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Event horizons and ergoregions in ^{3}He

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Event horizons for fermion quasiparticles naturally arise in moving textures in superconductors and Fermi superfluids. We discuss the example of a planar soliton moving in a superfluid ^{3}He-A, which is closely analogous to a charged rotating black hole. The moving soliton will radiate quasiparticles via the Hawking effect at a temperature of about 5 μK, and via vacuum polarization induced by the effective ''electromagnetic field'' and ''ergoregion.'' The superfluid ^{3}He-A thus appears to be a useful system for experimental and theoretical simulations of quantum effects related to event horizons and ergoregions. [S0556-2821(98)05818-4]

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I. INTRODUCTION

A black hole event horizon is the causal boundary of the exterior region of spacetime. According to quantum mechanics, an event horizon emits Hawking radiation [1] and possesses Bekenstein entropy [2]. These phenomena lie at the intersection of gravity and quantum mechanics, and have played a major role in efforts to understand quantum gravity. However, the nature of the fundamental degrees of freedom and the physics at short distances is still not understood, so basic questions about the statistical meaning of black hole entropy, the back reaction to Hawking radiation, the origin of the outgoing modes, and unitarity, remain unresolved. Moreover, the effect of Hawking radiation is negligible for the physics of solar mass black holes, since the temperature of Hawking radiation T_H is extremely small ($\sim 10^{-7}$ K). The only conceivable experimental consequences of Hawking radiation at present would arise from evaporation of a (hypothetical) population of primordial black holes.

For this reason models simulating event horizons in condensed matter can be useful. The first attempt at a model of this kind was made with a moving liquid [3–5]. The propagation of sound waves on the background of a moving inhomogeneous liquid is similar to the propagation of light in $(3+1)$-dimensional Lorentzian geometry, and is governed by the relativistic wave equation

$$\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Psi)=0. \tag{1.1}$$

The ''acoustic'' metric $g^{\mu\nu}$, in which the sound wave is propagating, is determined by the inhomogeneity and local flow velocity of the liquid. If the liquid moves supersonically a sonic ''event horizon'' can arise. A drawback of this model for the simulation of black hole physics is that ordinary liquids are essentially dissipative systems and are very far from the condition where quantum effects can be of any importance: this smears the effects that, similar to the Hawking effect, are related to quantum fluctuations.

Better candidates are superfluids, which allow nondissipative motion of the vacuum (superfluid condensate, or ground state), and which also support well defined elementary excitations that propagate in a ''curved space'' of the inhomogeneous moving condensate. The Fermi superfluids (including superconductors) are appealing candidates because their low-temperature dynamics are described by quantum field theories similar to those in high-energy physics [6]. Among them the superfluid ^{3}He-A has the advantage that this superfluid supports an effective gravity caused by some components of the superfluid order parameter [7].

There is one important obstacle to the formation of a horizon in a moving condensate: superfluidity collapses, i.e., the condensate disappears, before the corresponding speed of light is reached. For example in the superfluid ^{4}He the Landau velocity at which the condensate is unstable to roton excitation is smaller than the speed of sound and thus the supersonic flow cannot be established. For fermionic systems the collapse of the superfluid-superconducting state due to ''superluminal'' motion of the condensate was discussed in Ref. [8]. Therefore we have looked for a model in which the condensate is at rest with respect to the container.

We show here that a ''superluminally'' moving inhomogeneity of the order parameter (soliton, vortex, or other texture) in the superfluid ^{3}He-A provides such a model and can simulate the physics of an event horizon and ergoregion for ''relativistic'' massless fermions—the Bogoliubov-Nambu quasiparticles. The ''superluminally'' moving soliton produces dissipation due to quantum radiation of the fermions via several mechanisms, which decreases the soliton velocity. Similar processes occur also for a charged, rotating, black hole, where Schwinger pair production, pair production in the ergoregion outside the horizon, and Hawking radiation lead to discharge, spin-down, and evaporation of the black hole. So both the superluminally moving soliton and the black hole are quasiequilibrium, unstable inhomogeneous states exhibiting an event horizon.

II. RELATIVISTIC FERMIONS IN ^{3}He-A

The spontaneous breaking of symmetry in the superfluid condensate in ^{3}He-A is characterized in part by a unit vector

$\hat{l}$, which points along the spontaneous angular momentum of the Cooper pairs and determines the direction of the zeroes in the energy spectrum of the Bogoliubov-Nambu fermion quasiparticles [9,7]

$$E(\vec{p}) = \pm \sqrt{v_F^2(p-p_F)^2 + \frac{\Delta_A^2}{p_F^2}(\hat{l}\times\vec{p})^2}. \tag{2.1}$$

Here $v_F(p-p_F)$ is the quasiparticle energy in the normal Fermi liquid above transition, with p_F the Fermi momentum and $v_F = p_F/m^*$; m^* is the effective mass, which is of order the mass m_3 of the ^{3}He atom; Δ_A is the so-called gap amplitude.

The energy in Eq. (2.1) is zero at two points $\vec{p} = e\vec{A}$ with $\vec{A} = p_F\hat{l}$ and $e = \pm 1$. Close to the two zeroes of the energy spectrum one can expand in $\vec{p} - e\vec{A}$ and the spectrum $E(\vec{p})$ becomes that of a charged, massless relativistic particle propagating in a curved spacetime in the presence of an electromagnetic vector potential:

$$g^{\mu\nu}(p_\mu - eA_\mu)(p_\nu - eA_\nu) = 0. \tag{2.2}$$

The ''four-momentum'' p_μ, ''electromagnetic vector potential'' A_μ, and inverse ''metric tensor'' $g^{\mu\nu}$ in this covariant expression are specified by giving their components in the coordinate system (x^0, x^i) where x^0 is the Newtonian time t and $x^i =$ are Cartesian spatial coordinates at rest with respect to the superfluid:

$$(p_0,\ p_i) = (-E,\ p_i), \tag{2.3a}$$

$$(A_0,\ A_i) = (0,\ p_F l_i), \tag{2.3b}$$

$$g^{00} = -1, \quad g^{0i} = 0, \quad g^{ik} = c_\perp^2(\delta^{ik} - l^i l^k) + c_\parallel^2 l^i l^k. \tag{2.3c}$$

E and p_i on the right hand side of Eq. (2.3a) are the Newtonian energy and momentum, and the index on p_i and l_i on the right hand sides of Eqs. (2.3a) and (2.3b) is lowered with the Euclidian metric δ_{ij}. [Once the ''relativistic'' quantities are defined by Eq. (2.3), the Euclidean metric plays no further explicit role in the dynamics.] The fermion quasiparticles actually satisfy the curved spacetime Weyl equation for massless charged chiral spinors [7,10], although for our purposes here all that is needed is the dispersion relation (2.2). In general there is an additional term in the vector potential [10] which is proportional to the gradient of the $\hat{l}$ vector. Also, the square of the Weyl equation contains extra terms: in symbolic form it is of the type $(p-eA)^2 + R + \sigma\cdot F = 0$. Here R is the Ricci scalar, F is the electromagnetic field strength, and σ is the spin (in our case it is the Bogoliubov-Nambu ''spin'' of quasiparticles in particle-hole space). We ignore all these extra terms in the dispersion relation (2.2), since they are proportional to the gradients of the $\hat{l}$ vector and thus are small in the $\hat{l}$ texture discussed here: $F \propto \nabla\hat{l}$, $R \propto (\nabla\hat{l})^2$. In principle these terms affect the propagation of the field, however, they play no essential role in the particle production processes studied here. Hereafter we omit the quotes when referring to the quasi gravitational and electromagnetic fields, since no actual such fields enter our problem.

The quantities $c_\perp = \Delta_A/p_F$ and $c_\parallel = v_F$ in the inverse metric (2.3c) are the ''speed of light'' propagating transverse to $\hat{l}$ and along $\hat{l}$ correspondingly. The magnitudes of the ^{3}He-A parameters at zero pressure are $m^* \simeq 3m_{\text{He-3}}$, $\Delta_A \simeq 1.7$ mK, $v_F \simeq 55$ m/s, $\Delta_A/p_F \simeq 3$ cm/s. As a result the speed of light is very anisotropic (in Cartesian coordinates): $c_\perp \simeq 0.5 \times 10^{-3} c_\parallel$. The relativistic approximation (2.2) is valid provided $p - p_F \ll p_F$ and $p_\perp = |\vec{p}\times\hat{l}| \ll m^* c_\perp$. The condition $E \ll m^* c_\perp^2 \sim 0.5\times 10^{-3} T_c \sim 0.5\ \mu$K is thus sufficient for this approximation. This upper limit is still significantly lower than the lowest confirmed temperature reached so far in the superfluid ^{3}He experiments, about 100 μK. The actual lower bound is probably lower than this but at the moment there is no reliable thermometry below 100 μK [11]. If the energy is higher than $m^* c_\perp^2$, ''nonrelativistic'' higher order corrections must be added in general. However there are many examples (such as axial anomaly and zero charge effect) where only the propagation along the $\hat{l}$ axis is important, in which case the only restriction is that $T \ll T_c$ so that the thermal fermions are concentrated in the vicinity of the nodes.

If the $\hat{l}$ texture moves with constant velocity $\vec{v}$, then to obtain manifest time independence of the background one must use the coordinate system which is at rest with respect to the texture. Let us from now on denote the coordinates in the texture frame by the unprimed letters (t, x^i), and those in the superfluid frame by the primed letters $(t', x'^i) = (t, x^i + v^i t)$, where v^i is the velocity of the texture. The dispersion relation in the moving frame is obtained from Eqs. (2.2) and (2.3a)–(2.3c) simply by finding the components of the tensors p_μ, A_μ, and $g^{\mu\nu}$ in the new coordinate system:[1]

$$(p_0,\ p_i) = (-E' + p_i' v^i, p_i'), \tag{2.4a}$$

$$(A_0,\ A_i) = (p_F l_i v^i, p_F l_i), \tag{2.4b}$$

$$g^{00} = -1, \quad g^{0i} = v^i, \quad g^{ik} = c_\perp^2(\delta^{ik} - l^i l^k) + c_\parallel^2 l^i l^k - v^i v^k. \tag{2.4c}$$

Note that $-p_0 = E = E' - p_i' v^i$ is just the energy E of the quasiparticle in the moving frame. In the moving frame the metric tensor, and electromagnetic vector potential do not depend on time and thus the quasiparticle energy E is *conserved*. The condition $|E'| \ll m^* c_\perp^2$ which ensures the validity of the relativistic approximation becomes, in terms of E, $|E + p_i v^i| \ll m^* c_\perp^2$.

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[1]Under the Galilean transformation of coordinates, the tensor transformation law for the covariant (*not* contravariant) four-momentum agrees with the Galilean transformation law for the energy and momentum of quasiparticles, so the resulting components of p_μ are in fact the correct Galilean components. That is, it is not necessary to transform back to the rest frame of the superfluid in order to correctly identify the Galilean energy and momentum.

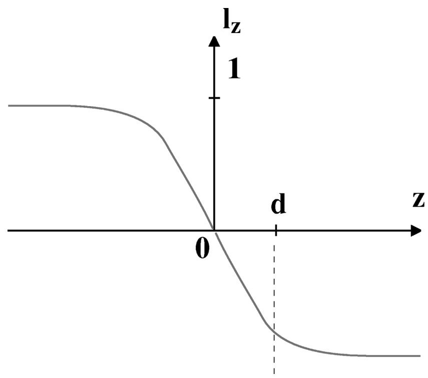

FIG. 1. Profile of the z projection of the vector $\vec{l}$ across the splay soliton wall in a superfluid ^{3}He-A. This soliton is topologically stable and can propagate in the liquid without destruction.

III. MOVING SOLITON

According to Eq. (2.2) the low-energy fermions move in gravitational and electromagnetic fields simulated by the $\hat{l}$ texture and its velocity $\vec{v}$. As explained above, to avoid the collapse of the condensate it is important to have the superfluid at rest with respect to the container, so we do not introduce the velocity field $\vec{v}_s$. While the superflow is limited by the critical velocity $\sim c_\perp$ above which the ^{3}He-A phase is destroyed and the broken symmetry is restored, the velocity v of the texture can exceed $c_\perp$ and can approach v_F [12,13]. The $\hat{l}$ solitons are well resolved in NMR experiments [14], and pulsed NMR can be used to accelerate them.

Here we consider a topologically stable texture, a ‘‘domain wall’’ soliton moving with velocity v in the z direction. We choose the so-called splay soliton [9]

$$\hat{l} = \hat{z}\cos\alpha(z) + \hat{x}\sin\alpha(z), \quad z = z' - vt, \tag{3.1}$$

where z is the coordinate comoving with the soliton and z' is the coordinate in the superfluid frame. Since the exact structure of the realistic soliton [9] is not important for our purposes, we consider a simplified profile for this soliton,

$$\hat{l} = -\hat{z}\tanh\frac{z}{d} + \hat{x}\,\mathrm{sech}\,\frac{z}{d}. \tag{3.2}$$

The thickness d of the soliton is on the order of the so-called dipole length $\xi_D \sim 10\ \mu$. The profile of the soliton is shown in Fig. 1.

The vector potential and metric tensor in the frame comoving with the soliton do not depend on time, nor do they depend on the x or y coordinates. Thus the energy $E = -p_0$ and momentum components p_x and p_y of the fermions are conserved quantities. The equations (2.4) and (3.1) give the following nonzero components for the vector potential:

$$A_0 = v p_F \cos\alpha, \quad A_x = p_F \sin\alpha, \quad A_z = p_F\cos\alpha, \tag{3.3}$$

and, for the inverse metric,

$$g^{00} = -1, \quad g^{0z} = v, \quad g^{yy} = c_\perp^2,$$

$$g^{zz} = c_\perp^2 \sin^2\alpha + v_F^2\cos^2\alpha - v^2, \tag{3.4}$$

$$g^{xx} = c_\perp^2\cos^2\alpha + v_F^2\sin^2\alpha, \quad g^{zx} = (v_F^2 - c_\perp^2)\sin\alpha\cos\alpha.$$

IV. PAIR PRODUCTION IN ELECTROMAGNETIC FIELD

The vector potential associated with the moving soliton gives rise to both magnetic and electric fields. In the soliton frame the electromagnetic field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ has the nonzero components

$$F_{zx} = p_F \partial_z \sin\alpha, \quad F_{0z} = v p_F \partial_z \cos\alpha. \tag{4.1}$$

The invariant combination

$$\text{‘‘}\mathbf{B}^2 - \mathbf{E}^2\text{’’} = \frac{1}{2} F_{\mu\nu} F_{\alpha\beta} g^{\mu\alpha} g^{\nu\beta} = v_F^2 c_\perp^2 F_{zx}^2 \left(1 - \frac{v^2}{v_F^2\cos^2\alpha}\right) \tag{4.2}$$

does not depend on the coordinate frame. For any velocity v there are two planes $z = \pm z_p$,

$$\cos^2\alpha(z_p) = \frac{v^2}{v_F^2}, \tag{4.3}$$

where the magnitude of the electric field equals the magnitude of the magnetic field. In the region between these planes, where $\mathbf{E}^2 > \mathbf{B}^2$, the electric field induces Schwinger production of pairs of fermions [15]. This leads to dissipation during the motion of the soliton, which gives rise to a friction force on the soliton even at zero temperature and the soliton will decelerate.

For textures where $A_0(z)$ has equal asymptotes $A_0(\infty) = A_0(-\infty)$ at both infinities the situation is different. In this case the potential $\Phi(z) = A_0(z) - A_0(\pm\infty)$ represents a potential well for the fermions. The fermions formed by Schwinger radiation finally occupy all the negative energy states in this potential well. After that the radiation stops. The filling of the negative energy levels will lead to a modification of the vacuum in the vicinity of the soliton. After that the soliton with the modified structure will move without friction. In our case the potential well is unbounded, $A_0(\infty) = -A_0(-\infty) \neq A_0(-\infty)$. The negative energy levels cannot be filled, thus the radiation will lead to the deceleration of the soliton until it reaches zero velocity.

V. HORIZON AND ERGOREGION FOR THE FERMIONS

If the velocity v of the soliton exceeds $c_\perp$, the metric (3.4) describes a planar ‘‘black hole’’ with an ergoregion outside the horizon, or rather a black hole/white hole pair. In the frame of the soliton, the horizons are lightlike surfaces at fixed position $z = \pm z_h$. These are given by an equation $f(z) = 0$, where the gradient $\partial_\mu f$ is lightlike: $0 = g^{\mu\nu}\partial_\mu f \partial_\nu f = g^{zz}(\partial_z f)^2$. That is, at the horizons one has $g^{zz} = 0$, or

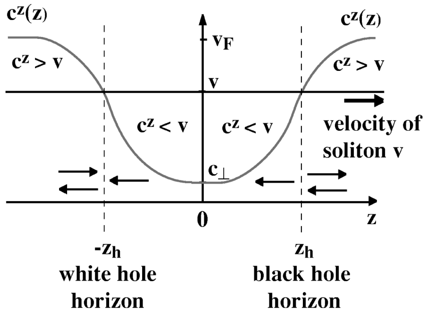

FIG. 2. The speed of light in the z direction in the superfluid frame $c^z(z)$. Quasiparticles in the region $-z_h<z<z_h$, where the speed of light is less than the velocity v of the soliton, cannot propagate to the right in the frame of the moving texture (the arrows show the possible directions of quasiparticle motion in the z direction). This region is bounded by the black and white hole horizons.

$$c_\perp^2\sin^2\alpha(z)+v_F^2\cos^2\alpha(z)=v^2. \tag{5.1}$$

Thus

$$\cos^2\alpha(z_h)=\frac{v^2-c_\perp^2}{v_F^2-c_\perp^2}. \tag{5.2}$$

The physical meaning of these horizons is revealed if one introduces c^z, the speed of light in the z direction in the superfluid rest frame. Any planar lightlike surface (''wave front'') is given by an equation $f(t,z')=0$, where $z'=z+vt$ is the coordinate in the superfluid frame, and c^z is defined by $(\partial_t+c^z\partial_{z'})f=0$. The lightlike condition on the gradient $\partial_\mu f$ yields $c^z=(g^{z'z'})^{1/2}$. Since $g^{z'z'}=g^{zz}+v^2$, the horizons [at $g^{zz}=0$ in Eq. (5.2)] occur where the speed of light equals the velocity of the soliton, $c^z(z_h)=v$. The speed of particles from the region between the horizons is less than v in the z direction and thus they cannot propagate out across the leading horizon. Furthermore no quasiparticle can enter the trailing horizon from the left because as it approaches its speed drops to the speed of the soliton. So the leading (future) event horizon is the black hole and the trailing (past) event horizon is the white hole (Fig. 2).

The horizons appear in the moving soliton if $v>c_\perp$. Note that the $\mathbf{E}^2>\mathbf{B}^2$ region (4.3) extends a bit outside the horizon, and exists even when $v<c_\perp$ and there is no horizon.

The ergoregion is the region where a particle must go faster than light in order to remain at the same value of (x,y,z). This occurs where $g_{tt}>0$. In the ergoregion the time translation Killing field ∂_t is *spacelike*, so the conserved ''energy'' can be negative even for a future pointing timelike four-momentum. As a result the vacuum in the ergoregion is unstable to creation of pairs of particles, both with future pointing momenta, with total energy zero. Put differently, the conserved energy can be *positive* even for a *past* pointing timelike four-momentum. For ^{3}He this means that a state in the occupied valence band [i.e., with the negative sign of the square root in Eq. (2.1)] has positive energy and thus can tunnel out away from the ergoregion, leaving behind a negative energy hole state.

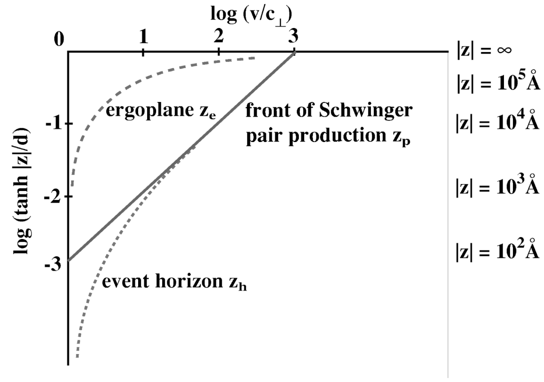

FIG. 3. Locations of the horizons, the ergoplanes, and the boundaries of the region of pair production by the electric field, as a function of the velocity v of the soliton.

The ergoplanes—boundaries of the ergoregion—occur at $z=\pm z_e$ where $g_{tt}=0$, which yields

$$\cos^2\alpha(z_e)=\frac{1-c_\perp^2/v^2}{1-c_\perp^2/v_F^2}. \tag{5.3}$$

An ergoregion exists for this soliton only if $v>c_\perp$, so there is an ergoregion if and only if there is an event horizon. The ergoplanes lie outside the event horizons (5.2) and outside the Schwinger pair region (4.3) unless v is extremely close to $c_\perp$ [i.e., unless $v<c_\perp(1-c_\perp^2/v_F^2)^{-1/2}$].

The locations of the boundary of the Schwinger pair production region z_p, event horizon z_h, and ergoplane z_e depend on the velocity v of the soliton. To get an idea of these locations and their scale we have plotted in Fig. 3 the coordinate z versus $\log(v/c_\perp)$ for each of these three positions. The Fermi velocity is at the abscissa $\sim 3(v_F/c_\perp\sim 10^3)$. Recall that $d\sim 10^5$ Å, so Fig. 3 shows that z_h is smaller than 10^3 Å until $v\sim 10c_\perp$.

The horizon has a ''transverse velocity'' because the light rays on the horizon are actually moving in the x direction. This is because the $\hat{l}$ vector has an x component, so the speed of light is faster in the x direction on the horizon. This is analogous to the rotational velocity of the horizon of a rotating black hole. To compute this velocity it is helpful to introduce the ''horizon generating Killing field'' χ^μ, which is tangent to the lightlike curves that generate the horizon. Since it is spacelike on the horizon, the vector field ∂_t is clearly *not* the horizon generating Killing field. Rather, we have

$$\chi=\partial_t+w\partial_x, \tag{5.4}$$

where w is some constant which we call the *transverse velocity* of the horizon. Since the horizon is a lightlike surface, χ must be orthogonal to ∂_x, so $0=g_{\mu\nu}\chi^\mu(\partial_x)^\nu=g_{tx}+wg_{xx}$. Thus $w=-(g_{tx}/g_{xx})_h=(g^{zx})_h/v$, or

$$w=v_F\sqrt{\frac{1-v^2/v_F^2}{1-c_\perp^2/v^2}}. \tag{5.5}$$

A lightlike surface generated by a Killing field is called a Killing horizon, so our black hole horizon is a Killing horizon. The surface gravity κ of a Killing horizon can be defined by the equation $\partial_a(\chi^2) = -2\kappa\chi_a$, evaluated on the horizon. Direct but tedious computation yields

$$\kappa = \left. \frac{dg^{zz}/dz}{2v} \right|_h = (dc^z/dz)|_h \tag{5.6}$$

or

$$\kappa = \frac{v_F}{d}\left(1 - \frac{v^2}{v_F^2}\right)\sqrt{\frac{1-c_\perp^2/v^2}{1-c_\perp^2/v_F^2}}. \tag{5.7}$$

VI. HAWKING RADIATION

The Hawking temperature is determined by the surface gravity as

$$T_H = \frac{\hbar}{2\pi k_B}\kappa, \tag{6.1}$$

where κ is given by Eq. (5.6) or (5.7). Note that, as in the case of Unruh's sonic black hole model, the Hawking temperature is given by the gradient of a velocity at the horizon. However, in the sonic case it was the velocity of the fluid, whereas in the present case it is the (anisotropic) velocity of the fermion quasiparticles. As long as the soliton velocity v is not too close to either v_F or $c_\perp$, then $\kappa \simeq v_F/d$, which gives $T_H \simeq 5\ \mu$K. This is an order of magnitude lower than the lowest confirmed temperature reached in the superfluid ^{3}He experiments today, but is an order of magnitude higher than the temperature$\simeq 0.5\ \mu$K above which the nonrelativistic corrections become important.

The Hawking flux for fermions has the form [1]

$$\Gamma[\exp[(E-\mu)/k_B T_H] + 1]^{-1}, \tag{6.2}$$

where Γ and μ are the emission coefficient and "chemical potential" for the mode in question. In our case, μ is given by

$$\mu = p_x w + eA_0(z_h) \tag{6.3}$$

(neglecting the spin energy), where w is the transverse velocity of the horizon (5.5) and $A_0(z_h)$ is the scalar potential (3.3) evaluated at the event horizon. By way of analogy, for a rotating charged black hole one has $\mu = J\Omega + e\Phi$ where J and e are the the angular momentum and charge of the mode, and Ω and Φ are the angular velocity and the electric potential of the horizon. The quantity E in Eq. (6.2) is $-p_0$, the conserved energy in the comoving frame of the soliton, which according to Eq. (2.4a) is equal to $E' - p_z' v$ where E' and $p_z'(=p_z)$ are the energy and momentum in the frame of the superfluid. We remind the reader that there is a constraint $|E'| \ll m^* c_\perp^2$ on the quasiparticle energy in order for the relativistic description to be generally valid. In terms of E this constraint becomes $|E + p_z v| \ll m^* c_\perp^2$. Since p_z is not conserved this condition may be satisfied at one point of a quasiparticle trajectory and not at another. A complete analysis of the particle creation processes will therefore require the nonrelativistic treatment in general, although the extent of the nonrelativisitic corrections will depend on the type of texture and other parameters of the system.

The existence of Hawking radiation in the black hole case follows from the assumption that near the horizon the high frequency outgoing modes of the quantum field are in the ground state as defined in a frame falling freely across the horizon. When the temperature of the heat bath (normal component of the liquid) is very low, this assumption holds for the moving soliton in ^{3}He, since the "freely falling frame" is the frame of the superfluid which is at rest with respect to the container (and thus to the heat bath). The passage of the moving texture through this frame is essentially adiabatic for the high frequency modes. As a result, the distribution of the fermions in the soliton frame remains thermal and is given by the Fermi function $f(E')$ with $E' = E - \vec{p}\cdot\vec{v}_n$, where $\vec{v}_n$ is the velocity of the heat bath and E and $\vec{p}$ are in the soliton frame.

The vacuum is therefore not excited directly by any time dependent forcing, but it is unstable to tunneling processes arising from both "level crossing" and the Hawking effect. The level crossing leads to Schwinger pair production in the "electric" field, as well as pair production in the ergoregion that would occur even in the absence of electric charge in analogy with the process outside a rotating black hole. What happens is that the Fermi sea is "tilted" in space, and some states under the Fermi surface near the soliton have positive energy relative to the Fermi surface far from the soliton. Quasiparticles in these states may tunnel out leaving behind quasiholes (or vice versa) that are swept past the horizon.

It was realized [16] shortly after Hawking's discovery that the flux from pair creation due to level crossing outside the horizon corresponds to the contribution from states with $E < \mu$ in the flux formula (6.2). As $T_H \to 0$, the flux is extinguished for states with $E > \mu$, whereas for states with $E < \mu$ it approaches the nonzero value Γ, the tunneling probability. At finite Hawking temperature the flux is modified as indicated by Eq. (6.2).

To determine the actual magnitude of the Hawking flux and "level crossing flux" it is necessary to evaluate the emission coefficients (or so-called "gray-body factors") $\Gamma(E, p_x, \sigma, e)$ (σ is the spin). These indicate the fraction of each mode that is "transmitted" from its high frequency form near the horizon out to infinity, while the rest is scattered back across the horizon. These coefficients have not yet been calculated.

VII. QUANTUM MECHANICS OF QUASIPARTICLES NEAR THE HORIZON

The temperature and chemical potential of the Hawking radiation were inferred above by exploiting the analogy with Hawking's calculation. It may be helpful here to exhibit the essential physics in a simple way [17]. Neglecting the spin degrees of freedom, the wave equation for the fermions is the same as in the bosonic case, which (neglecting the electromagnetic field) is governed by the wave equation (1.1). The outgoing waves oscillate rapidly near the horizon. If we choose eigenmodes

$$\Psi=\Psi_{E,p_x}(z)e^{-iEt}e^{ip_x x}e^{ip_y y}, \tag{7.1}$$

and neglect all terms without at least one z derivative, Eq. (1.1) becomes

$$\{2i[-vE+p_x g^{xz}(z_h)]\partial_z+\partial_z[g^{zz}(z)\partial_z]\}\Psi_{E,p_x}=0. \tag{7.2}$$

The general outgoing solution is

$$\Psi_{\tilde{E}}(z)=a\ \exp\left(2iv\tilde{E}\int^z dz'/g^{zz}(z')\right)$$
$$\simeq a\ \exp[i(\tilde{E}/\kappa)\ln(z-z_h)], \tag{7.3}$$

with $\tilde{E}=E-p_x g^{xz}/v=E-p_x w$, where w is the translational velocity of the horizon (5.4) and κ is the surface gravity (5.6).

The outgoing modes that have purely negative frequency with respect to the free-fall frame (i.e., the superfluid frame) are states below the Fermi sea, which is in (or near) the quantum ground state, as discussed above. Some of these modes have *positive* energy in the (stationary) soliton frame, however, so they may tunnel out away from the soliton. These modes fall into two classes according to whether $\tilde{E}$ is less than or greater than zero. When $\tilde{E}<0$, the positive energy states below the Fermi sea can be located *outside* the horizon. Tunneling of these states is identified as due to level crossing in the ergoregion, and includes the Schwinger pairs when an electric field is included. When $\tilde{E}>0$, the positive energy states below the Fermi sea can only exist *behind* the horizon, so it might seem that they could never tunnel out. However, this is not true because these states always have an exponential tail that spills out across the horizon. That they must have such a tail follows from the fact that a purely negative frequency wave packet must be analytic in the upper half complex time plane. Equivalently, the mode function $\Psi_{\tilde{E}}(z)$ must be analytic in the lower half complex z plane. Analytic continuation of Eq. (7.3) across the horizon in the lower half complex z plane yields

$$\theta(-u)\Psi_{\tilde{E}}(z_h-u)+e^{-\pi\tilde{E}/\kappa}\theta(u)\Psi_{\tilde{E}}(z_h+u), \tag{7.4}$$

where $u=z-z_h$. The second term is the tail term. The associated probability current is the ratio of the squared norm of this piece to the total squared norm, i.e., it is $(e^{2\pi\tilde{E}/\kappa}\pm 1)^{-1}$, where $\pm=+$ for fermions and $-$ for bosons. (For bosons, the term inside the horizon has negative norm in the relevant inner product.) The distribution of particles that tunnel across the horizon (in this sense) is thus a thermal one at the Hawking temperature (restoring $\hbar$) $T_H=\hbar\kappa/2\pi k_B$ and with the chemical potential $p_x w$, in agreement with Eqs. (6.1) and (6.3). After tunneling across the horizon (so to speak), the particles are partially scattered back across the horizon. The fraction that propagate out to the asymptotic region is the emission coefficient Γ of Eq. (6.2).

VIII. DISCUSSION

With this moving texture model it should now be possible to study some of the questions presented by black hole horizons. Hawking radiation, pair production in the ergoregion, and Schwinger pair production are driven by the kinetic energy of the moving soliton, and the back reaction will be to slow the soliton. The Hawking temperature is fairly constant until, as v approaches $c_\perp$, T_H goes to zero. This is unlike the evaporation of a neutral black hole which gets hotter as it shrinks. It is rather similar to Hawking radiation from a black hole with a large magnetic charge which cannot be discharged and so cools as it evaporates and approaches an extremal black hole.

The radiation from level crossing in the ergoregion may be observable with current technology, and the Hawking flux at $\sim 5\ \mu$K is probably not too low to be observed eventually. The Hawking temperature can be significantly higher if instead of the soliton one takes a moving planar interface between ^{3}He-A and ^{3}He-B. The A-B interface has many advantages: it can be moved with high velocity especially at low T [18] and the thickness d of the interface is much shorter $d\sim 500$ Å. This essentially increases the corresponding ''surface gravity'' and the Hawking temperature. But the ''nonrelativistic'' corrections also become more important and this requires further investigations. On the other hand, for understanding some of the principal issues related to event horizons, it is not necessary to consider a real system. Gedanken experiments can be made on model ^{3}He-A-like systems, in which the ''nonrelativistic'' corrections can be made arbitrarily small.

Horizons can occur in other moving topological and nontopological textures in superfluids and superconductors, and for the bosonic degrees of freedom as well as for the fermions. In particular, the orbital waves in ^{3}He-A—oscillations of the $\hat{l}$ vector—are analogous to electromagnetic waves. At low T their dynamics becomes relativistic [7] and one can discuss the propagation of such relativistic bosons in textures with event horizons.

As for other topological objects, an interesting analogy occurs in the case of quantized vortices, which correspond at large distances to spinning cosmic strings [19]. The vortex has fermion zero modes bound to the vortex core. There is a connection between the statistics of these fermion zero modes and the fermionic zero modes on fundamental strings, which simulate the thermodynamics of extreme black holes [20]. The Hawking radiation is absent if the vortex is stationary with respect to the heat bath: a stationary vortex corresponds to a local minimum of the energy and thus no radiation is possible from this state. If a vortex moves with respect to the heat bath or if a nonaxisymmetric vortex core rotates with respect to the heat bath, the spectral flow of the fermion zero modes lead to dissipation of the vortex motion and to an additional transverse force on the moving vortex [21–23]. In some cases this corresponds to the appearance of a horizon with nonzero surface gravity [20,23].

Another interesting texture is a domain wall in a thin film of ^{3}He-A, where the $\hat{l}$ vector which is perpendicular to the film changes sign [24]. If this texture is moving the fermion quasiparticles see an effective $(2+1)$-dimensional spacetime with black hole and white hole horizons and a curvature singularity in between [25]. This is in many ways a much simpler system than the one discussed in this paper, since there is no ergoregion or (pseudo)electromagnetic field.

To have a horizon in a condensed matter system, it is not necessary to create curvature singularities as inside black holes, since the metric describing the horizon does not follow from the Einstein equations. Moreover, if curvature singularities do occur, as in the thin film domain wall texture just mentioned, the physics is still under control. The quantum fermions propagating in the texture obey relativistic dynamics in the low-energy limit and thus fully exhibit the quantum physics of the horizon, including both the Hawking radiation and the entropy of the fermion zero modes. For high enough energies, or inevitably near singularities in the texture, the fundamental nonrelativistic description takes over. Thus the superfluid ^{3}He is a promising model for experimental and theoretical simulations of quantum effects related to the event horizon, and may offer useful ideas about resolving the physics near a singularity.

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