locations of *Design 2*. The system is stable and the pole locations match the designed values (and they are independent of the current). It can be seen that the flux-observer dynamics are decoupled from the speed-adaptation dynamics.

B. Experimental Results

1) *Control System*: The two observer designs are experimentally evaluated using the 6.7-kW SyRM drive. A sensorless control method was implemented on a dSPACE DS1006 processor board. The actual rotor speed ω_m is measured using an incremental encoder for monitoring purposes. The stator currents and the DC-link voltage are sampled in the beginning of each PWM period; both the switching and sampling frequencies are 5 kHz. The inverter nonlinearities are compensated for using a simple current feedforward method.

The control scheme shown in Fig. 1 was augmented with a speed controller, which provides the torque reference $T_{e,ref}$ based on the speed reference $\omega_{m,ref}$ and the estimated speed $\hat{\omega}_m$. Instead of controlling the measured current $\hat{i}_s$, the estimated flux linkage $\hat{\psi}_s$ is controlled. This choice makes it easy to take the magnetic saturation effects into account, since the incremental inductances are not needed at all in the control system. The flux-linkage controller and its tuning is based on the discrete-time controller presented in [13]; only the state variable to be controlled has been changed from the current to the flux. The flux reference $\psi_{s,ref}$ is determined from $T_{e,ref}$ and $\hat{\omega}_m$ using an optimal state reference calculation scheme [14], which includes the MTPA locus, field weakening, and MTPV limit. In the MTPA mode, the minimum value of 0.77 p.u. for the d-axis flux is used, corresponding to [12].

The inductances of the SyRM vary significantly due to the magnetic saturation. In the control system, the inductances are modeled to depend on the flux estimates as $L_d = L_d(\hat{\psi}_d, \hat{\psi}_q)$ and $L_q = L_q(\hat{\psi}_d, \hat{\psi}_q)$ according to the algebraic magnetic model described in [15]. This magnetic model is consistently used everywhere in the control system. The discretized observer is based on the form (12): the open-loop observer is discretized using the exact hold-equivalent model [13] and the correction term is discretized using the Euler method. This discretization approach allows to reach very low ratios between the sampling frequency and fundamental frequency.

2) *Fast Acceleration Tests*: The initial rotor position was found by supplying a constant current in the a -phase magnetic axis, causing the rotor to rotate into this direction, and the rotor position estimate $\hat{\theta}_m$ was reset to zero. Alternatively,

the initial rotor position could be obtained by using a signal-injection method, without causing the rotor to move.

The motor is accelerated from zero to 2 p.u. at the maximum torque, with the stator current magnitude limited to 1.5 p.u. Fig. 4(a) shows the results for *Design 1*. Clearly, the system becomes unstable soon above the rated speed. This result agrees well with the stability analysis in Fig. 3(b). As mentioned, the selection of g and the speed-adaptation gains affects the stability. If the current limit is decreased, the system becomes stable, but acceleration time naturally increases. It is to be noted that this instability problem is neither caused by the changing speed nor the speed controller, but it is caused by the unstable estimation-error dynamics; the system would become unstable even in the torque-control mode at constant speed, if the operating point appears in the unstable region.

Fig. 4(b) shows the results for *Design 2*. It can be seen that the system is stable. The estimated speed $\hat{\omega}_m$ behaves smoothly and follows the measured speed ω_m . It is worth mentioning that $\hat{\omega}_m$ comes directly from the speed-adaptation law without any low-pass filtering.

VI. CONCLUSIONS

The unified design framework can be used for the design, analysis, and comparison. The classical voltage-model-based flux observer structure, expressed in estimated rotor coordinates, has been adopted as a basis. A previously proposed stabilizing gain design has been extended to different error signals of the speed-adaptation loop. The stabilizing gain helps to simplify the design procedure and to avoid unstable regions. Detailed design guidelines were given. The observer designs were experimentally evaluated using a 6.7-kW SyRM drive.

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Fig. 3. Pole locations for $\omega_{m0} = 0 \dots 2$ p.u. at the maximum positive torque, when the current magnitude is limited to 1.5 p.u.: (a) open-loop poles from $\det(s\mathbf{I} - R_s \mathbf{L}^{-1} - \omega_{m0} \mathbf{J}) = 0$; (b) closed-loop poles for $\mathbf{K} = g\mathbf{I}$; (c) closed-loop poles for (21). The flux-observer poles are marked with the red line and the speed-adaptation poles with the blue line. The diamonds, crosses, and stars mark the speeds of 0, 1, and 2 p.u., respectively. The motor parameters are: $R_s = 0.55 \Omega$, $L_d = 45.6$ mH, and $L_q = 6.84$ mH. The dashed line corresponds to the damping ratio of 0.4.

Fig. 4. Experimental results of a fast-acceleration test to the 2-p.u. speed at the maximum motoring torque, having the current magnitude limited to 1.5 p.u.: (a) observer gain $\mathbf{K} = g\mathbf{I}$; (b) observer gain (21). First subplot: reference speed $\omega_{m,ref}$, actual speed ω_m , and estimated speed $\hat{\omega}_m$. Second subplot: estimated flux components and their references in estimated rotor coordinates. Last subplot: measured current components in estimated rotor coordinates.

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