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Kopnin and Volovik Reply: In his Comment [1] to our paper [2], Ao points out the main reason why he thinks our calculations are not correct. The reason is not specific to the particular paper [2] but rather refers to all of our microscopic calculations (see also [3], etc.) of vortex dynamics. It is, as he claims, the incorrect use of the τ approximation. Ao agrees that the τ approximation works well for calculations of conductivity but states, strangely enough, that it fails when applied for calculating resistivity. Instead, Ao suggests to use the approach developed in Ref. [4]. Being intrigued by the possibility to discover the truth, we turn to Ref. [4] and find that the starting point is exactly the same as in all of our work on the subject: One looks for a response of the superconductor to a time-dependent displacement of the vortex; i.e., the problem under consideration is exactly the calculation of the conductivity tensor σ_{xx} and σ_{xy} in the mixed state. Why then does one need to speak of a mysterious insufficiency of the τ approximation? Further analysis of Ref. [4] shows that all of the calculations refer to the case when the relaxation time is infinite. One would then expect that the results of Ref. [4] coincide with ours without impurities. They really do in the limit $\tau \to \infty$ but only for zero temperature. For a finite temperature, however, there is no Iordanskii force in Ref. [4]. The reason is easily seen if one follows how the Bogoliubov–de Gennes equation in Ref. [4] is transformed into Eq. (16). Here the phase factor $\exp 2i(\delta_k - \delta_{k'})$ is lost, where δ_k is the scattering phase shift for a particle traveling near a vortex. It is this phase factor which determines both the transverse and longitudinal scattering cross sections [5], the former being responsible for the Iordanskii force [6]. Turning to a finite relaxation, Ao speculates that it should affect the longitudinal vortex response but would not change the transverse response. The argumentation is based on Eq. (15) which has been derived in the absence of impurities. The transformation uses the Schrödinger equation which does not include interaction with either impurities or other sources of relaxation. Thus there is the important step missing in Ao's calculations: One has to incorporate the interaction with impurities into the starting equations and proceed carefully to get the correct result. The conductivity (*not resistivity*) roughly obeys the semicircle law in an isotropic superconductor [3]:

$$\sigma_{xx} \propto \omega_0\tau/(1 + \omega_0^2\tau^2), \qquad \sigma_{xy} \propto \omega_0^2\tau^2/(1 + \omega_0^2\tau^2).$$

The crossover is determined by the lowest energy scale in superconductor: the minigap ω_0 of core fermions. That is why the Hall conductivity σ_{xy} is extremely small in the dirty limit and in the Ginzburg-Landau theory, where $\omega_0\tau \to 0$—the points which the Ao theory fails to explain.

Ao presents handwaving arguments that everything is wrong which is not as simple as he wants: (1) Experiments in which the forces on ^{3}He vortices have been measured in a wide temperature range [7] are too complicated and thus can be wrong; (2) microscopic calculations using the Green function formalism [3] are wrong; (3) the spectral flow phenomenology in terms of the Landau-type theory for the Fermi system in vortex cores [8] uses τ approximation and thus is wrong, etc. Then there is a puzzle: Why do all three sources agree in the temperature dependence of both longitudinal and Hall conductivities?

The confusion regarding the transverse force on a vortex is typical for those who start to consider this problem. Here the simple results advocated by Ao can be dangerous: It is a strong temptation to take a simple formula and use it without precaution. But once the importance of the quasiparticle transport in the core and outside the core is realized, one can move further. One finds a lot of interesting things this way: Landau damping coming from the gapless excitations in d-wave superconductors [2]; rotational dynamics of the nonaxisymmetric vortices [9]; relation to event horizon [9]; mesoscopic effects and Zener tunneling in the core [10,11]; nonlinear transport, etc.

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