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Comment on “Upper Critical Dimension of the Kardar-Parisi-Zhang Equation”

In a recent Letter [1], Lässig and Kinzelbach consider the existence of an upper critical dimension for the strong coupling regime of the Kardar-Parisi-Zhang (KPZ) equation [2]

$$\frac{\partial \mathbf{h}(\mathbf{r}, t)}{\partial t} = v \nabla^2 \mathbf{h}(\mathbf{r}, t) + \frac{\lambda}{2} [\nabla \mathbf{h}(\mathbf{r}, t)]^2 + \eta(\mathbf{r}, t), \quad (1)$$

where \( h(\mathbf{r}, t) \) is a height variable in \( d+1 \) dimensions, and \( \eta \) is white noise with short-range correlations. By mapping this problem onto directed polymers, they study interactions between the polymers and argue that there is a transition that corresponds to an upper critical dimension \( d_u \approx 4 + 1 \) for the KPZ equation. However, this is not supported by numerical simulations of a model that should be in the universality class of Eq. (1). To demonstrate this, I have performed simulations of the restricted solid-on-solid growth model [3] in dimensions \( d \approx 4 + 1 \). This model has been shown to give nontrivial scaling exponents in complete agreement with direct numerical solutions of the KPZ equation up to dimensions \( d \approx 3 + 1 \) [4,5]. In Fig. 1 I show results for the width \( w^2(t) \equiv \langle (h - \overline{h})^2 \rangle \) at \( d = 4 + 1 \) for a system of size 100\(^4\). After the finite-size oscillations have decayed, there is a well defined power-law scaling regime that gives an estimate of \( \beta(4 + 1) = 0.16(1) \) (from two independent runs) which is slightly larger than that presented previously [4]. I have also calculated \( \chi \) from an independent fit to the saturated width \( w(L) \) up to 35\(^4\) systems, which gives \( \chi(4 + 1) = 0.141(1) \) with excellent power-law scaling even for these smaller systems (see the inset of Fig. 1 and also Ref. [4]). In addition, I have improved the estimates of \( \beta \) for \( 5 + 1 \) and \( 6 + 1 \) dimensions to obtain 0.11(1) and 0.09(1), respectively [6].

To summarize, numerical results show no evidence of an upper critical dimension for a model that should be in the universality class of the KPZ equation, and thus do not support the arguments in Ref. [1].

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[6] It should be noted that for \( d > 4 + 1 \), the oscillations in \( w(t) \) do not die out before saturation in the systems studied here.

FIG. 1. Surface width \( w^2(t) \) for a 100\(^4\) system at \( d = 4 + 1 \) (one run). Fitting between 40–1000 Monte Carlo steps gives \( \beta = 0.164(5) \). The inset shows data for the saturated surface width \( w(L) \), where the last data point for \( L = 100 \) is a rough estimate obtained by assuming that the saturation occurs around \( t \approx 10^{1.5} \) Monte Carlo steps.