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Comment on ''Scaling of the Quasiparticle Spectrum for d-wave Superconductors''

In a recent Letter, Simon and Lee (SL) [1] suggested a scaling law for thermodynamic and kinetic properties of superconductors with lines of gap nodes. For example, the heat capacity as a function of temperature and magnetic field for $T \ll T_c$ and $H \ll H_{c2}$ is

$$C(T,H) = aT^2 G(x), \qquad x = \alpha T/H^{1/2}, \qquad (1)$$

where $G(x)$ is a dimensionless function of the dimensionless parameter x. Equation (1) is in agreement with our calculations [2–4] if the scaling parameter is $x_{\mathrm{KV}} = (H_{c2}/H)^{1/2}(T/T_c)$ (apart from a logarithmic factor). Indeed, according to [2], the function $G(x) \sim 1 + (1/x^2)$ for large values of x; the first term is the bulk heat capacity $C(T,H) \propto T^2$, while the second term results in a temperature-independent vortex contribution $C(T,H) \propto H$. For small x, the function $G(x) \sim 1/x$ which gives [3,4] $C(T,H) \propto T\sqrt{H}$. The crossover value between these two regimes is $x_{\mathrm{KV}} \sim 1$. However, SL have obtained the crossover parameter $x_{\mathrm{SL}} \sim (H_{c2}/H)^{1/2}(T/T_c)\sqrt{E_F/T_c}$. The difference between our x and that obtained by SL is thus by the large factor $\sqrt{E_F/T_c}$. We discuss the origin of this disagreement.

Let us introduce the anisotropic Fermi momentum $p_F(\theta)$ which depends on the angle θ in the a-b plane. If $T \ll T_c$ the quasiparticles which are close to the gap node $\theta \ll 1$ are important. Their spectrum is

$$E(\mathbf{p}) = \sqrt{v_F^2[p - p_F(\theta)]^2 + (\Delta')^2\theta^2}, \qquad (2)$$

where v_F is the Fermi velocity and Δ' is the angular derivative of the gap, $\Delta(\theta) \approx \theta\Delta'$; both are in a vicinity of the node. Equation (2) was the starting point in [2,4].

In contrast, SL used the linearized spectrum

$$E(\mathbf{p}) = \sqrt{c_\parallel^2(\delta p_x)^2 + c_\perp^2 p_y^2}, \qquad (3)$$

where $\mathbf{p} = p_y\hat{y} + (p_F + \delta p_x)\hat{x}$, $c_\parallel = v_F$, and $c_\perp = \Delta'/p_F$. This is justified when the nonlinear contributions to

$$\epsilon(\mathbf{p}) - E_F = p_x^2/2m_x + p_y^2/2m_y - E_F$$
$$\approx v_F\delta p_x + p_y^2/2m_y$$

can be neglected, i.e., when $p_y^2/2m_y \ll c_\perp p_y$ where $p_y = p_F\theta$. This requires much stronger restrictions both on the angle $\theta \ll T_c/E_F$ and on the energy and temperature $T \ll T_c^2/E_F$.

At first glance, one might expect that the temperature of the order of T_c^2/E_F marks the boundary between our scaling and that by SL. However, this is not the case. Our quasiclassical approach is valid down to the temperature at which a discreteness of fermion bound states in the vortex background becomes important. For s-wave superconductors, the interlevel spacing of core fermions is of the order of T_c^2/E_F [5]; thus the quantum limit is reached at $T \sim T_c^2/E_F$. In d-wave superconductors, in a vicinity of the gap nodes, the interlevel distance is smaller; it is determined by a large dimension of the wave function which, for low energies, is limited by the intervortex distance R [2,3]. Thus the discreteness of the levels becomes important at lower temperatures $T \sim (T_c^2/E_F)(\xi/R) \sim (T_c^2/E_F)\sqrt{B/B_{c2}}$.

Therefore, one expects two changes of the regime with the crossover parameters as follows: (1) At $x_{\mathrm{KV}} \sim 1$ [i.e., at $(H_{c2}/H)^{1/2}(T/T_c) \sim 1$] the single-vortex contribution to the thermodynamic quantitity is comparable with the bulk contribution per one vortex. (2) At $x_{KV} \sim T_c/E_F$ [i.e., at $(H_{c2}/H)^{1/2}(T/T_c)(E_F/T_c) \sim 1$] the quasiclassical regime changes to the quantum one. However, there is no change in the regime at the SL scale, $x_{\mathrm{SL}} \sim 1$ [i.e., at $(H_{c2}/H)^{1/2}(T/T_c)\sqrt{E_F/T_c} \sim 1$]. This is because the high anisotropy of the conical spectrum in Eq. (3) was not taken properly into account in [1]: The rescaling of coordinates to get an isotropic spectrum of fermions with the average "speed of light" $c = \sqrt{c_\parallel c_\perp}$ [1] leads to a high deformation of the potential well produced by vortices.

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