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Charge sensitivity of the inductive single-electron transistor

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We calculate the charge sensitivity of a recently demonstrated device where the Josephson inductance of a single Cooper-pair transistor is measured. We find that the intrinsic limit to detector performance is set by oscillator quantum noise. Sensitivity better than $10^{-6} \text{e}/\sqrt{\text{Hz}}$ is possible with a high $Q$ value $\sim 10^3$, or using a superconducting quantum interference device amplifier. The model is compared to experiment, where charge sensitivity $3 \times 10^{-5} \text{e}/\sqrt{\text{Hz}}$ and bandwidth 100 MHz are achieved. © 2005 American Institute of Physics. [DOI: 10.1063/1.2034096]

Remarkable quantum operations have been demonstrated in the solid state. As exotic quantum measurements based on gate charge dependence of the Josephson inductance of a single Cooper-pair transistor (SCPT) forms a parallel oscillator. We further shunt the oscillator, mainly for practical convenience, by an inductor. SCPT is compared to experiment, where charge sensitivity $3 \times 10^{-5} \text{e}/\sqrt{\text{Hz}}$ and bandwidth 100 MHz are achieved.

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**Figure 1.** The L-SET resonator (a), and its equivalent circuit (impedance $Z$) coupled to cabling (b).
is a system ground-state property and hence it contributes no noise. Typically, $k_BT_2 \leq \hbar \omega_p$, and thus sample noise is already in the quantum limit.

The noise of contemporary rf amplifiers, however, remains far from the quantum limit, i.e., $T_B \gg T_s$. The best demonstrated superconducting quantum interference device (SQUID)-based rf amplifiers have reached $T_B \sim 100\text{–}200$ mK. Therefore, added noise from the sample can be safely ignored when analyzing detector performance.

The charge sensitivity for amplitude modulation (AM) of the rf-SET was calculated in detail in Ref. 11 assuming detection of one sideband. It was assumed that the sensitivity is limited by the general equivalent noise temperature similarly as here, and hence the formula applies as such:

$$s_q = \sqrt{2 k_B T_s Z_0} \left( \frac{V_0}{\tilde{\Gamma}} \frac{\partial \tilde{\Gamma}}{\partial \tilde{q}} \right).$$

(2)

In the linear regime, the best sensitivity of the L-SET is clearly at the largest acceptable value of $V_0$, where linearity still holds reasonably well. This is the case when an ac current of critical current peak value flows through the SCPT, and the phase swing is $\pi p - p$. Then, voltage across the SCPT, and the resonator (later we discuss important quantum corrections to this expression), $V_p = |2 \tilde{g}_p/(Z + Z_0)|V_0$ equals a universal critical voltage of a Josephson junction, $V_C = \pi \hbar \omega_s / (4e) \approx 3 \mu V$ at $f_p = 1$ GHz. Here, $Z_R$ is impedance of the parallel resonator.

We decompose the derivative in Eq. (2) into terms due to the circuit and SCPT: $\partial \tilde{\Gamma} / \partial \tilde{q} = (\partial \tilde{\Gamma} / \partial \omega_p)/(\omega_p / \partial \tilde{q}) \times (\partial L_j / \partial q)$. We define a dimensionless transfer function $g = (\partial L_j / \partial q)(1 / L_0)$ scaled according to a minimum (with respect to the gate) of $L_j$. The gate value that yields the maximum of $g$, denoted $g_\text{opt}$, is the optimum gate dc operation point of the charge detector. In what follows, $L_j$ should be understood as its value at this point. With a given $E_J / E_C$, we compute the values of $g$ and $L_j$ numerically from the SCPT band structure ($g$ is plotted in Fig. 4 in Ref. 5).

If $E_J / E_C \ll 1$, one can use the analytical result $L_0 = (\Phi_0 / \pi)^2 / (2E_J$).

With a general choice of parameters of the tank resonator, Eq. (2) needs to be evaluated numerically. However, when the system is critically coupled, $Z = Z_0$, a simple analytical formula can be derived. Numerical calculations of Eq. (2) over a large range of parameters show that the best sensitivity occurs when $Z = Z_0$. This is reasonable because it corresponds to the best power transfer. All the following results are for critical coupling. Later, we examine the effects of detuning from the optimum. Initially, we also suppose the oscillator is classical, i.e., its energy $E \gg \hbar \omega_p$.

The optimal value of the coupling capacitor is calculated using $Q_L = 1 / 2Q_p$, and we get $C_c = \sqrt{C/(\omega_s Q_0Z_0)}$.

Since it was assumed $Z = Z_0$, it holds that $Z_R = Z_0 + i/(\omega_s C_3)$. Voltage amplification by the resonator then becomes $V_p = V_0 \sqrt{Q_c / \omega_s C_3}$, which holds for a reasonably large $Q_c$. We thus have $V_p \approx \pi \hbar \omega_s / (Z_0 C/(4e \sqrt{Q_c}))$.

With $\omega_s = (L_0 C_3)^{-1/2}$, we get immediately $(\partial \omega_p / \partial L_j)^{-1} = 2 \sqrt{L_j} / (1 + L_j)$, and the fact 13 that the full width at half maximum (FWHM) of the loaded resonance absorption dip at critical coupling is $\omega_p / (2Q_p)$, we get $\partial \tilde{\Gamma} / \partial \omega_p = 2Q_p / \omega_p = Q_0 / \omega_p$.

Inserting these results into Eq. (2), we get an expression for the AM charge sensitivity in the limit the oscillator is classical

$$s_q = \frac{8eL_j^2 \sqrt{\frac{1}{L} + \frac{1}{L_j} - 2k_BT_s}}{g \pi \hbar L_0 \sqrt{\omega_p Q_c}}$$

(3)

in units of $[e / \sqrt{Hz}]$. Clearly, the shunting inductor is best omitted, i.e., $L \to \infty$. The classical result, Eq. (3), improves without limit at low $E_J / E_C$.

We will now discuss quantum corrections to Eq. (3). Although the spectral density of noise in the resonator is negligible in output, the integrated phase fluctuations even due to quantum noise can be large. Integrated phase noise in a high-$Q$ oscillator is $(\Delta \phi^2) = 2 \pi \hbar L_0 \omega_p / \Phi_0$. When $(\Delta \phi)$ exceeds the linear regime $\sim \pi$, which happens at high inducance (low $E_J / E_C$), plasma resonance “switches” into the nonlinear regime, and the gain due to the frequency modulation vanishes. If $L \gg L_j$, and $f_p \sim 1$ GHz, we have ultimate limits of roughly $E_J / E_C \sim 0.06$, or $\sim 0.02$ for a SCPT made out of Al or Nb, respectively.

Before even this happening, the quantum noise in the oscillator $E_Q = \frac{1}{2} \hbar \omega_p$ has an adverse effect because less energy can be supplied in the form of drive; that is, $V_0$ is smaller. This can be calculated in a semiclassical way as follows. Energy of the oscillator is due to drive ($E_D$) and noise (we stay in the linear regime): $E = (\Phi_0 \phi^2 / (8 \pi^2 L_0)) = E_D + E_Q = (\Phi_0 \phi^2 / (8 \pi^2 L_0)) + \frac{\pi}{2} \hbar \omega_p$, where the phases are in RMS, $\phi$ is the total phase swing, and $\phi_D$ is that due to drive. Solving for the latter, we get $\phi_D = \phi^2/2 \pi = \phi_0$, and hence the maximum probbing voltage $V_0$ is reduced by a factor $\beta = \sqrt{1 - E_D / E_C}$, which happens at high $E_J / E_C = 10$ K$^2$ (Al) and $E_J / E_C = 10$ K$^2$ (Nb). The results are plotted in Fig. 2 together with corresponding power dissipation ($V_C \sqrt{2})^2 / R = \pi \hbar^2 \omega_s / (32 e^2 Q_L)$.

The optimal sensitivity is reached around $E_J / E_C = 0.1, \ldots, 0.3$, where the curves in Fig. 2 almost coincide with Eq. (4). $C_3$ should be chosen so that critical coupling results. Typically it should also hold $L \gg L_j$ (see the analytical curve in Fig. 2). However, sensitivity decreases only weakly if these values are detuned from their optimum (Fig. 3).

By numerical investigation we found that readout of $\arg (\tilde{\Gamma})$, with mixer detection, offers within accuracy of numerics the same numbers than the discussed AM (readout of $|\tilde{\Gamma}|$).

In experiment, we measured the charge sensitivity for the following sample and resonator: $R_T \approx 11$ k$\Omega$, $E_J \approx 0.7 \text{K}$, $E_C \approx 2.6 \text{K}$, $E_J / E_C \approx 0.3$, $Q_0 \approx 16$, $L \approx 28 \text{ nH}$, $C_3 \approx 1.2 \text{ pF}$, $C_1 \approx 0.5 \text{ pF}$. In all samples so far, $Q_0 \approx 20$, which is currently not understood. The measurements were done as described in Ref. 5, with $T_N \sim 5$ K. We measured $s_q = 7$.
FIG. 2. Charge sensitivity of the L-SET optimized from Eq. (2) (black lines). The analytical result [Eq. (3) multiplied by $\beta^{-1}$], with $L=\infty$, is shown with dashed lines. The gray lines are the corresponding power dissipation. All the graphs have the same scales, which are indicated for $s_q$ (left) and dissipation (right). The curves are for different $Q_1$ as marked. All graphs have $Z=Z_0$.

$\times 10^{-5} \, e/\sqrt{\text{Hz}}$ by AM at 1 MHz, while a prediction with the present parameters is $s_q = 3 \times 10^{-5} \, e/\sqrt{\text{Hz}}$ (see also Fig. 3).

Theory and experiment thus agree reasonably. The somewhat lower sensitivity in experiment is likely to be due to external noise, which forces a lower $V_0$ and also smoothes out the steepest modulation. Its origin is not clear. Also the 25% higher values of $L_j$ than expected agree qualitatively with noise.

In the “anharmonic” mode, we measured $s_q = 3 \times 10^{-5} \, e/\sqrt{\text{Hz}}$ with a usable bandwidth of about 100 MHz ($s_q \sim 10^{-4} \, e/\sqrt{\text{Hz}}$ at 100 MHz). Considering both $s_q$ and band, a performance comparable to the best rf-SETs (Refs. 6 and 16) has been reached with the L-SET, though here at more than two orders of magnitude lower power dissipation ($\sim 10 \, \text{fW}$).

In the linear regime, the power lost $P_Z$ from drive frequency $m=1$ to higher harmonics is determined by the sum, for $m \geq 2$, of Josephson junction admittance components $|Y_m|=2J_m(2e/h\omega)V_f$. At the critical voltage $V_f=V_C$, this amounts to $Y_S/Y_f=0.3\% \sim 30\%$. Since charge sensitivity is proportional to square root of power, it thus decreases only $\sim 15\%$ due to nonlinearity. Further corrections due to slightly nonsinusoidal lowest band of the SCPT, as well as asymmetry due to manufacturing spread in junction resistance, we estimate as insignificant.

Next we discuss nonadiabaticity. Interband Zener transitions might make the SCPT jump off from the supposed ground band 0. We make a worst case estimate by assuming that the drive is $2\pi \varphi$ (partially due to noise). The probability to cross the minimum $\Delta_m$ of band gap $\Delta=\varphi_1-\varphi_0$ is:

$P_Z=\exp[-\pi\Delta_m^2/(2\hbar\varphi)]$, where we evaluate the dependence of the band gap on phase $\varphi=\pi/2$. $\varphi=2\omega_0t$ is determined by the drive.

Zener tunneling is significant if it occurs sufficiently often in comparison to $1\rightarrow 0$ relaxation. The threshold is when $P_Z\sim\Gamma/(2f_0)$, where $\Gamma(\approx (1 \, \text{ms})^{-1}$ is the relaxation rate. Operation of the L-SET can thus be affected above $P_Z \sim 10^{-4}$.

Numerical calculations for $P_Z$ show that Zener tunneling is exponentially suppressed, at the L-SET optimal working point, in the interesting case of low $E_j/E_C$. This is because $\Delta_m$ becomes large and $D$ small. For instance, if $E_j=1 \, \text{K}$ and $f_0=1 \, \text{GHz}$, we get that Zener tunneling is insignificant below $E_j/E_C \sim 3$. With $E_j=0.5 \, \text{K}$ and $f_0=5 \, \text{GHz}$, the threshold is $E_j/E_C=1$.

We conclude that with sufficiently high $Q_1$ and using an amplifier close to the quantum limit, even $s_q \sim 10^{-3} \, e/\sqrt{\text{Hz}}$, order of magnitude better than the shot-noise limit of rf-SET, is intrinsically possible for the L-SET. So far, the sensitivity has been limited by $Q_1 \leq 20$.

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REFERENCES