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Nonwetting Conditions for Coherent Quantum Precession in Superfluid $^3\text{He-B}$

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We consider the interaction of the “magnetically superfluid” phase of superfluid $^3\text{He-B}$ —the coherently and homogeneously precessing domain (HPD)—with the walls of container. We show that the HPD does not wet the vertical walls, and in a small applied field gradient an extreme nonwetting takes place: the walls are completely coated by the “magnetically nonsuperfluid” phase. The relation of this phenomenon to the experiments with HPD at low temperature is discussed.

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Recently a new class of coherent quantum states has been found. They occur only as dynamic states which we refer to as magnetic (or spin) superfluidity. The first representative of the magnetic superfluids is the homogeneously precessing domain (HPD) [1], observed in superfluid $^3\text{He-B}$. The crucial property of the HPD is that the Larmor precession spontaneously acquires coherent phase throughout the sample even in inhomogeneous external magnetic field. This is equivalent to the phase-coherent Bose condensate in (mass) superfluids and superconductors. Many consequences of the phase coherence have been experimentally verified, such as supercurrent transport of magnetization, phase slippage at critical current, spin-current Josephson effect, quantized spin-current vortex, etc. It was found that in an applied gradient of magnetic field the “magnetically superfluid” phase (HPD) can coexist with the “magnetically normal” phase, represented by the stationary state of $^3\text{He-B}$ [stationary domain (SD)]. Two phases are separated by a domain boundary. We discuss the wetting phenomenon of these quantum phases of $^3\text{He-B}$.

The main feature of the spin superfluidity is the off-diagonal long-range order (ODLRO), first introduced by Penrose and Onsager [2] and Yang [3] for the mass superfluidity. In the latter case the ODLRO is represented by the order parameter which is a finite expectation value of the particle creation or annihilation operators $\Psi = \langle \mathbf{a} \rangle = |\Psi\rangle e^{i(\Phi - \mu t)}$ in superfluid [4] He and $F_{\mathbf{k}} = \langle \mathbf{a}_{\mathbf{k}} \mathbf{a}_{-\mathbf{k}} \rangle = |F_{\mathbf{k}}\rangle e^{i(\Phi - 2\mu t)}$ in superconductors, where μ is the chemical potential and Φ is the coherent phase. The ODLRO in the system of the Larmor precessing spins is obtained by the following analogy: (i) The role of particle number operator \mathbf{n} is played by the projection S_z of spin operator on the external magnetic field H . This is a conserved quantity if one neglects the spin-orbit interaction. (ii) The spin creation and annihilation operators S_{\pm} substitute the particle-nonconserving operators. (iii) The precession frequency ω corresponds to the chemical potential, while (iv) the Zeeman energy $E_Z = -\gamma H S_z$ corresponds to the particle energy nU in an external scalar

potential. As a result the ODLRO in a spin system is given by the expectation value of the spin-lowering operator $\langle S_- \rangle = \sqrt{S^2 - S_z^2} e^{i(\Phi + \omega t)}$, where Φ now is the coherent phase of the precession.

The precession may be stable and coherent only if the following two conditions are satisfied: (v) The internal energy of the spin liquid $E_D(S_z)$ (usually this is the dipole-dipole interaction) is the concave function of S_z , in the same way as the concave shape of the internal energy $\epsilon(n)$ prevents phase separation in a liquid. (vi) The phase coherence is supported by the spin rigidity: the energy depends on the gradient of phase $E_G = (K/2)(\nabla\Phi)^2$, where the stiffness K plays the part of the superfluid density in mass superfluids. These conditions are satisfied for Larmor precession in $^3\text{He-B}$ in which the dipole energy has the concave form (see review book [4])

$$E_D = 4(\chi/\gamma^2)\Omega_L^2(\cos\lambda + 1/4)^2\Theta(-\cos\lambda - 1/4), \quad (1)$$

where λ is the tipping angle of precessing magnetization $S_z = S \cos\lambda$, χ the magnetic susceptibility of $^3\text{He-B}$, γ the gyromagnetic ratio for the ^3He nucleus, $S = \chi H/\gamma$, Ω_L the Leggett frequency, and Θ the step function. The magnetic stiffness is supported by the stiffness of the superfluid order parameter. This state was called HPD. The existence of the other magnetically superfluid phases both in $^3\text{He-B}$ and in $^3\text{He-A}$ has also been proposed [5]. Moreover, the magnetic superfluidity can arise in substances which do not display the spin rigidity in stationary state: in the $^3\text{He-}^4\text{He}$ solution the Larmor precession spontaneously evolves into the phase coherent 2D structure [6].

The HPD state is obtained by application of pulsed NMR or cw NMR techniques. In the latter case a small rf field compensates the dissipation of S_z caused by dipole interaction. The frequency ω of the rf field plays the part of the chemical potential. It serves as a Lagrange multiplier in the term $E_L = -\omega S_z$ and defines the steady state S_z which corresponds to the resonance frequency ω :

$$\omega - \gamma H = -\frac{\partial E_D}{\partial S_z}. \quad (2)$$

In the same way the fixed chemical potential determines the equilibrium particle density in fluids.

The coherent precession represents a state with spontaneously broken symmetry: the relevant symmetry, which is broken, is related to the Larmor theorem (see below). The symmetry can be broken in a different way producing different phases of magnetic superfluidity, which can coexist with each other and with the magnetically non-superfluid state. In ${}^3\text{He-B}$ the presence of a small field gradient $H(z) = (\omega/\gamma) - z|\nabla H|$, which is equivalent to the gravity field, separates the precessing state into two domains, HPD with $\cos \lambda \approx -1/4$ in the low-field region, $z > 0$, and the stationary state with $S_z = S$ in the high-field region, $z < 0$. This horizontal interface between domains turns into a meniscus because the boundary conditions at the side walls of the container prefer the stationary domain. The conventional analysis of the meniscus in terms of the wetting angle cannot be applied here because, due to specific form of the dipole energy in Eq. (1), the interface has no internal length scale. The thickness is determined by the applied ∇H , as in the case of phases separated by the second order phase transition, and one must consider the texture—the continuous distribution of the ${}^3\text{He-B}$ order parameter across the interface. Here we consider such an extreme case that, close to the wall, the HPD-SD interface is nearly parallel to the vertical wall and forms the 1D texture (see Fig. 1).

In a conventional experimental situation the dipole energy has the intermediate value $\gamma HS_z \gg E_D \gg |(\gamma H - \omega)S_z|$ between the Zeeman energy and the uncompensated energy of the magnetic field, which is usually called the

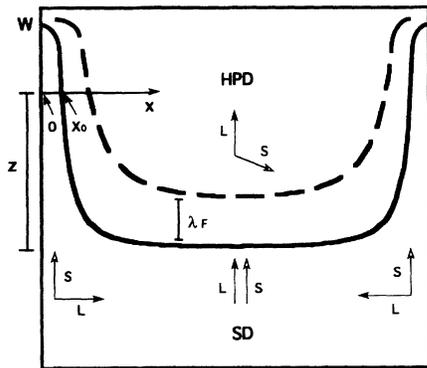


FIG. 1. The spin \vec{S} and orbital momentum \vec{L} distribution inside the cylindrical chamber within the considered model of the interface between the magnetically superfluid phase of ${}^3\text{He-B}$ (HPD) and the magnetically normal phase (SD) in the limit of small gradient of the magnetic field applied along z . Near the vertical wall the SD is more favorable since its orbital momentum \vec{L} can be oriented along the normal to the wall to minimize the surface energy. The solid line marks the surface where the dynamical texture of the HPD contacts the stationary texture of SD. The dashed line illustrates the extension of the dynamical \vec{n} texture within the HPD state. The thickness of the SD region near the vertical wall and the thickness of the adjacent dynamical \vec{n} texture are defined by the length $\lambda(z) = c_{\parallel}/\sqrt{\omega[\omega - \gamma H(z)]}$.

spectroscopic energy. Thus, on one hand, E_D may be considered as perturbation, and, on the other hand, it essentially exceeds the energy of inhomogeneity, produced by the field gradient. When E_D is neglected compared to the Zeeman energy, the general solution for the free precessing state can be obtained using the Larmor theorem. It states that in a system rotating with the Larmor frequency $\omega = \gamma H$, the effect of the magnetic field is removed and the symmetry group of the solid-body spin rotations SO_3^S is restored (“tilde” means that the rotations are in the rotating frame). The full group of symmetry transformations $\mathbf{G} = \text{SO}_3^S \times \text{SO}_3^L$ includes also the orbital rotations SO_3^L in the laboratory frame and this symmetry \mathbf{G} is spontaneously broken in the magnetically superfluid phases of ${}^3\text{He-B}$ or ${}^3\text{He-A}$.

All the degenerate states of the coherent Larmor precession are found by applying the symmetry operations \mathbf{G} to some initial state. If one chooses as the initial state the equilibrium state with the stationary spin density $\vec{S}^{(0)} = \chi \vec{H}/\gamma$ and with an equilibrium value of the 3P_0 order parameter of the ${}^3\text{He-B}$, $A_{\alpha i}^{(0)} = \delta_{\alpha i}$, one obtains the general solution for the free Larmor precession [5]:

$$A_{\alpha i}(t) = O_{\alpha\beta}(\tilde{z}, -\omega t) R_{\beta\gamma}^{(S)} O_{\gamma\mu}(\tilde{z}, \omega t) R_{i\mu}^{(L)}, \quad (3)$$

$$S_{\alpha}(t) = O_{\alpha\beta}(\tilde{z}, -\omega t) R_{\beta\gamma}^{(S)} O_{\gamma\mu}(\tilde{z}, \omega t) S_{\mu}^{(0)}. \quad (4)$$

Here $R_{ik}^{(L)}$ is the time-independent matrix of orbital rotations SO_3^L in the laboratory frame and $R_{\beta\gamma}^{(S)}$ is the time-independent matrix of spin rotations SO_3^S in the precessing frame. The time-dependent matrix $O_{\gamma\mu}(\tilde{z}, \omega t)$ of rotation about the axis z by the angle ωt describes the transition from laboratory frame to rotating one. Another important quantity, the orbital momentum of Cooper pairs $L_i = A_{\alpha i} S_{\alpha}$, is the dynamical invariant of precession: since the dipole coupling is neglected, this orbital vector does not depend on time.

The five-dimensional degeneracy of the Larmor precession in Eq. (3) is lifted by the dipole energy $E_D \propto (A_{ii}A_{jj} + A_{ij}A_{ji})$ (see [4]), which chooses two manifolds.

(i) The stationary state (SD) has $\mathbf{R}^{(S)} = 1$ and $\mathbf{R}^{(L)} = \mathbf{R}(\vec{n}, \theta_0)$, where $\mathbf{R}(\vec{n}, \theta_0)$ is the conventional B -phase matrix which describes the rotation by a “magic” angle $\theta_0 = \arccos(-1/4) \approx 104^\circ$ about arbitrary axis \vec{n} . In this state the magnetization is fixed, $\gamma S = \chi \vec{H}$, while the orbital momentum $\vec{L} = \mathbf{R}(\vec{n}, \theta_0) \vec{S}$ has an arbitrary orientation within the cone of 104° .

(ii) The precessing (magnetically superfluid) phase is the mirror image of the stationary state: $\mathbf{R}^{(L)} = 1$, $\mathbf{R}^{(S)} = \mathbf{R}(\vec{n}, \theta_0)$. This means that now the orbital momentum, which is fixed, $\vec{L} = \chi \vec{H}/\gamma$, while the magnetization in the precessing frame, $\vec{S} = \mathbf{R}(\vec{n}, \theta_0) \vec{L}$, is oriented arbitrarily within the cone, $\lambda < 104^\circ$. The $L \leftrightarrow S$ cross symmetry between the precessing and stationary states is the manifestation of the broken relative spin-orbital symmetry in ${}^3\text{He-B}$ [7].

In both phases the coset space of degenerate states is a unit sphere of the vector \vec{n} . For the magnetically

superfluid phase the vector \vec{n} is time independent in the precessing frame, while in the laboratory frame $\vec{n}(t) = [\sin\beta \cos(\Phi + \omega t); \sin\beta \sin(\Phi + \omega t); \cos\beta]$, which means that the phase Φ of coherent precession is an azimuthal angle of the degeneracy parameter \vec{n} .

The spherical degeneracy of the HPD state is further lifted by the smaller spectroscopic energy

$$E_\omega = (\omega - \gamma H)S_z = \frac{\chi}{4\gamma^2} \omega(\omega - \gamma H)(5 \cos^2 \beta - 1), \quad (5)$$

which discriminates between different \vec{n} : in the region $\gamma H < \omega$ the equilibrium state has $\beta = \pi/2$ which corresponds to HPD, while the region $\gamma H > \omega$ supports the state with $\beta = 0$, which already belongs to the manifold of the stationary states.

Another energy which discriminates between the magnetically superfluid HPD and the magnetically nonsuperfluid SD is the surface energy at the container wall orienting the orbital momentum along the normal $\vec{\nu}$ to the wall:

$$F_S = d(\gamma/\chi)^2 (\vec{L} \times \vec{\nu})^2. \quad (6)$$

This energy makes the HPD phase with its fixed $\vec{L} \parallel \hat{z}$ energetically unfavorable on the side wall where $\nu \perp \hat{z}$. For the SD phase Eq. (6) is completely minimized on both vertical and horizontal walls by appropriate orientation of \vec{L} (this corresponds to the following orientation of \vec{n} on the side wall: $\cos\beta = 1/5$, $\Phi = \pi/3$). This produces the surface tension of the HPD phase at the side wall,

$$\sigma_{\text{HPD}} = dH^2, \quad (7)$$

and suggests that even in the region where $\gamma H < \omega$ and the HPD state is favorable in bulk due to the spectroscopic energy in Eq. (5), the side wall can be coated by the SD phase. We consider the structure of the surface layer in such an extreme limit of nonwetting, that the bulk HPD phase transforms into the SD phase at the vertical wall.

The relevant energies which determine such surface texture are of the smallest scale: the spectroscopic energy (5) and the gradient energy $E_G\{\vec{n}\}$ of \vec{n} field. In the region of SD, at $0 < x < x_0$, where x is along the normal to the wall, one has the conventional gradient energy (see [8])

$$E_G(\text{SD}) = \frac{\chi}{\gamma^2} \left\{ c_\perp^2 (1 - \cos\theta_0) (\partial_x \vec{n})^2 - \frac{1}{2} (c_\perp^2 - c_\parallel^2) \times [\sin\theta_0 \partial_x n_x + (1 - \cos\theta_0) (n_z \partial_x n_y - n_y \partial_x n_z)]^2 \right\}. \quad (8)$$

Further we use the relation for spin wave velocity anisotropy, valid for the Ginzburg-Landau approach: $c_\perp^2 = \frac{3}{4} c_\parallel^2$. The gradient energy in the HPD phase at $x > x_0$ is obtained from Eq. (8) after averaging over the fast precession of $\vec{n}(t)$:

$$E_G(\text{HPD}) = \frac{\chi}{\gamma^2} \frac{5}{256} c_\parallel^2 (53\beta'^2 + 3\beta'^2 \cos^2 \beta + 51\Phi'^2 \sin^2 \beta + 5\Phi'^2 \sin^2 \beta \cos^2 \beta - 2\sqrt{15}\Phi'\beta' \sin^3 \beta), \quad (9)$$

where Φ' and β' are spatial derivatives.

Now one can solve in each domain the Euler-Lagrange equations, which follow from the minimization procedure. At the wall ($x = 0$) the boundary condition for the stationary \vec{n} vector of SD is determined by the surface energy (6). The condition on the boundary between the HPD and SD textures (solid line in Fig. 1) is $\beta(x_0) = 0$ to match the precessing and nonprecessing \vec{n} vectors. The resulting dependence of $n_z = \cos\beta$ is shown in Fig. 2. The point x_0 , at which $n_z = \cos\beta = 1$, marks the surface where the precessing vector \vec{n} of HPD at $x > x_0$ transforms to the stationary vector \vec{n} of SD at $x < x_0$. The thickness x_0 of the SD layer is determined by the condition of equal pressures on HPD and SD sides:

$$x_0 = 1.088 c_\parallel / \sqrt{\omega(\omega - \gamma H)}; \quad (10)$$

it decreases as $z^{-1/2}$, thus the height of the meniscus increases as $z \propto 1/x^2$. The textural energy in the SD region is $2.72(\chi/\gamma^2)c_\parallel[\omega(\omega - \gamma H)]^{1/2}$, while the excess of energy in the HPD phase due to texture is $2.32(\chi/\gamma^2) \times c_\parallel[\omega(\omega - \gamma H)]^{1/2}$, which together give the surface tension of the HPD at the wall in the nonwetting regime:

$$\sigma_{\text{nonwet}} = 5.04(\chi/\gamma^2)c_\parallel \sqrt{\omega(\omega - \gamma H)}. \quad (11)$$

We considered the regime of the extreme nonwetting of HPD at the wall when the SD layer, which wets the side wall, is almost vertical. Let us find the condition when this can occur. On one hand the surface tension in Eq. (11) should be smaller than the tension $\sigma_{\text{HPD}} = dH^2$ of pure HPD. At the point $z = W$ where $\sigma_{\text{nonwet}}(z = W) = \sigma_{\text{HPD}}$ the HPD starts wetting the walls. This point marks the height of the meniscus. On the other hand the vertical slope of the HPD-SD interface means that $dx_0/dz \ll 1$. This gives the following limitation on the extreme nonwetting regime:

$$\left(\frac{c_\parallel}{\omega}\right)^{2/3} \left(\frac{\nabla H}{H}\right)^{-1/3} \ll z < \left(\frac{d\omega}{5.04\chi c_\parallel}\right)^2 \left(\frac{\nabla H}{H}\right)^{-1}, \quad (12)$$

which requires the following condition on the field gradient:

$$\frac{\nabla H}{H} \ll \left(\frac{d}{5\chi}\right)^3 \left(\frac{\omega}{c_\parallel}\right)^4. \quad (13)$$

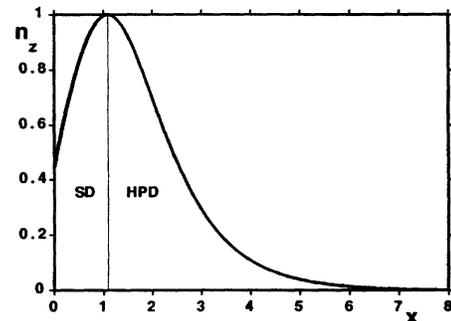


FIG. 2. The calculated profile of the order parameter component n_z within the vertical interface in the units of length $\lambda = c_\parallel / \sqrt{\omega(\omega - \gamma H)}$. The vector \vec{n} is stationary in the SD region and is precessing in the HPD region. In the place of the contact between the phases the \vec{n} vector is parallel to \vec{H} to match the precessing and nonprecessing textures.

Thus we have two competing extreme regimes of HPD distinguished by the surface states: when Eq. (13) is fulfilled the HPD-SD interface has a huge meniscus with almost zero wetting angle. As the field gradient is increased this state continuously evolves: both the thickness of the texture and the meniscus height W decrease and in the opposite extreme regime of high gradients the HPD-SD interface becomes horizontal. This horizontal HPD-SD boundary is represented by the dynamical texture of thickness $\lambda_F = (\frac{c_{\parallel}}{\omega})^{2/3} (\frac{\nabla H}{H})^{-1/3}$ earlier calculated by Fomin [9]; it is shown schematically as the region between the horizontal parts of the dashed and solid lines in Fig. 1.

There are two experiments, which together allow us to estimate d in Eq. (13). The first one measures the g shift in the transverse NMR on $^3\text{He-B}$ [10]; this shift is related to the B -phase magnetic anisotropy energy $-a(\vec{n} \cdot \vec{H})^2$ by $g = (4/5)a/\chi$ [11]. Another one measures the ratio d/a from the textural transitions in rotating $^3\text{He-B}$ [12]. Together they give $d/\chi = (5/4)(d/a)g$. Let us estimate this at low pressure $P \approx 0$ bar. According to [10] the g shift was found to decrease from 7.5×10^{-6} at $T = T_c$ to 4.5×10^{-6} at low T , while according to [12] d/a is $\approx 1.7(1 - T/T_c)$ cm in the intermediate T , which being extrapolated to low T gives $d/a \sim 1.7$ cm. Thus at low T one has an estimate $d/\chi \approx 10^{-5}$ cm. With the value of $c_{\parallel} \sim 2 \times 10^3$ cm/s and for the frequency $\omega = 2\pi \times 10^6$ Hz, the condition (13) gives the upper limit of the field gradient: $\nabla H/H < 10^{-3}$ cm $^{-1}$. This is on the order of magnitude of the experimental field gradients.

An extreme nonwetting is achievable only in the region of the lowest temperatures, high NMR frequencies, and relatively small gradients. Exactly these conditions have been applied to the experiments, where an extremely long induction signal has been observed [13]. Actually there are two regimes with different relaxation times observed at low temperature. It is tempting to identify that with a very long relaxation time [13] with the HPD completely coated by SD at the wall: the absence of the contact between the HPD and the wall would eliminate the surface relaxation of HPD, which is thought to be the dominating source of relaxation at low T . Estimation of the right-hand side of Eq. (13) can support this identification. However, the wetting arguments alone cannot explain the new regime with very long relaxation. Moreover, in the low-temperature experiments in Moscow [14] the peculiar but possibly similar behavior has been observed at lower frequency which, according to Eq. (13), corresponds to essentially higher relative field gradients. Therefore this anomalously long relaxation can signify a new class of magnetic superfluidity. Nevertheless, the wetting phenomenon should be an essential element of any theory of magnetic superfluidity, and the approach developed here on the example of the "conventional" HPD state should be extended to new classes.

A small (nonextreme) meniscus explains the results of experiments with the spin supercurrent through the

relatively thin channel [15]. It was observed that some extra frequency difference $\delta\omega$ should be applied to push the HPD phase through the channel. Our result shows two important conditions at which the wetting effects of the magnetic superfluidity can be measured experimentally: these are the relatively small gradient of magnetic field and the high NMR frequency. The effects can be observed by filling the thin channels with the HPD or by measuring the capillary waves on the HPD-SD interface.

In conclusion, we found that the magnetically superfluid phase of $^3\text{He-B}$ —HPD—does not wet the side wall of the container. At low pressure and in a low applied field gradient there is crossover to an extreme regime of the coherent precession: the side wall of container is coated by the magnetically nonsuperfluid phase with the almost zero wetting angle.

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