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Comment on “Upper Critical Dimension of the Kardar-Parisi-Zhang Equation”

In a recent Letter [1], Lässig and Kinzelbach consider the existence of an upper critical dimension for the strong coupling regime of the Kardar-Parisi-Zhang (KPZ) equation [2]

$$\frac{\partial h(\mathbf{r}, t)}{\partial t} = \nu \nabla^2 h(\mathbf{r}, t) + \frac{\lambda}{2} [\nabla h(\mathbf{r}, t)]^2 + \eta(\mathbf{r}, t), \quad (1)$$

where $h(\mathbf{r}, t)$ is a height variable in $d + 1$ dimensions, and η is white noise with short-range correlations. By mapping this problem onto directed polymers, they study interactions between the polymers and argue that there is a transition that corresponds to an upper critical dimension of $d_u \leq 4 + 1$ for the KPZ equation. However, this is not supported by numerical simulations of a model that should be in the universality class of Eq. (1).

To demonstrate this, I have performed simulations of the restricted solid-on-solid growth model [3] in dimensions $d \geq 4 + 1$. This model has been shown to give nontrivial scaling exponents in complete agreement with direct numerical solutions of the KPZ equation up to dimensions $d = 3 + 1$ [4,5]. In Fig. 1 I show results

FIG. 1. Surface width $w^2(t)$ for a 100^4 system at $d = 4 + 1$ (one run). Fitting between 40–1000 Monte Carlo steps gives $\beta = 0.164(5)$. The inset shows data for the saturated surface width $w(L)$, where the last data point for $L = 100$ is a rough estimate obtained by assuming that the saturation occurs around $t \approx 10^{3.5}$ Monte Carlo steps.

for the width $w^2(t) \equiv \overline{(h - \bar{h})^2}$ at $d = 4 + 1$ for a system of size 100^4 . After the finite-size oscillations have decayed, there is a well defined power-law scaling regime that gives an estimate of $\beta(4 + 1) = 0.16(1)$ (from two independent runs) which is slightly larger than that presented previously [4]. I have also calculated χ from an independent fit to the saturated width $w(L)$ up to 35^4 systems, which gives $\chi(4 + 1) = 0.141(1)$ with excellent power-law scaling even for these smaller systems (see the inset of Fig. 1 and also Ref. [4]). In addition, I have improved the estimates of β for $5 + 1$ and $6 + 1$ dimensions to obtain $0.11(1)$ and $0.09(1)$, respectively [6].

To summarize, numerical results show no evidence of an upper critical dimension for a model that should be in the universality class of the KPZ equation, and thus do not support the arguments in Ref. [1].

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