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Comment on "Transverse Force on a Quantized Vortex in a Superfluid"

In a recent Letter [1] Thouless *et al.* (TAN) suggested an exact expression for the nondissipative transverse force on a vortex line and claimed that it contained no contribution from fermions localized in the vortex core.

The forces on the vortex have been recently measured in superfluid ^{3}He-B in a broad temperature range [2]. A general expression for a balance of forces acting on the vortex with circulation κ moving with velocity $\mathbf{v}_V$ is [3]

$$\rho_s \kappa \hat{\mathbf{z}} \times (\mathbf{v}_V - \mathbf{v}_s) + D' \hat{\mathbf{z}} \times (\mathbf{v}_n - \mathbf{v}_V) + D(\mathbf{v}_n - \mathbf{v}_V) = 0, \quad (1)$$

where the first two terms represent the transverse force on the vortex, while the parameter D is responsible for the dissipative friction (ρ_s and $\mathbf{v}_s$ are superfluid density and velocity, $\mathbf{v}_n$ is the normal or heat-bath velocity). The measured ratio of two reactive parameters $d_\perp = D'/\kappa\rho_s$ [2] reproduces at low T the result $d_\perp \approx 0$ observed in the limit $T \ll T_c$ [4]. When T increases, $d_\perp$ first becomes negative, then after reaching the minimum at $T \sim 0.4 T_c$ it increases, changes sign, and smoothly approaches $d_\perp(T_c) = 1$. Equation (1) of TAN [1] suggests $d_\perp(T) = \rho_n(T)/\rho$, while the rest of the TAN Letter implies that $d_\perp(T) = 0$ at all T. Both results are in disagreement with experiment and with correct theory.

The reason is that the formalism of TAN does not incorporate the kinetic properties of fermions localized in the vortex core and interacting with heat-bath fermions. This kinetics, determined by the level spacing ω_0 and the lifetime τ of the core fermions [5–8], leads to

$$D \approx \kappa C_0 \tanh \frac{\Delta(T)}{2T} \frac{\omega_0 \tau}{1 + \omega_0^2 \tau^2}, \quad (2)$$

$$D' \approx \kappa \left[C_0 - \rho_n(T) - \frac{\omega_0^2 \tau^2}{1 + \omega_0^2 \tau^2} C_0 \tanh \frac{\Delta(T)}{2T} \right]. \quad (3)$$

The friction parameter D is completely determined by the core fermions: the experimental bell shape of $d_\parallel(T) = D/\kappa\rho_s$ in Ref. [2] follows the T dependence of $\omega_0\tau$ in Eq. (2) with $\omega_0\tau \gg 1$ at $T \ll T_c$ and $\omega_0\tau \ll 1$ close to T_c. The negative sign of $d_\perp(T)$ observed by [2] at low T is produced by a dominating contribution of the Iordanskii force, $D' \approx -\kappa\rho_n(T)$ at $T \ll T_c$, while the core fermions with the T-independent spectral-flow parameter $C_0 = m p_F^3 / 3\pi^2$ are responsible for the observed upturn and change of sign of $d_\perp(T)$ at $T > 0.5 T_c$ [6].

The formalism used by TAN [1] and that in [5–8] lead to different results due to the effect similar to the axial anomaly in quantum field theory. Since the core fermions are nearly gapless one should be extremely careful in which order to take different limits. However, within their theory, TAN cannot resolve between two different regions of the kinetic parameter, $\omega_o\tau \ll 1$ and $\omega_o\tau \gg 1$. If $\omega_o\tau \ll 1$ the spectral flow of "chiral" core fermions leads to an extra force on a vortex, which almost cancels the Magnus force in superfluid/superconducting systems, where an approximate particle-hole symmetry leads to $\rho - C_0 \ll \rho$ [9]. At low T in many (but not all) systems $\omega_o\tau \gg 1$: in this regime the discrete character of the core spectrum becomes relevant, the spectral flow is suppressed, and the full-size Magnus force discussed by TAN is restored. A similar effect of gapless fermions is responsible for linear and angular momentum paradoxes in the gapless ^{3}He-A. An intrinsic dynamical angular momentum is a small fraction $(\rho - C_0)/\rho$ of the value obtained in the similar density-matrix formalism (see Refs. [10,11]). The linear momentum paradox in ^{3}He-A is directly related to the axial anomaly: The direct derivation of the momentum exchange from the anomaly equation $\partial_\mu j^\mu \sim F F^*$ shows that the effective Magnus force on a continuous vortex in ^{3}He-A is reduced by the factor $(\rho - C_0)/\rho$ [12]. For such a continuous vortex, ω_0 is very small and this reduction ceases only at very low T [13]. Such anomaly is apparently missing in [1].

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