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Linear-scaling self-consistent implementation of the van der Waals density functional

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An efficient linear-scaling approach to the van der Waals density functional in electronic-structure calculations is demonstrated. The nonlocal correlation potential needed in self-consistent calculations is derived in a practical form. This enables also an efficient determination of the Hellmann-Feynman forces on atoms. The numerical implementation employs adaptive quadrature grids in real space resulting in a fast and an accurate evaluation of the functional and the potential. The approach is incorporated in the atomic orbital code SIESTA.

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The van der Waals (vdW) interaction plays an important role in various processes in biochemistry, in surface science, and in physics of layered materials. Although the basic underlying physics of dispersion forces is understood, an accurate and efficient calculation of the vdW interaction is still challenging.

The post-Hartree-Fock approaches, based on perturbation theory, are able to treat the vdW interaction. For instance, the coupled-cluster methods\(^1\) [especially coupled-cluster with single and double and perturbative triple excitations, CCSD(T)] are very accurate and often used in benchmark calculations. Unfortunately, the high precision comes with enormous computational expenses, making the method fully applicable only for small complexes with less than a few dozen of atoms. The second-order Møller-Plesset perturbation theory (MP2), although less expensive and less accurate, also contains the essential physics for describing the vdW interaction. Both of the two methods are not straightforward to apply to extended systems and the existing approaches to periodic systems, as those in Refs.\(^2\) and \(^3\), are fairly expensive and not in common use.

Another class of methods for electronic-structure calculations is based on the density-functional theory (DFT). The standard approximations to the exchange-correlation energy in DFT, such as the local-density approximation (LDA) and the generalized-gradient approximation (GGA), are relatively cheap and easily applicable also to periodic systems. Unfortunately, these methods cannot describe the dispersion interaction properly. The problem has been addressed by Langreth, Lundqvist and co-workers,\(^4–6\) who constructed a nonlocal correlation functional aiming to preserve the essential mechanisms behind the vdW interaction. This approach has been successfully utilized for selected test systems and applications.\(^7–10\) However, most of these results are obtained either using an \textit{a posteriori} energy correction scheme or a self-consistent calculation with a restricted capability of relaxing the structures. The reason for this is the computational complexity of the self-consistent approach of Ref.\(^11\), which was employed in the earlier self-consistent calculations.

In this Rapid Communication, we demonstrate our alternative highly efficient real-space approach. We derive the functional derivative of the vdW density functional (vdW-DF) and obtain a representation, which enables a simpler evaluation of the nonlocal correlation potential and forces for self-consistent calculations with full atomic relaxation. Our implementation employs numerical quadratures on adaptive grids, which vary depending on the local properties of the electron density and are optimized to calculate the vdW-DF. We perform calculations on the widely used molecular benchmark set S22.\(^12\) The results for its molecular complexes provide an extensive test for the reliability of the vdW-DF. Simple examples such as noble gas dimers are not suitable for these purposes as they do not provide a complete picture with respect to the variation in the nature of the bonding. Extended systems are either not discussed as a systematic discussion would require a versatile test set, which does not exist.

Starting from the adiabatic connection-fluctuation dissipation theorem, Dion \textit{et al.}\(^4\) derived the nonlocal part of the correlation energy in the compact form

\[
E_{\text{nl}}(n) = \frac{1}{2} \int \int n(r)\varphi(r,r')n(r')d^3rd^3r',
\]

(1)

where \(n(r)\) is the electron density and \(\varphi(r,r')\) is a kernel. Following Dion \textit{et al.},\(^5\) we write the kernel as a function of the dimensionless variables \(D\) and \(\Delta\) defined as

\[
D = \frac{q_0(r) + q_0(r')}{2} |r - r'|,
\]

(2)

\[
\Delta = \frac{q_0(r) - q_0(r')}{q_0(r) + q_0(r')},
\]

(3)

where \(q_0(r)\) is a characteristic wave number, which depends on the density and its gradient at the given point \(r\). Additionally it can be related to the radius of the sphere, where the kernel function is mostly localized.

For self-consistent calculations, an algorithm to evaluate also the correlation potential is required. One method is introduced in Ref.\(^11\). However, we suggest a different approach. The functional derivative \(v_{\text{nl}}(r) = \delta E_{\text{nl}}(\Delta)/\delta n(r)\) is evaluated using the variables \(D\) and \(\Delta\). Employing the chain rule, after straightforward but rather tedious steps, we obtain the expression

\[
E_{\text{nl}}[n] = \frac{1}{2} \int \int n(r)\varphi(r,r')n(r')d^3rd^3r',
\]

(1)
which is removed by the Jacobian factor in the spherical coordinates. The choice of spherical instead of Cartesian coordinates is inspired by the shape of the kernel function. First, it is localized in space within a radius that can be defined locally. Second, it has a logarithmic singularity at the point \( D = 0 \), which is removed by the Jacobian factor in the spherical coordinates.

The nonlocal correlation energy density can be approximated by the sum

\[
e^{nl}_{c}(r) = \sum_{ij} \varphi(r, r + r_{ij}) n(r + r_{ij}) w_{rad}(\bar{r}_{ij}) w_{ang}(\bar{\Omega}_{ij}),
\]

where the weights \( w_{rad}(\bar{r}_{ij}) \) and \( w_{ang}(\bar{\Omega}_{ij}) \) correspond to the radial and angular parts of the integral, respectively. The latter is calculated on a Lebedev grid \( \bar{\Omega}_{ij} \). The Gauss-Chenyshev quadrature of the second kind yields the radial integral weights \( w_{rad}(\bar{r}_{ij}) \). The substitution,

\[
\bar{r} = r_{m} \sqrt{\frac{1 - x}{1 + x}},
\]

is used to map the initial integration range \( 0 < r < \infty \) to the range \( -1 < x < 1 \). The parameter \( r_{m} \) scales the grid and can be selected so that the quadrature grid \( \bar{r}_{ij} \) adapts to the behavior of the kernel function. The kernel function experiences most rapid variations at small \( D \) values up to \( D_{\text{max}} \approx 2 \). This defines the range of \( \bar{r} \), where the contribution to the integral [Eq. (5)] is the largest and also the most difficult to calculate. If \( r_{m} = 1 / q_{0}(r) \), on average half of the grid points corresponds to \( D < 1 \) and \( \sim 70\% \) to \( D < 2 \).

The above-described approach requires an algorithm to evaluate the density \( n(r) \) and the wave number \( q_{0}(r) \) at any point in the space. In many electronic-structure codes, the valence electron density is represented as a smooth function on a Cartesian grid, which is used also for representing the potential and integrating \( e^{nl}_{c}(r) \) to obtain \( E^{nl}_{c} \). Trilinear or tricubic interpolation can be used to evaluate this function on the spherical quadrature grid on which the more rapidly varying kernel function can be represented accurately enough.

The explicit expression for \( \varphi(r, r') \) involves a two-dimensional integral and is not suitable for practical calculations due to its complexity. Similarly to the previous approaches,\(^{4,11} \) we use a look-up table to evaluate \( D^{2}\varphi(D, \Delta) \) in Eq. (4). We do this by bivariate cubic spline interpolation that allows us to obtain smooth first derivatives and is constructed by assuming continuous piecewise-bilinear second derivatives.
In order to perform an extensive test of the vdW-DF and its present implementation, we have investigated the S22 set of molecules. The structures of the molecules are completely optimized with the force convergence threshold of 0.01 eV/Å. The real-space grid cutoff of 250 Ry is chosen to diminish the “egg-box” effects due to the representation of the exchange-correlation potential on the Cartesian grid. When calculating the interaction energies, the basis set superposition error is estimated by using the counterpoise correction method. The typical size of the correction is 20 % of the total interaction energy.

The original approach by Dion et al. implies the use of the revised Perdew-Burke-Ernzerhof (revPBE) exchange. Such an approach is based on the notion that this exchange functional provides little or no bonding for van der Waals systems. On the other hand, it is known that revPBE systematically overestimates experimental bond lengths and gives smaller binding energies than the PBE or the Perdew-Wang 91 (Ref. 19) functional, which are more frequently used for different applications. Thus, it is important to understand the performance of the vdW-DF depending on the choice of the exchange functional. This has been addressed previously by Vydrov et al. who found for a limited set of examples that the use of the revPBE exchange matches experimental or highly accurate quantum-chemical data better than the PBE or the exact exchange functional.

In the present Rapid Communication, we consider the revPBE and PBE exchange functionals. The results for the binding energies of the S22 molecular complexes are shown in Table I and visualized in Fig. 2. Table I shows a few systematic trends for the interaction energies obtained with different exchange functionals within the vdW-DF scheme. The complexes are bound stronger when employing the PBE than the revPBE exchange functional reflecting the more repulsive exchange interaction at the relevant intermolecular distance range when the latter functional is used. For the systems with predominant dispersion contributions and mixed complexes, as grouped in Table I, the revPBE exchange functional is somewhat more advantageous, as justi-

![FIG. 1. (Color online) Test system of a Ne dimer. The positions of the nuclei with the distance of 3.0 Å are shown in part (a). (a)–(c) The distribution of sizes of quadrature grids used for the evaluation of ε_{n}^{(r)}(r) and t_{n}^{(r)}(r) on different levels of adaptation is shown. (a) The quadrature grid size is not varied. (b) Only the number of radial shells is adjusted. c) Both the radial and angular grids are adapted. (d) The nonlocal correlation energy contributions to the binding energy are shown. The plotted quantity is the difference in the function 10^{3}n(r)e_{n}^{(r)}(r) (in atomic units) between the dimer and the two free atoms at the dimer distance. The solid, long-, and short-dashed isolines correspond to negative, zero, and positive values, respectively.

**TABLE I. Calculated binding energies (in meV) for the S22 model systems of Ref. 12. Complexes 1–7 are hydrogen bonded. Complexes 8–15 are with predominant dispersion interaction. The remaining ones are complexes with mixed bonding character.**

<table>
<thead>
<tr>
<th>Complex</th>
<th>revPBE a</th>
<th>PBE a</th>
<th>CCSD(T) c</th>
</tr>
</thead>
<tbody>
<tr>
<td>(NH₃)₂</td>
<td>106</td>
<td>161</td>
<td>137</td>
</tr>
<tr>
<td>(H₂O)₂</td>
<td>177</td>
<td>242</td>
<td>218</td>
</tr>
<tr>
<td>Formic acid dimer</td>
<td>610</td>
<td>793</td>
<td>807</td>
</tr>
<tr>
<td>Formamide dimer</td>
<td>542</td>
<td>698</td>
<td>692</td>
</tr>
<tr>
<td>Uracil dimer</td>
<td>701</td>
<td>893</td>
<td>895</td>
</tr>
<tr>
<td>2-pyridoxine-2-aminopyridine</td>
<td>608</td>
<td>778</td>
<td>725</td>
</tr>
<tr>
<td>Adenine-thymine</td>
<td>586(659b)</td>
<td>762</td>
<td>710</td>
</tr>
<tr>
<td>(CH₄)₂</td>
<td>38</td>
<td>67</td>
<td>23</td>
</tr>
<tr>
<td>(C₂H₄)₂</td>
<td>61</td>
<td>116</td>
<td>65</td>
</tr>
<tr>
<td>Benzene-CH₄</td>
<td>60(68b)</td>
<td>109</td>
<td>65</td>
</tr>
<tr>
<td>Benzene dimer (slip-parallel)</td>
<td>119(119b)</td>
<td>215</td>
<td>118</td>
</tr>
<tr>
<td>Pyrazine dimer</td>
<td>168</td>
<td>271</td>
<td>192</td>
</tr>
<tr>
<td>Uracil dimer (stack)</td>
<td>356(305b)</td>
<td>560</td>
<td>439</td>
</tr>
<tr>
<td>Indole-benzene (stack)</td>
<td>188</td>
<td>318</td>
<td>226</td>
</tr>
<tr>
<td>Adenine-thymine (stack)</td>
<td>425(414b)</td>
<td>639</td>
<td>530</td>
</tr>
<tr>
<td>Ethene-ethine</td>
<td>67</td>
<td>103</td>
<td>66</td>
</tr>
<tr>
<td>Benzene-H₂O</td>
<td>118(118b)</td>
<td>180</td>
<td>142</td>
</tr>
<tr>
<td>Benzene-NH₃</td>
<td>81</td>
<td>138</td>
<td>102</td>
</tr>
<tr>
<td>Benzene-NCN</td>
<td>168</td>
<td>238</td>
<td>193</td>
</tr>
<tr>
<td>Benzene dimer (T-shape)</td>
<td>89</td>
<td>173</td>
<td>119</td>
</tr>
<tr>
<td>Indole-benzene (T-shape)</td>
<td>205</td>
<td>299</td>
<td>248</td>
</tr>
<tr>
<td>Phenol dimer</td>
<td>252</td>
<td>369</td>
<td>306</td>
</tr>
</tbody>
</table>

aPresent Rapid Communication.
bReference 7.
cReference 12.
and pseudopotentials differ from the previous ones, and also therefore one cannot expect a perfect matching with the earlier results. Nevertheless, as shown in Table I the binding energies are in rather good agreement with the exception of the hydrogen-bonded adenine-thymine complex for which the difference is 74 meV.

In conclusion, we have demonstrated an efficient real-space algorithm to evaluate the vdW correlation energy and potential. The application to the noncovalently bound complexes of the S22 set reveals a remarkable agreement between the vdW-DF and accurate quantum chemistry methods. The dependence on the exchange functional used with vdW-DF has been confirmed. While it has been believed that the revPBE exchange functional is the most appropriate one for vdW-DF calculations, we have shown a class of counterexamples demonstrating a systematic underbinding when using the revPBE, while the PBE exchange functional succeeds well. Such observations imply that none of the popular GGA exchange functionals can be considered as universal for problems where noncovalent bonds play the major role.

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