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## Composite Defect Extends Analogy between Cosmology and $^3\text{He}$

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Spin-mass vortices have been observed to form in rotating superfluid  $^3\text{He-B}$ , following the absorption of a thermal neutron and a rapid transition from the normal to the superfluid state. The spin-mass vortex is a composite defect which consists of a planar soliton (wall) which terminates on a linear core (string). This observation fits well within the framework of a cosmological scenario for defect formation, known as the Kibble-Zurek mechanism. It suggests that in the early Universe analogous cosmological defects might have formed.

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Experiments with superfluid  $^3\text{He}$  [1,2] have shown that quantized vortex lines are formed in the aftermath of a neutron absorption event, during the subsequent rapid transition from the normal to the superfluid state. These observations agree with a theory of defect formation, the Kibble-Zurek (KZ) mechanism [3,4], which was developed for the phase transitions of the early Universe. In this scenario a network of cosmic strings is formed during a rapid nonequilibrium second order phase transition, in the presence of thermal fluctuations. The real experimental conditions in the neutron irradiation experiment of  $^3\text{He-B}$  (and also probably in the early Universe) do not coincide with the perfectly homogeneous transition assumed in the KZ scenario: The temperature distribution within the “neutron bubble” is nonuniform, the transition propagates as a phase front between the high- and low-temperature phases, and the phase is fixed outside the bubble. This requires modifications to the original KZ scenario [5–7] and even raises concerns whether the KZ mechanism is responsible for the defects which are extracted from the neutron bubble and observed in the experiment [8,9]. New measurements now demonstrate that a more unusual composite defect is also formed and directly observed in the neutron experiment. This strengthens the importance of the KZ mechanism and places further constraints on the interplay between it and other competing effects.

Composite defects exist in continuous media and in quantum field theories, if a hierarchy of energy scales with different symmetries is present [10]. Examples are strings terminating on monopoles and walls bounded by strings. Many quantum field theories predict heavy objects of this kind, which could appear only during symmetry-breaking phase transitions at an early stage in the expanding Universe [11,12]. Various roles have been envisaged for them. For example, domain walls bounded by strings have been suggested as a possible mechanism for baryogenesis [13]. Composite defects also provide one possible mechanism for avoiding the monopole overabundance problem [14].

In high-energy physics it is generally assumed that composite defects can exist after two successive symmetry-breaking phase transitions, which are far apart in energy [10]. An example of successive transitions in grand unification theories is  $\text{SO}(10) \rightarrow \text{SU}(4) \times \text{SU}(2)_R \times \text{SU}(2)_L \rightarrow \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \rightarrow \text{SU}(3)_C \times \text{U}(1)_Q$ . In condensed matter physics composite objects are known to result even from a single transition, provided that at least two distinct energy scales are involved, such that the symmetry at large lengths can become reduced [15]. An example is the spin-mass vortex in superfluid  $^3\text{He-B}$ . It was discovered in rotating NMR measurements, after a slow adiabatic first order  $^3\text{He-A} \rightarrow ^3\text{He-B}$  transition had taken place in the rotating liquid [16]. Our new observations show that the spin-mass vortex is also formed in a rapid nonequilibrium quench through the second order transition from the normal phase to  $^3\text{He-B}$ .

Superfluid  $^3\text{He-B}$  corresponds to a symmetry-broken state  $\text{U}(1) \times \text{SO}(3)_L \times \text{SO}(3)_S \rightarrow \text{SO}(3)_{L+S}$ , where  $\text{SO}(3)_L$  and  $\text{SO}(3)_S$  are groups of rotations in orbital and spin spaces, respectively. In this state two topologically distinct linear defects with singular cores are possible [15]. Their structure can be seen from the  $B$ -phase order parameter [17], a  $3 \times 3$  matrix  $A_{\alpha j} = \Delta_B e^{i\phi} R_{\alpha j}(\hat{\mathbf{n}}, \theta)$ . It is a product of the energy gap  $\Delta_B$ , the phase factor  $e^{i\phi}$ , and a rotation matrix  $R_{\alpha j}$ . The latter is an abstract rotation which reflects the broken relative  $\text{SO}(3)$  symmetry between spin and orbital spaces. The unit vector  $\hat{\mathbf{n}}$  points in the direction of the rotation axis while  $\theta$  is the angle of rotation.

A conventional vortex is the result of broken gauge  $\text{U}(1)$  symmetry, which is common to all superfluids and superconductors. It has  $2\pi\nu$  winding in the phase  $\phi$  of the order parameter around the singular core, with integer  $\nu$ . This vortex belongs to the homotopy group  $\pi_1(\text{U}(1)) = \mathbb{Z}$  and its quantum number  $\nu$  obeys a conventional summation rule  $1 + 1 = 2$ . The phase winding translates to a persistent quantized supercurrent circulating around the central core and thus this vortex is called a “mass vortex.”

The second type of defect appears in the order parameter matrix  $R_{\alpha j}(\hat{\mathbf{n}}, \theta)$  (Fig. 1, top). On moving once around its core,  $\hat{\mathbf{n}}$  reverses its direction twice: first by smooth rotation while the angle  $\theta$  remains at the equilibrium value  $\theta_D \approx 104^\circ$ , which minimizes the spin-orbit interaction energy, and later by increasing  $\theta$  to  $180^\circ$ , where both directions of  $\hat{\mathbf{n}}$  are equivalent, and then decreasing back to  $\theta_D$ . The second leg in the direction reversal does not minimize the spin-orbit interaction and hence it becomes confined in space within a planar structure, a soliton sheet,

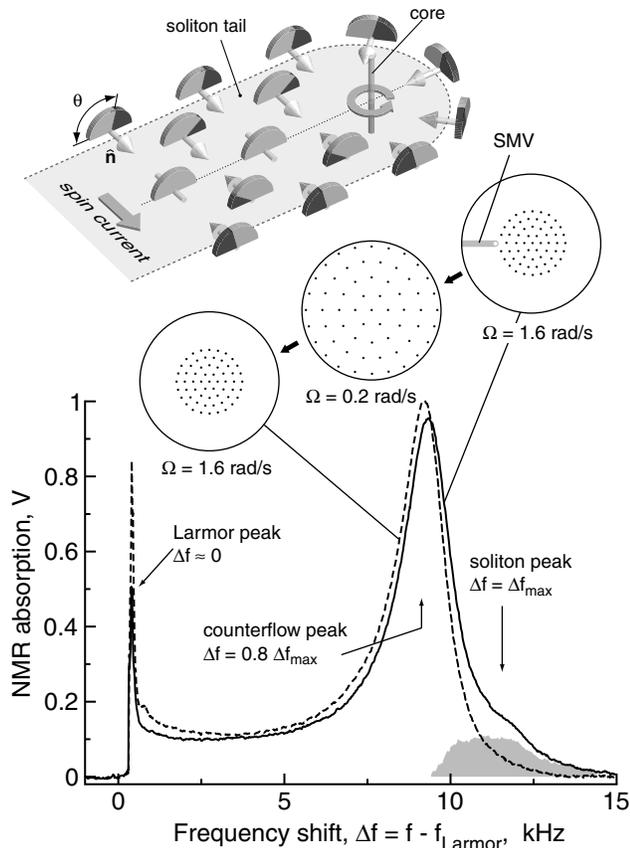


FIG. 1. (Top) The spin vortex in  $^3\text{He-B}$  is a disclination in the spin-orbit rotation field  $R_{\alpha j}(\hat{\mathbf{n}}, \theta)$ . It has a singular core which is encircled by a spin current and which serves as a termination line to the planar  $\theta$  soliton. (Middle) Cross sections through rotating container perpendicular to the rotation axis. A spin-mass vortex (SMV) is formed by combining a spin and mass vortex to a common core. Its equilibrium position is slightly outside the cluster of usual mass vortex lines (right). By decreasing  $\Omega$  to just above the annihilation threshold (center), the SMV is selectively removed (left). (Bottom) A NMR spectrum measured with a SMV in the container (solid line) shows an absorption peak at the maximum possible frequency shift  $\Delta f_{\text{max}}$ . This component in absorption originates from regions where  $\hat{\mathbf{n}}$  is oriented perpendicular to the magnetic field  $\mathbf{H}$ , which occurs only in and around the soliton tail of the SMV (sketch on the top;  $\mathbf{H}$  is oriented parallel to the rotation axis). After the SMV has been selectively removed, the only significant change in the spectrum (dashed line) is the absence of the soliton contribution (shaded area).

which terminates on the linear singular core or on the wall of the container. This structure becomes possible through the existence of two different energy (and length) scales: The superfluid condensation energy defines the scale of the coherence length  $\xi \sim 10\text{--}100$  nm, which is roughly the radius of the singular core. The much weaker spin-orbit interaction defines the scale of the dipolar healing length  $\xi_D \sim 10$   $\mu\text{m}$ , at which the angle  $\theta$  becomes fixed. This length determines the thickness of the soliton sheet. Since the matrix  $R_{\alpha j}$  spans the space  $\text{SO}(3)$ , this defect belongs to the homotopy group  $\pi_1(\text{SO}(3)) = \mathbb{Z}_2$ , a two-element group with a summation rule  $1 + 1 = 0$  for its topological charge. Such a defect with the charge 1 is identical to its antidefect and represents a nonzero (but not quantized) circulation of current in the spin part of the order parameter around a singular core. It is named a “spin vortex.” By itself the spin vortex is an unstable structure: The surface tension of its soliton tail leads to its annihilation.

Mass and spin vortices do not interact significantly—they “live in different worlds”; i.e., their order parameters belong to different isotopic spaces. The only instance where the spin vortex has been found to remain stable in the rotating container arises when the cores of a spin and a mass vortex happen to get close to each other and it becomes energetically preferable for them to form a common core. Thus by trapping the spin vortex on a mass vortex, the combined core energy is reduced [18] and a composite object— $\mathbb{Z}_2$  string + soliton + mass vortex, or “spin-mass vortex”—is formed. Its equilibrium position in the rotating container, which has a deficit of the usual mass vortices, is slightly outside of the cluster of mass vortex lines (Fig. 1, middle right). This is determined by the balance of the Magnus force from the externally applied normal-superfluid counterflow and the surface tension of the soliton.

Details about the experiment are given in Refs. [1,8]. The stable configuration of the spin-mass vortex in the rotating container can be observed with different types of NMR methods. One signature from the spin-mass vortex in the neutron irradiation measurement is illustrated in Fig. 2, where the height of the NMR absorption peak, used for monitoring the number of vortex lines, is plotted as a function of time. This accumulation record shows one oversize downward jump in the absorption amplitude. The total number of vortex lines accumulated by the end of the irradiation session can be determined by different independent methods. These include (a) measurement of the annihilation threshold, i.e., of the rotation velocity at which vortex lines start to annihilate at the wall of the container during deceleration [19], (b) measurement of the relative heights of the two peaks in the NMR spectrum of Fig. 1, known as the counterflow and Larmor peaks [20], and (c) comparison to measurements at other rotation velocities using an empirically established  $\Omega$  dependence of the vortex formation rate [1,8]. These comparisons prove

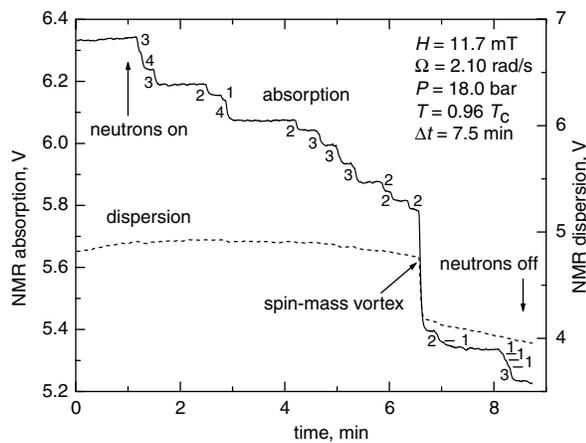


FIG. 2. Neutron-irradiation record of  ${}^3\text{He-B}$  and the spin-mass vortex. The height of the counterflow peak in the NMR absorption spectrum is plotted as a function of time during irradiation of the sample with thermal neutrons. Each of the small downward steps in the absorption record marks a neutron absorption event and its height measures the number of newly formed mass-current vortex lines [1], also given by the number next to each step. In such events the (out-of-phase) dispersion signal remains unchanged (dashed line). In contrast the single large oversized step is recorded by both the absorption and dispersion signals. It is attributed to one SMV where the soliton tail becomes responsible for the large jumps in both signals. These are proportional to the length of the soliton sheet.

that the large jump can include at most a few ( $\leq 5$ ) circulation quanta.

A large reduction in the peak height of the NMR absorption, together with a small number of circulation quanta, can be attributed only to a soliton sheet which is trapped on the spin-mass vortex. This identification is based on the change in the line shape of the NMR spectrum with and without the spin-mass vortex (Fig. 1, bottom). The first spectrum was recorded right after the neutron irradiation and shows the shifted absorption at the maximum possible frequency shift, the characteristic signature of the soliton sheet. The second spectrum, recorded after reducing the rotation briefly to a sufficiently low value where the spin-mass vortex is selectively removed by pushing it to the container wall, displays no soliton signal. Such a recovery of the NMR spectrum to the line shape of an axially symmetric configuration, with a central vortex cluster surrounded by a coaxial region of vortex-free counterflow (Fig. 1, middle left), provides a most tangible demonstration of the initial presence of the spin-mass vortex at the edge of the cluster. In neutron absorption events where only mass vortex lines are formed, the reduction in the NMR peak height is not accompanied by a frequency shift in the location of the maximum, while the soliton produces discontinuities in both the absorption and dispersion signals (Fig. 2).

The spin-mass vortex is a rare product from the neutron absorption event, compared to the yield of vortex lines. In the conditions of Fig. 2 their ratio is roughly 1:100.

Nevertheless, its presence is thought to convey an important signal. The spin-mass vortex is the only other type of defect, besides mass-current vortices, which so far has been observed to form in a neutron absorption event. (Indirect experimental evidence for the creation of  ${}^3\text{He-A}$ - ${}^3\text{He-B}$  interfaces has been discussed in Ref. [8].) It demonstrates that more than one type of order-parameter defect can be created. This limits the possible scenarios of defect formation which work within or around the small volume of about  $100\ \mu\text{m}$  in diameter which is heated to the normal state by the energy of the decay products from the neutron absorption reaction. The formation of defects occurs during the rapid cooling back to the superfluid state on a time scale of microseconds.

At the moment the only presently viable general principle by which defects can be created under such constraints and which would give rise to different types of order-parameter defects is the quench cooling of thermal fluctuations within the KZ scenario. Two possible routes can be suggested for the formation of the spin-mass vortex: One possibility is that the spin and mass vortices are formed independently, since the random order-parameter fabric after the quench may contain discontinuity in both the phase  $\phi$  as well as in the relative rotation of the spin and orbital axes of the order parameter. Where these two types of vortices happen to fuse, the combined spin-mass vortex appears. The long-range attractive force between the spin and mass vortices is due to Casimir-type effects: In the vicinity of the spin vortex the order parameter amplitude and thus the superfluid density  $\rho_s$  is reduced. This reduces the kinetic energy of superflow  $v_s$  around the mass vortex,  $\frac{1}{2} \int dV \rho_s v_s^2$ . The magnitude of this force is smaller by the factor  $\xi^2/d^2$  than the interaction between two vortices of the same type, where  $d$  is the distance between the vortices.

A second possibility is a similar process as that after which the spin-mass vortex was first observed [16]: In addition to the neutron absorption event, so far the only other effective method for forming spin-mass vortices in larger numbers is from  $A$ -phase vortex lines when the  $A \rightarrow B$  transition is allowed to propagate slowly through the rotating container. In a neutron absorption event,  $AB$  interfaces are also among the objects which should be formed in the KZ process [8,21]. The present measurements favor this second explanation: In neutron absorption events spin-mass vortices are formed only at high pressure close to the  $AB$  transition line. Earlier measurements [8] have established that also the yield of vortex lines from a neutron absorption event is reduced in the vicinity of the stable  $A$ -phase regime. The most straightforward explanation is to assume that  $AB$  interfaces, which are formed as additional defects within the rapidly cooling neutron bubble, intervene in the formation of vortex lines.

Within the neutron bubble a spin-mass vortex is initially formed as a loop which traps both the spin and mass currents, with the soliton spanned like a membrane across

the loop. This loop expands in sufficiently strong applied counterflow to a rectilinear vortex line, in the same manner as other loops formed from mass-current vortices. The threshold velocity for the expansion corresponds to the largest possible loop size, limited by the diameter of the neutron bubble. This threshold velocity is higher for a spin-mass vortex than for a mass-current vortex, because the energy of the composite object is larger. In the logarithmic approximation the energy of a spin-mass vortex is the sum of the energies of the constituent mass and spin vortices. The energy of the spin vortex is about 0.6 of that of the mass vortex [18]. Thus one obtains the estimate  $E_{\text{SMV}}/E_{\text{MV}} \sim 1.6$  and the same ratio for their threshold velocities. This is consistent with the measurements: The threshold velocity for the creation of the mass vortices at the experimental conditions of Fig. 2 is 0.75 rad/s, while spin-mass vortices were not observed below 2 rad/s. However, the actual threshold velocity for the SMV might be smaller because the irradiation time at such high rotation velocities cannot be extended indefinitely due to the rapid accumulation of mass vortices.

In addition to the KZ mechanism also other sources might give rise to the mass vortex lines which are counted in Fig. 2. While the measurements clearly point to a volume effect [1,8], recent numerical simulations of the experiment [9] conclude that a surface phenomenon dominates as the origin for these directly observed vortices. Using the thermal diffusion equation to describe the cooling neutron bubble and a one-component order parameter in the time-dependent Ginzburg-Landau equation to model the order-parameter relaxation, this calculation confirms the appearance of a tangled vortex network within the bubble volume via the KZ mechanism. However, in the presence of the externally applied counterflow from the rotation, vortex rings are also formed on the bubble surface, due to the classical corrugation instability of the normal/superfluid interface. This flow instability at the warm boundary of the neutron bubble does not require the presence of thermal fluctuations [9] and may be interpreted as an instability of a vortex-sheet-like intermediate state [22]. These vortex rings around the boundary of the neutron bubble screen the externally applied counterflow and allow the random vortex network inside the bubble volume to be dissipated.

The formation of a spin-mass vortex in the flow instability seems unlikely, since the applied counterflow does not significantly interact with the spin degrees of the order parameter. More likely, fluctuations must be an essential ingredient in its formation process. Thus here again the experiment prefers the fluctuation-dominated KZ mechanism as a more plausible explanation. For the description of composite defects simulation calculations should be extended to the multicomponent order parameter of  $^3\text{He-B}$ . However, the interplay between the KZ mechanism within

the bubble volume and the flow instability at the bubble surface might depend on the details of the transition process, which cannot be described by the time-dependent Ginzburg-Landau equations, but requires a microscopic treatment of quasiparticle dynamics in the presence of a rapidly changing order parameter.

The observation of the spin-mass vortex, as a product from neutron irradiation of  $^3\text{He-B}$ , strengthens the importance of the fluctuation-mediated mechanisms as the source of defect formation in nonequilibrium transitions. It shows that composite objects do not necessarily require for their creation two phase transitions at very different energies.

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