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**Self-tuning vacuum variable and cosmological constant**

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A spacetime-independent variable is introduced which characterizes a Lorentz-invariant self-sustained quantum vacuum. For a perfect (Lorentz-invariant) quantum vacuum, the self-tuning of this variable nullifies the effective energy density which enters the low-energy gravitational field equations. The observed small but nonzero value of the cosmological constant may then be explained as corresponding to the effective energy density of an imperfect quantum vacuum (perturbed by, e.g., the presence of thermal matter).

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**I. INTRODUCTION**

A first step towards the elusive theory of “quantum gravity” [1,2] is to identify the microscopic constituents of space [3–6]. A next step which may be strongly correlated with the first step is to properly describe the dynamics of these constituents.

In the last few years, a new approach to the above-mentioned program has emerged, which has been driven by both experimental and theoretical considerations. Astronomical observations suggest the existence of a cosmological constant [7] or vacuum energy density [8–11] with a typical energy scale of the order of  $10^{-3}$  eV [12–14]. But current theories do not provide a good symmetry explanation for the smallness of this value [15–19]. This last observation suggests that power-divergent quantum corrections to the vacuum energy density would not be regulated by a dynamical principle [20]. Instead, the vacuum energy density would have to be regulated by a more general principle.

The aim of this article is to present *thermodynamic* arguments for the behavior of the effective vacuum energy density which enters the low-energy gravitational field equations. These arguments will be formulated in the context of emergent-gravity models [21–24] but may have a more general applicability.

Let us give, right from the start, a heuristic argument [25] in favor of the emergent-symmetry approach to the cosmological constant problem. Namely, if gravitation would be a truly fundamental interaction, it would be hard to understand why the energies stored in the quantum vacuum would not gravitate at all [15–19]. If, however, gravitation would be only a low-energy effective interac-

tion [21–24], it could be that the corresponding gravitons as quasiparticles do not feel *all* microscopic degrees of freedom (gravitons would be analogous to small-amplitude waves at the surface of the ocean) and that the gravitating effect of the total vacuum energy density would be effectively *tuned away*. In fact, it is at this point that elementary thermodynamic arguments [26] might be useful, even if the details of the underlying microscopic physics (the ocean in the above analogy) remain unknown.

A brief outline of this article runs as follows. In Sec. II, a particular Lorentz-invariant vacuum variable is introduced which allows us to discuss thermodynamic properties of the vacuum such as stability, compressibility, and thermodynamic response to perturbations. In Sec. III, it is argued that this vacuum variable adjusts itself so as to nullify the relevant vacuum energy density which contributes to the effective gravitational fields equations and the effective cosmological constant of a perfect quantum vacuum would be zero. In Sec. IV, it is shown how the presence of thermal matter shifts the value of this effective vacuum energy density away from zero (the same happens with other types of perturbations, such as the existence of a spacetime boundary of the volume considered). In Sec. V, the possible origin of the vacuum variable is discussed in general terms, but two specific examples are also presented (one with a four-form field strength and another with a four-velocity field). In Sec. VI, our results are summarized and an outlook is given.

**II. VACUUM VARIABLE AND THERMODYNAMICS****A. New variable for the Lorentz-invariant quantum vacuum**

The quantum vacuum (or “aether” in the old terminology [27–29]) is a medium of a peculiar nature. The most

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important property of the quantum vacuum is its Lorentz invariance. Current experimental bounds on Lorentz-violating modifications of Maxwell theory at the level of  $10^{-18}$  or better [30,31] suggest that a hypothetical Lorentz-violating energy scale  $E_{LV}$  [32] exceeds the Planck energy scale  $E_{\text{Planck}} \equiv \sqrt{\hbar c^5/G}$  by at least 9 orders of magnitude, for Lorentz-violating modifications of order  $(E_{\text{Planck}}/E_{LV})^2$ . The Lorentz invariance of the quantum vacuum imposes constraints on its kinematics and dynamics. According to Einstein [29], “this aether may not be thought of as endowed with the quality characteristic of ponderable media” and “the idea of motion may not be applied to it.” These considerations suggest that the momentum density of the quantum vacuum is strictly zero. With Lorentz invariance playing a crucial role, it will be natural to use units with  $c = 1$  in the rest of this article. The same holds for quantum mechanics and we also set  $\hbar = 1$ .

Another important property of the quantum vacuum is the apparent discrepancy between the magnitude of the theoretically expected contributions to the vacuum energy density and the experimentally observed value of the effective vacuum energy density. The formal one-loop contribution is proportional to  $M^4$ , where  $M$  is the characteristic mass scale of the particles and fields which make up the vacuum and where a relativistically-invariant regularization must be used in order to recover a Lorentz-invariant equation of state for the quantum vacuum (see, e.g., Appendices II and VIII of the well-known review [9(b)] and the more recent article [33]). The particular value of  $M$  depends on the detailed theory. If the vacuum is made solely from the elementary particles and fields of the electroweak standard model, the characteristic vacuum energy density  $\epsilon_{\text{vac}}$  is of order  $M_W^4 \approx (10^{11} \text{ eV})^4$ . In the Frolov–Fursaev scheme [34] of Sakharov-like induced gravity [35], the vacuum is made from constituent bosonic and fermionic fields whose masses are of the order of the Planck scale. This gives rise to a “natural” value of the vacuum energy density  $\epsilon_{\text{vac}}$  of order  $E_{\text{Planck}}^4 \approx (10^{28} \text{ eV})^4$ , unless there is a special cancellation between the different species caused by, for example, supersymmetry. For the case of supersymmetric cancellations, the remaining vacuum energy is determined by the mass scale at which supersymmetry is broken,  $M_{\text{SUSYbreaking}} \geq 1 \text{ TeV} = 10^{13} \text{ eV}$ . Anyway, regardless of which extension of the electroweak standard model turns out to be correct, the theoretical estimate of the vacuum energy,  $\epsilon_{\text{vac}}^{\text{theo}} \geq (10^{11} \text{ eV})^4$ , is very much larger than the vacuum energy density indicated by astronomical observations [12–14],  $\epsilon_{\text{vac}}^{\text{obs}} \approx (10^{-3} \text{ eV})^4$ .

Still, this need not present an insurmountable problem. Having a zero value (or almost zero value) of the relevant vacuum energy density can be the property of a *self-sustained medium*, that is, a medium with a definite macroscopic volume even in the absence of an environment. An example of such a self-sustained medium is a droplet of water falling in empty space.

The observed near-zero value of the cosmological constant compared to a Planck-scale value suggests that the quantum vacuum of our universe belongs to this class of systems, namely, the class of self-sustained media.<sup>1</sup> As any other medium of this kind, the equilibrium vacuum would be homogeneous and extensive. The homogeneity assumption is indeed supported by the observed flatness and smoothness of our universe [12–14]. The implication is that the energy of the equilibrium quantum vacuum would be proportional to the volume considered.

Usually, a self-sustained medium is characterized by an *conserved extensive quantity* whose total value determines the actual volume of the system [36–38]. For this reason, we suggest that the quantum vacuum is *also* characterized by such a variable [39]. The Lorentz invariance of the quantum vacuum imposes, however, strong constraints on the possible form this variable can take. In particular, any global charge such as the fermionic charge  $B - L$  of the standard model must be zero in the Lorentz-invariant vacuum. (In the standard model,  $B$  and  $L$  are the baryon and lepton number, with  $B - L$  conserved and  $B + L$  anomalously violated [40].) The reason for allowing only  $B - L = 0$  in the Lorentz-invariant vacuum is that  $B - L$  is the integrated fourth component of the corresponding 4-current, so that  $B - L$  transforms nontrivially under Lorentz transformations if  $B - L \neq 0$ .

In order to be specific, we choose the vacuum variable to be a symmetric tensor  $q^{\mu\nu}(x)$  satisfying the following conservation law:

$$\nabla_{\mu} q^{\mu\nu}(x) = 0, \quad (1)$$

where  $\nabla_{\mu}$  is the standard covariant derivative defined in terms of the metric  $g_{\mu\nu}(x)$ , its inverse  $g^{\mu\nu}(x)$ , and their derivatives [10,11]. In a homogeneous vacuum, one has  $q^{\mu\nu}(x) = q g^{\mu\nu}(x)$  with  $q$  constant over space and time. This new degree of freedom  $q$  would be responsible for the equilibration of the quantum vacuum: the equilibrium value of  $q$  readjusts if the vacuum is perturbed towards a new equilibrium state (see Sec. II C). The quantum vacuum can now be considered as a reservoir of trans-Planckian energies stored in the  $q$ -field (see Sec. V for further discussion on the possible origin of  $q$ ).

As an elementary *Gedankenexperiment*, take a large portion of quantum vacuum which is supposed to be isolated from its environment. This implies having vanishing

<sup>1</sup>A related suggestion is to consider “empty spacetime” as a type of condensate [3]. Here, however, we require that such a condensate should be self-sustained, i.e., that it should support an equilibrium state even in the absence of an external environment. Conventional Bose–Einstein condensates do not satisfy this requirement, since they exist as equilibrium states only under applied external pressure. It is to be emphasized that the analysis of the present article does not rely on a condensed-matter-physics analogy but aims to be self-contained, being guided only by thermodynamics and Lorentz invariance.

external pressure,  $P = 0$ . For simplicity, consider only two fields: the low-energy effective matter field  $\Psi(x)$  and the vacuum field  $q(x)$ . Specifically, choose the classical field  $\Psi(x)$  to be a complex scalar field. The volume of the isolated portion of quantum vacuum is variable, but its total “charge”  $Q(t) \equiv \int d^3r q(\mathbf{r}, t)$  must be conserved,  $dQ/dt = 0$ . Throughout this article, we neglect the possible spacetime dependence of  $q$  and consider  $q$  to be a genuine thermodynamic variable.

The energy of this portion of quantum vacuum at fixed charge  $Q = qV$  is then given by

$$E = \int d^3r \epsilon(\Psi, q) = \int d^3r \epsilon(\Psi, Q/V). \quad (2)$$

As the volume of the system is a free parameter, the equilibrium state of the system is obtained by variation over both the matter field  $\Psi$  and the volume  $V$ :

$$\frac{\delta E}{\delta \Psi} = 0, \quad \frac{dE}{dV} = 0. \quad (3)$$

The second equation in (3) actually corresponds to the condition of having no external pressure (cf. Sec. II B) and gives

$$P = -\frac{dE}{dV} = -\epsilon(\Psi_0, q) + q \frac{d\epsilon(\Psi_0, q)}{dq} = 0, \quad (4)$$

where  $\Psi_0$  corresponds to the equilibrium value of the classical matter field, which has been taken to be spacetime independent. The solution of (4) determines the equilibrium value  $q = q_0$  and the corresponding volume is given by  $V = V_0 = Q/q_0$ .

Consider, for the moment, the simplest possible case, where the energy density  $\epsilon(\Psi, q)$  splits into two independent parts, the Ginzburg–Landau functional for the macroscopic matter field  $\Psi(x)$  and the energy density of the microscopic vacuum  $q$ -field (both fields taken to be spacetime independent):

$$\epsilon(\Psi, q) = \epsilon_{\text{micro}}(q) + \epsilon_{\text{macro}}(\Psi), \quad (5a)$$

$$\epsilon_{\text{macro}}(\Psi) = \alpha |\Psi|^2 + \beta |\Psi|^4, \quad (5b)$$

for real constants  $\alpha$  and  $\beta$  with  $\beta > 0$ . Now, condition (4) becomes

$$P = -\epsilon_{\text{macro}}(\Psi_0) - \epsilon_{\text{micro}}(q) + q \frac{d\epsilon_{\text{micro}}(q)}{dq} = 0. \quad (6)$$

For  $\alpha > 0$  in (5b), one has a vanishing equilibrium value of the classical matter field,  $\Psi_0 = 0$ . As a result, the corresponding energy density vanishes,  $\epsilon_{\text{macro}}(\Psi_0) = 0$ , and condition (6) becomes

$$P = -\epsilon_{\text{micro}}(q) + q \frac{d\epsilon_{\text{micro}}(q)}{dq} = 0. \quad (7)$$

The solution of this last equation determines the actual equilibrium value  $q = q_0$  of the perfect (nondisturbed)

quantum vacuum. The special case with linear behavior  $\epsilon_{\text{micro}}(q) \propto q$  needs to be discarded, as it does not allow the vacuum to reach an equilibrium (such an energy density would anyway be unbounded from below).

## B. Vacuum under external pressure

If the portion of quantum vacuum considered is not isolated from the environment, the pressure of the vacuum in equilibrium equals the external pressure. Under external pressure  $P$ , the relevant thermodynamic potential (Gibbs free energy) at zero temperature is given by [36–38]

$$W = E + PV = \int d^3r \epsilon(\Psi, Q/V) + PV, \quad (8)$$

with  $V$  the volume of the portion of quantum vacuum considered,  $Q \equiv qV$  the fixed total “charge,” and  $\epsilon(\Psi, q)$  the energy density introduced in (2). The variational equations,

$$\frac{\delta W}{\delta \Psi} = 0, \quad \frac{dW}{dV} = 0, \quad (9)$$

give an integrated form of the Gibbs–Duhem equation:

$$P = -\epsilon(\Psi_0, q) + q \frac{d\epsilon(\Psi_0, q)}{dq}, \quad (10)$$

whose solution determines the equilibrium value  $q = q_0$  for a definite external pressure  $P$  and a fixed “charge”  $Q \equiv qV$ . The Gibbs–Duhem equation [38] in its simplest form,  $Nd\mu = VdP - SdT$ , relates an infinitesimal change  $d\mu$  in the chemical potential  $\mu$  to infinitesimal changes in the other thermodynamic variables of the system, the change  $dP$  in the pressure  $P$  and the change  $dT$  in the temperature  $T$ , with  $N$  the conserved particle number and  $S$  the entropy. Here, the system is kept at  $T = 0$  and the conserved quantity  $Q$  plays the role of the particle number  $N$ , so that  $dE/dQ = d\epsilon/dq$  corresponds to the chemical potential  $\mu$ .

It is seen from (10) that the thermodynamically relevant vacuum energy density is given by

$$\tilde{\epsilon}_{\text{vac}}(\Psi, q) \equiv \epsilon(\Psi, q) - q \frac{d\epsilon(\Psi, q)}{dq}, \quad (11)$$

for equilibrium values  $q = q_0 \equiv q_{\text{vac}}$  and  $\Psi = \Psi_0$ . The numerical values of  $q_0$  and  $\epsilon(\Psi_0, q_0)$  are determined by the detailed microscopic theory, so that  $\epsilon(\Psi_0, q_0)$  can be expected to be of order  $E_{\text{UV}}^4$ , where the “ultraviolet” energy scale  $E_{\text{UV}}$  is, for the moment, taken to be of order  $E_{\text{Planck}}$ . But the vacuum energy density  $\tilde{\epsilon}_{\text{vac}}$  is determined by macroscopic physics, namely, the external pressure  $P$ . It will be argued in Sec. III that it is the macroscopic vacuum energy density  $\tilde{\epsilon}_{\text{vac}}$  which plays the role of the cosmological constant rather than the microscopic energy density  $\epsilon$ .

It is furthermore assumed that the stationary point  $(\Psi_0, q_{\text{vac}})$  of the thermodynamic potential (8) corresponds to a minimum, be it local or global. This implies, in

particular, that

$$\left[ q^2 \frac{d^2 \epsilon(\Psi_0, q)}{dq^2} \right]_{q=q_{\text{vac}}} \geq 0. \quad (12)$$

With the standard definition [38] of the inverse of the isothermal compressibility,  $\chi^{-1} \equiv -VdP/dV$ , and the functional behavior  $P = P(q)$  from (10), it is possible to identify the left-hand side of (12) with the inverse vacuum compressibility and to obtain the following inequality

$$\chi_{\text{vac}}^{-1} = \left[ q^2 \frac{d^2 \epsilon(\Psi_0, q)}{dq^2} \right]_{q=q_{\text{vac}}} \geq 0. \quad (13)$$

From the low-energy point of view, the compressibility of the vacuum  $\chi_{\text{vac}}$  is a fundamental physical constant, just as Newton's gravitational constant  $G$ . The numerical value of  $\chi_{\text{vac}}$  is determined by the detailed microscopic theory and can be expected to be of order  $1/E_{\text{UV}}^4$ . A positive value of the vacuum compressibility is a necessary condition for the stability of the vacuum in the absence of an external environment (cf. Sec. 21 of Ref. [37]). It is, in fact, the stability of the vacuum which leads to the nullification of the cosmological constant, as will be discussed in Sec. III.

### C. Vacuum readjustment

The simple model (5) of Sec. II A allows us to discuss quantitatively the back reaction of the low-energy effective field  $\Psi(x)$  on the underlying microscopic subsystem, as described by the thermodynamic parameter  $q$  and for the case of zero external pressure.

Consider what happens to the equilibrium value  $q_0$  if a cosmological phase transition occurs in the low-energy effective theory, which is, for example, described by the electroweak standard model or quantum chromodynamics. Let us discuss only the simplified case, where the symmetry-breaking phase transition results from having  $\alpha < 0$  in (5b). Then, a nonzero value of  $\Psi_0$  results and the Ginzburg–Landau energy density  $\epsilon_{\text{macro}}(\Psi_0)$  becomes negative. It follows from condition (6) that the parameter  $q_0$  is renormalized to  $q'_0 = q_0 + \delta q_0$  after the phase transition. In words, the parameter value of  $q_0$  is readjusted to the zero value of the pressure in the new equilibrium state of the vacuum.

Specifically, the relative change of parameter  $q_0$  after the transition is given by

$$\frac{\delta q_0}{q_0} = \chi_0 \epsilon_{\text{macro}}(\Psi_0), \quad (14)$$

where the vacuum compressibility  $\chi_0$  is defined by the inverse of the left-hand side of (12) evaluated at the original equilibrium value  $q = q_0$ . From (14) with  $\chi_0 \sim E_{\text{UV}}^{-4}$ , one finds that a small change of the microscopic parameter suffices to compensate the Ginzburg–Landau energy density from a low-energy symmetry-breaking phase transition:

$$|\delta q_0/q_0| \sim \epsilon_{\text{macro}}/E_{\text{UV}}^4. \quad (15)$$

For the case of an electroweak phase transition, the energy density  $\epsilon_{\text{macro}}$  is determined by the energy scale  $E_{\text{ew}} \sim M_W \sim 10^2$  GeV and one has

$$|\delta q_0/q_0| \sim E_{\text{ew}}^4/E_{\text{UV}}^4 \sim 10^{-68}. \quad (16)$$

Result (16) quantifies the smallness of the relative change of the vacuum parameter for any physical process occurring at electroweak energies or lower. But the discussion based on (6) also suggests that the renormalization of the fundamental constants in the standard model from the readjustment of the “deep-vacuum variable”  $q$  to a new low-energy state would be negligibly small compared to mechanisms operating at the low-energy scale itself.

The fine-structure constant  $\alpha$ , for example, is determined both by the ultraviolet energy scale  $E_{\text{UV}}$  and the “infrared” electroweak scale  $E_{\text{ew}}$ . Essentially, the inverse of the fine-structure constant is given by the natural logarithm of the ratio of these two energy scales,

$$1/\alpha \sim \ln(E_{\text{UV}}/E_{\text{ew}}). \quad (17)$$

The relative change in  $\alpha$  due to changes  $\delta E_{\text{ew}} \sim E_{\text{ew}}$  of the infrared physics is of order  $\alpha$ , whereas the relative change in  $\alpha$  due to an adjustment of the deep vacuum is given by

$$\begin{aligned} |\delta \alpha/\alpha| &\sim |\delta E_{\text{UV}}/E_{\text{UV}}| \alpha \sim |\delta q_0/q_0| \alpha \\ &\sim (E_{\text{ew}}^4/E_{\text{UV}}^4) \alpha \sim 10^{-68} \alpha. \end{aligned} \quad (18)$$

Anticipating the discussion of gravity in Sec. III, consider also Newton's constant  $G$  which is mainly determined by the ultraviolet scale of the deep vacuum,

$$G^{-1} \sim E_{\text{UV}}^2 \pm E_{\text{ew}}^2. \quad (19)$$

Even though the second term on the right-hand side of (19) is small compared to the first one, it is precisely this term which is responsible for the main change of  $G$ . In fact, the first term on the right-hand side of (19) gives for the relative change of  $G$  due to a change  $\delta q_0$  in the equilibrium vacuum parameter  $q_0$  the value  $|\delta G/G|^{(1)} \sim |\delta q_0/q_0| \sim E_{\text{ew}}^4/E_{\text{UV}}^4$ , whereas the second term gives for the relative change of  $G$  due to changes  $\delta E_{\text{ew}} \sim E_{\text{ew}}$  from the low-energy physics the larger value  $|\delta G/G|^{(2)} \sim E_{\text{ew}}^2/E_{\text{UV}}^2$ . Hence, the total relative change in  $G$  would be of order

$$|\delta G/G| \sim E_{\text{ew}}^2/E_{\text{UV}}^2 \sim 10^{-34}, \quad (20)$$

which is still a very small number.

All this implies that the thermodynamics of the vacuum variable  $q$  (or, more generally, the dynamics of  $q$ ) does not influence processes occurring in the low-energy world. There is, however, one exception, namely, the cosmological constant which will be discussed in the next section.

### III. NULLIFICATION OF THE COSMOLOGICAL CONSTANT $\Lambda$

#### A. Effective vacuum energy density

In this section, we address the question of which type of vacuum energy density would be gravitating in the context of emergent-gravity models [21–24]. The local gauge invariance and local Lorentz invariance of our current theories (standard model and general relativity) would then be emergent symmetries in the low-energy limit of some unknown fundamental theory. With general covariance (i.e., local Lorentz invariance) in place, the effective low-energy equations of gravitation are the standard Einstein field equations [7,10,11], with Ricci curvature tensor  $R^{\mu\nu}(x)$  sourced by the matter energy-momentum tensor  $T_M^{\mu\nu}(x)$  and the cosmological constant  $\Lambda$ ,

$$\frac{1}{8\pi G} \left( R^{\mu\nu}(x) - \frac{1}{2} g^{\mu\nu}(x) R(x) \right) = -T_M^{\mu\nu}(x) - \Lambda g^{\mu\nu}(x). \quad (21)$$

Taking the signature of the metric  $g_{\mu\nu}(x)$  as  $(+---)$ , the quantity  $\Lambda$  in (21) corresponds to a vacuum energy density [8,9]. As mentioned in the Introduction, the problem is to understand the small but nonzero value of this vacuum energy density (that is, small compared to Planck-scale energy densities).

For the emergent theory envisaged in this article, the quantum field theory (QFT) of the low-energy fields must satisfy two requirements: (1) the resulting QFT must take into account the conservation of the global quantity  $Q$ , and (2) the energy-momentum tensor of the gravitating quantum vacuum must give the standard relativistic relation between energy and momentum from Lorentz invariance.

In fact, the general form of the relativistic energy-momentum tensor of a perfect fluid is given by

$$T_{\text{vac}}^{\mu\nu} = \tilde{\epsilon}_{\text{vac}} u^\mu u^\nu + P_{\text{vac}} (u^\mu u^\nu - g^{\mu\nu}), \quad (22)$$

where  $u^\mu$  is the 4-velocity of the fluid with respect to the spacetime coordinate system chosen. For the Lorentz-invariant quantum vacuum, this tensor must be the same in any coordinate system and cannot depend on the 4-velocity  $u^\mu$ . Hence, the equation of state for the vacuum must be

$$P_{\text{vac}} = -\tilde{\epsilon}_{\text{vac}} \quad (23)$$

and the vacuum energy-momentum tensor (22) becomes

$$T_{\text{vac}}^{\mu\nu} = \tilde{\epsilon}_{\text{vac}} g^{\mu\nu}. \quad (24)$$

It is then entirely appropriate to speak of a ‘‘Lorentz-invariant quantum vacuum,’’ as a suitable coordinate transformation brings (24) to the form  $\tilde{\epsilon}_{\text{vac}} \eta^{\mu\nu}$  in terms of the standard inverse Minkowski metric  $\eta^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ .

In order to construct the proper low-energy QFT satisfying the two requirements mentioned above, we introduce

the Hamiltonian operator corresponding to the simple classical model (5):

$$\mathcal{H}(\hat{\Psi}, q) = E_{\text{micro}}(q) + \mathcal{H}_{\text{macro}}(\hat{\Psi}), \quad (25)$$

where  $\hat{\Psi}(x)$  is now a quantum field and  $q$  at low energies can be considered as a constant classical parameter (its quantization will probably be required in the definitive theory and possibly also its spacetime dependence). The relevant low-energy Hamiltonian, which takes  $Q$ -conservation into account, is then represented by the quantum counterpart of (11):

$$\tilde{\mathcal{H}} = \mathcal{H} - q \frac{d\mathcal{H}}{dq}. \quad (26)$$

The vacuum energy experienced by the low-energy degrees of freedom is the vacuum expectation value of this Hamiltonian:

$$\begin{aligned} \langle \text{vac} | \tilde{\mathcal{H}} | \text{vac} \rangle &= \langle \text{vac} | \mathcal{H} | \text{vac} \rangle - q \frac{d}{dq} \langle \text{vac} | \mathcal{H} | \text{vac} \rangle \\ &= V \epsilon_{\text{vac}} - V q \frac{d\epsilon_{\text{vac}}(q)}{dq}. \end{aligned} \quad (27)$$

The corresponding vacuum energy density is found to be given by

$$\tilde{\epsilon}_{\text{vac}} = \epsilon_{\text{vac}} - q \frac{d\epsilon_{\text{vac}}}{dq} = -P_{\text{vac}}, \quad (28)$$

where the last equality follows from the Gibbs–Duhem relation (10) by equating the external pressure  $P$  with the internal pressure  $P_{\text{vac}}$ . The energy density and pressure (28) obtained by a thermodynamic argument are, indeed, seen to satisfy condition (23) from Lorentz-invariance.

The Hamiltonian for the low-frequency excitations is built upon the ground state of (26) and reads in quadratic approximation:

$$\begin{aligned} \tilde{\mathcal{H}} &= \langle \text{vac} | \tilde{\mathcal{H}} | \text{vac} \rangle + \sum_{n,\mathbf{p}} E_{\mathbf{p}}^{(n)} a_{\mathbf{p}}^{(n)\dagger} a_{\mathbf{p}}^{(n)} + \dots \\ &= \int d^3 r \tilde{\epsilon}_{\text{vac}} + \sum_{n,\mathbf{p}} E_{\mathbf{p}}^{(n)} a_{\mathbf{p}}^{(n)\dagger} a_{\mathbf{p}}^{(n)} + \dots, \end{aligned} \quad (29)$$

with a sum over quasiparticle type  $n$  and energy  $E_{\mathbf{p}}^{(n)} \equiv \sqrt{|\mathbf{p}|^2 + m_n^2}$  for a quasiparticle of momentum  $\mathbf{p}$  and effective mass  $m_n$ . Note that the quadratic term in (29) includes the two polarization modes of the massless graviton.

It is then the energy density  $\tilde{\epsilon}_{\text{vac}}$  from the vacuum expectation value of the low-energy Hamiltonian (26), and not  $\epsilon_{\text{vac}}$  from (25), which gravitates. Correspondingly, only the excitations above  $\tilde{\epsilon}_{\text{vac}}$  gravitate, as indicated by (29). The reason is that it is only  $\tilde{\epsilon}_{\text{vac}}$  which equals the negative of the pressure of the vacuum. Hence,  $\tilde{\epsilon}_{\text{vac}}$  is the vacuum energy density which corresponds to the cosmological constant  $\Lambda$  in the gravitational field equations (21):

$$\Lambda = \tilde{\epsilon}_{\text{vac}}(q_0) = -P_{\text{vac}}(q_0), \quad (30)$$

for an equilibrium value  $q_0$  of the vacuum parameter  $q$  (the numerical value of  $q_0$  needs to be determined by a further condition such as pressure equilibrium in Sec. III B). Conceptually, the identification of the cosmological constant  $\Lambda$  with the self-tuned vacuum energy density  $\tilde{\epsilon}_{\text{vac}}(q_0)$  may be one of the most interesting results of the present article.

The following two remarks elaborate on the reasoning of the previous paragraphs. The first remark concerns precisely the emergence of the effective cosmological constant (30) in our approach to the problem. In this article, we have discussed certain properties of a Lorentz-invariant self-sustained quantum vacuum in which gravity (fundamental or emergent) is already supposed to exist and have obtained the low-energy Hamiltonian (26) which is consistent with this kind of vacuum. Pressure in the effective theory must coincide with pressure in the microscopic theory and the same holds for temperature. The thermodynamic quantities  $P$  and  $T$  are, therefore, invariant quantities, that is, they do not depend on the detailed theory but are determined by the environment. In the low-energy effective theory with the single remaining microscopic variable  $q$ , the equation of state of the vacuum is given by  $P = -\langle \text{vac} | \mathcal{H}_{\text{eff}} | \text{vac} \rangle$ , and this particular equation of state is certainly consistent with gravity, i.e., with having a  $\Lambda$  term in (21). In the fully microscopic theory with variable  $q$  included, the Gibbs–Duhem relation (10) reads  $P = -\epsilon_{\text{micro}}(q) + q(d\epsilon_{\text{micro}}/dq)$ . Since these two pressures must be equal (that is, the one from the microscopic theory and the one from the effective theory), we must choose for the relevant effective Hamiltonian  $\mathcal{H}_{\text{eff}}$  of the low-energy theory with gravity the Hamiltonian  $\tilde{\mathcal{H}}$  from (26) and not  $\mathcal{H}$  from (25). It is, therefore, the vacuum expectation value of  $\tilde{\mathcal{H}}$  which gravitates and not the vacuum expectation value of  $\mathcal{H}$ . However, it must be admitted that the argument just given remains heuristic in the absence of the detailed microscopic theory. Still, a dynamic origin of  $\tilde{\epsilon}_{\text{vac}}(q)$  is, in principle, possible, as will be shown by the examples of Secs. V C and V D.

The second remark concerns the way how emergent gravity couples to the low-energy degrees of freedom. The crucial observation, here, is that emergent (or fundamental) Lorentz gauge invariance is known [41] to require the equivalence principle for consistency. The equivalence principle, in turn, underlies Einstein’s theory of gravitation, whose field equations (21) were already used in the first paragraph of this subsection. See also Sec. 10.8 of Ref. [10] and Sec. 13 of Ref. [11] for a general discussion of the connection between local Lorentz gauge invariance and the equivalence principle.

## B. Nullification of $\Lambda$ in a perfect quantum vacuum

For the perfectly homogenous equilibrium quantum vacuum, one has in the absence of an environment (i.e.,

in the absence of external pressure  $P$ ) a vanishing cosmological constant from (30):

$$\Lambda = \tilde{\epsilon}_{\text{vac}}(q_0) = -P_{\text{vac}}(q_0) = -P = 0. \quad (31)$$

Observe that the vanishing of the effective vacuum energy density  $\tilde{\epsilon}_{\text{vac}}$  for  $q = q_0$  results from the cancellation of the two terms in the middle part of (28), each of which are typically of order  $E_{\text{UV}}^4$ . Numerically, the nullification of  $\tilde{\epsilon}_{\text{vac}}$  (and, thereby, of  $\Lambda$ ) may be one of the most important results of the present article and a concrete example will be given in the last paragraph of Sec. V C.

This zero value for the effective energy density of the isolated quantum vacuum is consistent with the Lorentz invariance of a perfect vacuum. Indeed, the mixed terms  $T_0^i$  in (24) vanish for the inverse Minkowski metric  $\eta^{\mu\nu}$  and the 3-momentum of the vacuum is zero,  $\mathbf{P} = 0$ . The relativistic relation [26] between the system velocity  $\mathbf{v}$ , momentum  $\mathbf{P}$ , and energy  $\tilde{E}$  is given by (temporarily reinstating  $c$ )

$$c\mathbf{P} = \tilde{E} \frac{\mathbf{v}}{c}. \quad (32)$$

Having  $\mathbf{P} = 0$  then requires that the vacuum energy also vanishes,  $\tilde{E} \equiv \int d^3r \tilde{\epsilon}_{\text{vac}} = 0$ . Physically, this result can be understood as follows. If  $\tilde{E}$  would be nonzero, it would transform under a Lorentz boost and produce a nonzero momentum of the vacuum. This would imply the existence of a preferred reference frame in the vacuum, disagreeing with experimental fact [26]. But this disagreement does not arise if  $\tilde{E}$  vanishes, as the perfect equilibrium quantum vacuum with  $|\mathbf{P}| = \tilde{E} = 0$  does not have a preferred frame.

A preferred frame does appear if the quantum vacuum is perturbed by external pressure. The experimentally observed Lorentz invariance (cf. Refs. [30,31] and references therein) suggests that there is no external pressure and that the vacuum energy density must be zero. Lorentz invariance may still be violated by the existence of spacetime boundaries or the presence of matter, both of which introduce preferred reference frames. (With matter present, the preferred reference frame is the one in which the matter is, on average, at rest.) In all these cases, the vacuum energy density becomes nonzero but small compared to a Planck-scale energy density, as will be discussed further in the next section.

The violation of Lorentz invariance by the presence of matter, spacetime boundaries, or external pressure does not mean that Lorentz invariance, as a *law of physics*, is violated. Instead, it is the *state* of the universe which loses the property of Lorentz invariance in the presence of matter and/or external environment, precisely because of the appearance of a preferred frame. Throughout this article, we assume that Lorentz invariance is either a fundamental physical law or a symmetry which can only be violated at an energy scale  $E_{\text{LV}}$  far above the Planck energy scale (cf. Ref. [32]).

## IV. NONZERO VACUUM ENERGY DENSITY FROM PERTURBATIONS

### A. Preliminary remarks

In Sec. II C, we have discussed the response of the “deep vacuum” to a homogeneous change of the matter ground state by a low-energy phase transition, which is, for simplicity, described in terms of the Ginzburg–Landau functional of a single matter field. In this section, we consider the response of the deep vacuum to perturbations which themselves violate Lorentz invariance. The outstanding problem is to see how this response can be consistent with Lorentz invariance as a physical law. In Sec. II B, we have already discussed this issue by using the example of an external pressure which, indeed, violates Lorentz invariance. But external pressure is, by definition, not accessible to us and, here, we wish to consider other physically more relevant perturbations of the perfect quantum vacuum.

Two types of Lorentz-noninvariant perturbations will be discussed in this section, the presence of matter (e.g., homogeneous thermal matter) and the possibility of having a nonuniform setup (e.g., a droplet with a different content than the ambient space). In both cases, we will investigate how and under which conditions a Lorentz-invariant (relativistic) relation between energy and momentum arises.

### B. Vacuum in the presence of matter

As a first *Gedankenexperiment*, consider thermal matter enclosed in a box. This box is taken to have rigid walls which are impenetrable for the matter contained in the box and the box as a whole is considered to be moving in an empty spacetime (perfect quantum vacuum) with the Minkowski metric in standard coordinates. Furthermore, assume that the box is, on the one hand, sufficiently small so as to ignore its gravitational effects and, on the other hand, sufficiently large in size so as to ignore the energy of the walls of the box compared to the energy of the matter inside the box.

Let  $v^\mu$  be the 4-velocity of the box. The energy-momentum tensor of the matter inside the box is

$$T_M^{\mu\nu} = \rho_M v^\mu v^\nu + P_M (v^\mu v^\nu - g^{\mu\nu}), \quad (33)$$

where  $\rho_M$  and  $P_M$  are the energy density and pressure of matter in the frame moving along with the box. A naive consideration would suggest that the energy  $E$  and momentum  $P_i$  of the box with matter are given by integration of, respectively,  $T_M^{00}$  and  $T_M^{0i}$  over the box volume. But this cannot be correct, since the resulting energy  $E$  and momentum  $P_i$  would not satisfy the relativistic relation (32). In fact, one must also take into account the work needed to compress the matter into the box. For the correct energy-momentum tensor component  $T^{00}$ , one must therefore use  $\rho_M + P_M$  instead of  $\rho_M$  in (33).

However, there is a physical way to avoid wall pressure, namely, by having a phase boundary (interface) separating two states which are in equilibrium with each other. Specifically, consider having a perfect quantum vacuum outside the interface and a vacuum with matter inside the interface. Then, the outer vacuum has zero pressure and the partial pressure of the inner vacuum compensates the partial pressure of matter, as long as the surface tension can be neglected. The pressure is now zero both inside and outside of the box, so that an area element of the box wall (interface) experiences no net force.

The total energy-momentum tensor of the system inside the box then has two contributions:

$$\begin{aligned} T^{\mu\nu} &\equiv T_M^{\mu\nu} + T_{\text{vac}}^{\mu\nu} \\ &= (\rho_M + \tilde{\epsilon}_{\text{vac}}) v^\mu v^\nu + (P_M + P_{\text{vac}})(v^\mu v^\nu - g^{\mu\nu}). \end{aligned} \quad (34)$$

If the vacuum pressure compensates the matter pressure,  $P_M + P_{\text{vac}} = 0$ , one obtains

$$T^{\mu\nu} = (\rho_M + \tilde{\epsilon}_{\text{vac}}) v^\mu v^\nu, \quad (35)$$

with

$$\tilde{\epsilon}_{\text{vac}} = -P_{\text{vac}} = P_M. \quad (36)$$

The integration of  $T^{00}$  and  $T^{0i}$  over the box volume now gives the correct relativistic relation (32) between the energy and momentum of the box. It is, therefore, a genuine relativistic object whose rest mass is given by  $(\rho_M + \tilde{\epsilon}_{\text{vac}})V_0$ , where  $V_0$  is the volume of the box in a comoving frame. This effective rest mass is, in fact, the sum of matter energy and vacuum energy. But, as mentioned above, the box must be relatively small so as to ignore all gravitational effects.

It is also possible to have an interface between *vacua* with different energy densities, namely, by considering a cosmic domain wall separating two different types of vacua (for example, “false” and “true” vacua, as will be explained later on). Such an interface has a large energy itself and we shall take this surface energy into account in Sec. IV D.

### C. Universe with nongravitating matter

As a further *Gedankenexperiment*, expand the box wall of the previous subsection to a shell enclosing the entire visible universe, with the shell corresponding to what might be called the “boundary” of the universe. This droplet universe [42,43], containing both vacuum and matter, would again be moving in empty spacetime (perfect quantum vacuum) with the Minkowski metric in standard coordinates.

In this article, we are primarily interested in the properties of the quantum vacuum which are governed by Lorentz invariance. The presence of gravitating matter, however, leads to a deviation of the metric from the Minkowski

metric and Lorentz invariance loses its meaning. That is why, here, we will only consider the hypothetical case of a universe filled with “nongravitating” matter [42], where Newton’s constant  $G$  is effectively set to zero. In such a hypothetical universe, the metric  $g_{\mu\nu}$  remains constant and takes the Minkowski form in appropriate coordinates.

In a general coordinate frame, the energy and momentum density of matter are given by [10,26]

$$\rho_M(v) = \frac{\rho_M + (v^2/c^2)P_M}{1 - v^2/c^2}, \quad (37a)$$

$$\mathbf{p}_M(v) = \frac{\mathbf{v}}{c^2} \frac{\rho_M + P_M}{1 - v^2/c^2}, \quad (37b)$$

with  $c$  reinstated for clarity. One obvious consequence of (37) is that energy and momentum of matter do not satisfy the relativistic relation (32). As mentioned in Sec. IV B, the reason for this has to do with the presence of external forces acting on matter, which violate Lorentz invariance (these forces establish a preferred reference frame in which they are isotropic). The external forces manifest themselves in (37) through the pressure  $P_M$  of the matter, which is supported by the external pressure  $P$  (see, e.g., Part IV of Ref. [26]). If the universe would be completely isolated from the “environment,” the external pressure would be absent,  $P = P_M = 0$ , and the Lorentz-invariant equation (32) would be restored.

But the typical matter considered in realistic cosmological models [10], such as a relativistic plasma, cannot exist as an equilibrium state at zero pressure (except for the case of cold dust with  $P_M = 0$ ). According to special relativity, the equilibrium universe would appear to be necessarily empty of matter. However, in this consideration, we have not yet taken into account the contribution of the quantum vacuum with equation of state (23).

The equilibrium state of the system (quantum vacuum + nongravitating matter) is achieved when the partial pressure of matter is compensated by the partial pressure of the vacuum. In that case, the required external pressure  $P$  can be zero,

$$P = P_{\text{vac}} + P_M = 0, \quad (38)$$

and the system can be in equilibrium without external environment. For this equilibrium universe, the second term in (34) disappears and the relativistic relation (32) between the energy and momentum of the whole universe (droplet) is restored:

$$T^{\mu\nu} \equiv T_M^{\mu\nu} + T_{\text{vac}}^{\mu\nu} = (\rho_M + \tilde{\epsilon}_{\text{vac}})v^\mu v^\nu. \quad (39)$$

Using the equation of state (23) and the equilibrium condition (38), one obtains the effective vacuum energy density induced by nongravitating matter with pressure  $P_M$ :

$$\tilde{\epsilon}_{\text{vac}} = -P_{\text{vac}} = P_M, \quad (40)$$

which is of the same form as (36) but now for a droplet

universe with fictitious (nongravitating) matter. Since the vacuum momentum density is zero,

$$\mathbf{p}_{\text{vac}} = 0, \quad (41)$$

the total energy and momentum densities of the system (quantum vacuum + nongravitating matter) become

$$\rho_{\text{total}} = \rho_M(v) + \tilde{\epsilon}_{\text{vac}} = \frac{\rho_M + \tilde{\epsilon}_{\text{vac}}}{1 - v^2/c^2}, \quad (42a)$$

$$\mathbf{p}_{\text{total}} = \mathbf{p}_M(v) + \mathbf{p}_{\text{vac}} = \frac{\mathbf{v}}{c^2} \frac{\rho_M + \tilde{\epsilon}_{\text{vac}}}{1 - v^2/c^2}, \quad (42b)$$

where (37) and (40) have been used. These quantities are seen to satisfy the relativistic equation (32) and the corresponding rest-mass density  $\mu_{\text{rest}}$  is given by

$$\rho_{\text{total}} = \frac{\mu_{\text{rest}}c^2}{\sqrt{1 - v^2/c^2}}, \quad (43a)$$

$$\mu_{\text{rest}} = \frac{\rho_M + \tilde{\epsilon}_{\text{vac}}}{c^2\sqrt{1 - v^2/c^2}}, \quad (43b)$$

where  $c$  is again shown explicitly.

For the calculation of the total rest mass of the droplet, the extra factor  $\sqrt{1 - v^2/c^2}$  in the denominator of the rest-mass density  $\mu_{\text{rest}}$  is canceled by the relativistic transformation of the volume. Indeed, the volume element  $dV$  in the frame of measurement and the volume element  $dV_0$  in the comoving frame are related by  $dV = dV_0\sqrt{1 - v^2/c^2}$ , so that the total rest energy of the system is given by

$$\begin{aligned} M_{\text{rest}}c^2 &= \int dV \mu_{\text{rest}}c^2 = \int dV \frac{\rho_M + \tilde{\epsilon}_{\text{vac}}}{\sqrt{1 - v^2/c^2}} \\ &= (\rho_M + \tilde{\epsilon}_{\text{vac}})V_0. \end{aligned} \quad (44)$$

Even though each of the two subsystems (vacuum or matter) does not separately satisfy the relativistic relation (32) between energy and momentum, the whole droplet universe (vacuum + matter) does represent a genuine relativistic object whose effective rest mass is the sum of the energies of matter and quantum vacuum. This universe has a preferred reference frame which is precisely the rest frame of the matter. The state of the universe is no longer Lorentz invariant and this allows for having a nonzero value of the vacuum energy, as given by (40).

If the matter pressure vanishes, the connection between vacuum and preferred reference frame of matter is lost and the zero value of the Lorentz-invariant vacuum energy restored. Still, the general result (40) is of considerable interest as it relates a relatively small (i.e., sub-Planckian), and positive, effective vacuum energy density to the presence of thermal matter, even though the matter considered here was “nongravitating.” The approach of the present article (suitably extended to genuine gravitating matter) may provide a new point of view on the so-called “cosmic-coincidence” puzzle of current cosmological models [17,44].

### D. Droplet of false vacuum

The final *Gedankenexperiment* of this section takes into account possible surface-tension effects of the droplet universe. We consider a spherical droplet of false vacuum moving in true vacuum. We assume that the external true vacuum is in equilibrium, so that, with zero external pressure, its pressure and energy density vanish. The vacuum inside the droplet is taken to be “false,” meaning that it corresponds to a local minimum of the Ginzburg–Landau potential which is higher than the global minimum of the “true” vacuum. The false vacuum has, therefore, a nonzero positive energy density. (If the false vacuum were on the outside, it would have vanishing pressure and energy density due to the zero external pressure, whereas the true vacuum on the inside would have negative energy density.) We also assume that matter is massive in the true vacuum but massless in the false vacuum, so that this type of matter cannot pass through the interface. Hence, the matter is trapped inside the droplet.

Now, the interface between the vacua participates in the dynamics, along with the trapped matter and vacuum energy. This interface can be represented by fictitious matter which is distributed homogeneously over the interior of the droplet and has the following equation of state:

$$P_\sigma = -\frac{2}{3}\rho_\sigma = -\frac{2\sigma}{R}, \quad (45)$$

where  $\sigma$  is the surface tension and  $R$  the radius of the spherical droplet. Expression (45) follows from the free energy  $F = A\sigma$  and the identification  $\rho_\sigma \equiv F/V$  for a spherical volume  $V = (4\pi/3)R^3$  with surface area  $A = 4\pi R^2$ ; cf. Ref. [36]. As the interface with nonzero (positive) surface tension prefers to shrink, this fictitious matter has negative pressure.

If  $\mathbf{v}$  is the 3-velocity of the droplet with matter energy and momentum densities given by (37), the total droplet energy and momentum per unit volume are given by

$$\rho_{\text{total}} = \frac{\rho_\sigma + (v^2/c^2)P_\sigma + \rho_M + (v^2/c^2)P_M}{1 - v^2/c^2} + \tilde{\epsilon}_{\text{vac}}, \quad (46a)$$

$$\mathbf{p}_{\text{total}} = \frac{\mathbf{v}}{c^2} \frac{\rho_\sigma + P_\sigma + \rho_M + P_M}{1 - v^2/c^2}, \quad (46b)$$

with positive energy density  $\tilde{\epsilon}_{\text{vac}}$  of the interior (false) vacuum and vanishing vacuum momentum density according to (41). In equilibrium, one has pressure balance, so that [43]

$$\tilde{\epsilon}_{\text{vac}} = -P_{\text{vac}} = P_\sigma + P_M. \quad (47)$$

Observe that, in the absence of a false vacuum ( $\tilde{\epsilon}_{\text{vac}} = 0$ ), the matter pressure  $P_M$  is given by the Laplace pressure  $2\sigma/R$  of the droplet [38].

Inserting expression (47) for  $P_\sigma + P_M$  into (46), one obtains that the whole object (nongravitating matter + false vacuum + interface) obeys the relativistic relation

(32) between energy and momentum:

$$\mathbf{p}_{\text{total}} = \rho_{\text{total}} \frac{\mathbf{v}}{c^2} = \frac{\rho_\sigma + \rho_M + \tilde{\epsilon}_{\text{vac}}}{1 - v^2/c^2} \frac{\mathbf{v}}{c^2}, \quad (48)$$

whereas the subsystems do not separately obey the relativistic relation. Result (48) provides a concrete example of some of the ingredients which may enter a realistic description of the vacuum energy density indicated by recent astronomical results [12–14].

## V. NATURE AND ORIGIN OF THE VACUUM VARIABLE $q$

### A. Examples of spacetime-independent variables

In view of its origin, the proposed parameter  $q$  of the Lorentz-invariant self-sustained quantum vacuum may be strictly spacetime independent. There are known physical systems where such constant variables occur as extra degrees of freedom. In the thermodynamic limit, one then has to minimize over these variables, which leads to the phenomenon of self-tuning [45].

Let us give three examples of these constant variables. First, there is the homogeneous deformation  $u_m(\mathbf{x}) = u_{mn}x_n$  in solids, where all six components of the strain tensor  $u_{mn}$  are independent, even though, for the nonuniform case (momentum  $\mathbf{k} \neq 0$ ), only three components of the displacement field  $u_m$  determine the strain tensor by  $u_{mn} \sim ik_m u_n + ik_n u_m$ . There is thus a jump in the number of degrees of freedom at  $\mathbf{k} = 0$  compared to  $\mathbf{k} \neq 0$ . The macroscopic elastic mode with  $\mathbf{k} = 0$  plays, in fact, a crucial role in the magnetic phase transitions of crystals: the coupling of this mode to the order parameter transforms the second-order phase transition to a first-order one (the Larkin–Pikin effect [46]).

Second, there can, in principle, exist constant electric and magnetic fields *in vacuo*, with linear gauge potentials  $A_\mu(x) = -\frac{1}{2}F_{\mu\nu}x^\nu$  for constant  $F_{\mu\nu} = -F_{\nu\mu}$ . These constant electric and magnetic fields would be mutually independent, in contrast to the spacetime-dependent fields connected by the Maxwell equations.

Third, there exist so-called “discrete states” in certain string theories, which correspond to homogeneous degrees of freedom of the system [47].

### B. Possible dynamic origin of $q$ : General considerations

In Sec. II A, the basic vacuum variable  $q^{\mu\nu}$ , with vanishing covariant divergence (1), was simply postulated. But what kind of variable can this be? One possible scenario is the following.

In our approach, we have focused on only one thermodynamic degree of freedom out of the large number of degrees of freedom contained in the quantum vacuum. At the microscopic level, all degrees of freedom (including the low-energy effective ones) are described by some type of underlying QFT, which can be a continuous or discrete

theory. This microscopic QFT has its own microscopic energy-momentum tensor  $(t_{\text{micro}})^{\mu\nu}$ , which, in principle, has no direct relation to the energy-momentum tensor of the low-energy effective theory. For this reason, the tensor  $q^{\mu\nu}$  may very well correspond to the vacuum expectation value of the energy-momentum tensor of the microscopic quantum fields:

$$q^{\mu\nu} = \langle \text{vac} | (t_{\text{micro}})^{\mu\nu} | \text{vac} \rangle = q g^{\mu\nu}, \quad (49a)$$

$$q = \frac{1}{4} \langle \text{vac} | (t_{\text{micro}})^\mu_\mu | \text{vac} \rangle. \quad (49b)$$

The energy-momentum conservation law is assumed to be exact at the microscopic level, so that  $q^{\mu\nu}$ , defined by (49a) with constant  $q$ , automatically satisfies conservation law (1). However, the energy-momentum conservation law for the emergent effective quantum fields is only approximate, as exchange of energy and momentum is possible between matter and quantum vacuum. In other words, energy-momentum can be exchanged between the low-energy degrees of freedom visible to the ‘‘poor physicist’’ of Ref. [23] and the high-energy degrees of freedom from the deep vacuum.

In this article, we have considered the low-energy effective theory supplemented by a single thermodynamic parameter  $q$ , and calculated in Sec. II C the backreaction of the low-energy effective fields on this parameter  $q$ , after they have reached a new equilibrium. The challenge is to describe the complete process of equilibration, which allows us to discuss the dynamics of  $q$  and, hence, the dynamics of the ‘‘cosmological constant’’ [48]; see Sec. VI for further remarks.

According to (49), the vacuum variable  $q$  would be determined by the ultraviolet cut-off,  $q \sim E_{\text{UV}}^4$ , where the ultraviolet energy scale  $E_{\text{UV}}$  can be approximately equal to the energy scale  $E_{\text{Planck}}$  defined in Sec. II A or a Lorentz-violating energy scale  $E_{\text{LV}} \gg E_{\text{Planck}}$  [32]. These energy scales are, in fact, the natural energy scales for any quantity describing the quantum vacuum, but not for thermodynamic variables (such as pressure and temperature) which are determined by external conditions and do not depend on the details of the microscopic physics.

### C. Possible dynamic origin of $q$ : Four-form field strength

A specific suggestion for a possible dynamic origin of the vacuum variable  $q$  starts from a rank-three antisymmetric tensor gauge potential  $A_{\mu\nu\rho}$ , with field strength [49,50]

$$F_{\mu\nu\rho\sigma} \equiv \nabla_{[\sigma} A_{\mu\nu\rho]}, \quad (50)$$

where the square brackets around the spacetime indices denote total antisymmetrization. This gauge potential  $A_{\mu\nu\rho}$  may correspond to a microscopic degree of freedom from

the quantum vacuum, as discussed in the previous subsection.

The classical action of this four-form field strength coupled to gravity is given by

$$S = S_{\text{grav}} + S_{\text{M}} = \int_{\mathbb{R}^4} d^4x \sqrt{-g} \left( \frac{R}{16\pi G} - \epsilon(F) \right), \quad (51)$$

with  $R$  the Ricci scalar of the pure gravity action  $S_{\text{grav}}$  and  $\epsilon(F)$  a scalar function entering the matter action  $S_{\text{M}}$  with the following definition for the square of its argument:

$$F^2 \equiv -\frac{1}{24} F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma}, \quad (52)$$

where the right-hand side contains four factors of the inverse metric  $g^{\kappa\lambda}$  to convert one covariant tensor (50) into a contravariant tensor. A quadratic function  $\epsilon(F) = \frac{1}{2} F^2$  for the matter part of action (51) is often used in the literature (see, e.g., Refs. [49–51]), but, here, we keep the even function  $\epsilon(F)$  arbitrary. In general, this function  $\epsilon(F)$  involves an energy scale  $E_{\text{UV}}$  and one has explicitly  $\epsilon(F) = \hat{\epsilon}(F/E_{\text{UV}}^2) E_{\text{UV}}^4$  in terms of a dimensionless even function  $\hat{\epsilon}(f)$  of a dimensionless variable  $f$ .

The generalized Maxwell equations from (51) read

$$\nabla^\mu \left( \frac{1}{F} \frac{d\epsilon(F)}{dF} F_{\mu\nu\rho\sigma} \right) = 0. \quad (53)$$

For a flat spacetime and a Lorentzian metric signature  $(+ - - -)$ , these equations have the following solution (indicated by a bar):

$$\bar{F}_{\mu\nu\rho\sigma} = F e_{\mu\nu\rho\sigma}, \quad F = q, \quad (54)$$

with a constant  $q$  (given explicitly by  $q = \hat{q} E_{\text{UV}}^2$ , in terms of a dimensionless number  $\hat{q}$ ) and the totally antisymmetric Levi-Civita symbol  $e_{\mu\nu\rho\sigma}$  in a slightly unusual notation (i.e., having the Latin letter ‘‘e’’ instead of the Greek letter ‘‘e’’ which is reserved for the energy density). For the special case of a quadratic function  $\epsilon(F)$ , this solution has already been found by the authors of Refs. [49,50].

Solution (54) gives a constant (i.e., spacetime-independent) value  $\epsilon(q)$  for the energy density which enters action (51). However, the energy-momentum tensor obtained by variation over  $g^{\mu\nu}$  in the matter part of action (51) is given by

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{M}}}{\delta g^{\mu\nu}} = \epsilon(F) g_{\mu\nu} + \frac{1}{6F} \frac{d\epsilon(F)}{dF} F_{\mu\alpha\beta\gamma} F_{\nu}^{\alpha\beta\gamma}, \quad (55)$$

and the following expression results when evaluated for the particular solution (54):

$$\begin{aligned}\bar{T}_{\mu\nu} &= T_{\mu\nu}|_{F_{\mu\nu\rho\sigma}=\bar{F}_{\mu\nu\rho\sigma}, \bar{g}_{\mu\nu}=\bar{g}_{\mu\nu}} \\ &= \bar{g}_{\mu\nu} \left[ \epsilon(F) - F \frac{d\epsilon(F)}{dF} \right]_{F=q},\end{aligned}\quad (56)$$

where  $\bar{g}_{\mu\nu}$  stands for the flat spacetime metric. Result (56) corresponds to having the following cosmological constant in the gravitational field equation (21):

$$\Lambda = \epsilon(q) - q \frac{d\epsilon}{dq} \equiv \tilde{\epsilon}_{\text{vac}}(q), \quad (57)$$

which has precisely the form discussed in Sec. III A (see, e.g., Ref. [52] for a related but different discussion of  $\Lambda$  and four-form fields).

Two remarks are in order. First, the transmutation of the energy density  $\epsilon(q)$  of the action to the effective vacuum energy density  $\tilde{\epsilon}_{\text{vac}}(q)$  of the gravitational field equations occurs naturally. The reason is that the quantity  $d\epsilon/dF$  is constant according to (53) and (54), so that  $d\epsilon/dF$  indeed plays the role of a chemical potential; see the discussion on the Gibbs–Duhem equation in Sec. II B. For a quadratic function  $\epsilon(q)$ , the sign of  $\tilde{\epsilon}_{\text{vac}}(q)$  from (57) is simply the opposite of  $\epsilon(q)$ , as noted already in Ref. [52(b)].

Second, it is possible to have a nonzero equilibrium value  $q_0$  in the vacuum at vanishing external pressure,  $P = 0$ , for appropriate functions  $\epsilon(F)$  in the original action (51). The equilibrium value  $q_0$  is determined by condition (7), which corresponds to the nullification of cosmological constant (57). For example, the function  $\epsilon(F) = \epsilon_b + \frac{1}{2}\sin^2 F$ , with the energy scale  $E_{\text{UV}}$  set to unity and the “bare” cosmological constant  $\epsilon_b$  set to the arbitrary numerical value  $1/5$ , gives a possible equilibrium value  $q_0 \approx 3.20479$  [from the solution of the zero-pressure Gibbs–Duhem relation  $(1/5) + (1/2)\sin^2 q - q \sin q \cos q = 0$ ] and the corresponding inverse vacuum compressibility  $\chi_0^{-1} = q_0^2 \cos 2q_0 \approx 10.1887$ , which satisfies the vacuum stability condition (13).

#### D. Possible spacetime origin of $q$ : Aether velocity field

Another possible origin of  $q$  may be through a four-vector field  $u^\mu(x)$ . This vector field could be the four-dimensional analog of the concept of shift in the deformation theory of crystals. (Deformation theory can be described in terms of a metric field, with the role of torsion and curvature fields played by dislocations and disclinations, respectively; see, e.g., Ref. [53] for a review.) However, a better realization of  $u^\mu$  would be as a 4-velocity field entering the description of the structure of spacetime. In this case, one obtains a variant of the Einstein–aether theory discussed by Jacobson [54], in which the timelike vector field is constrained to have unit norm  $u^\mu u_\mu = 1$ , or the more general vector-tensor gravity theories studied by Will and Nordvedt [55] (see also Refs. [56,57]).

Here, we do not impose a constraint on the magnitude of  $u^\mu$  and assume that the action does not depend on  $u^\mu$  explicitly but only depends on its covariant derivatives  $\nabla_\nu u^\mu \equiv u^\mu_{,\nu}$ . This last assumption reflects the postulated Lorentz invariance of the quantum vacuum: the vacuum does not depend on the choice of the inertial reference frame in which one has constant  $u^\mu_\nu$  (see below). Furthermore, we assume that the action contains higher order terms, which allow for a nonzero value of  $u^\mu_\nu$  in the equilibrium vacuum. Specifically, the action is taken to have the following form:

$$S = S_{\text{grav}} + S_{\text{vel}} = \int_{\mathbb{R}^4} d^4x \sqrt{-g} \left( \frac{R}{16\pi G} - \epsilon(u^\mu_\nu) \right), \quad (58)$$

with an energy density containing even powers of  $u^\mu_\nu \equiv \nabla_\nu u^\mu$ :

$$\epsilon(u^\mu_\nu) = K + K_{\mu\nu}^{\alpha\beta} u^\mu_\alpha u^\nu_\beta + K_{\mu\nu\rho\sigma}^{\alpha\beta\gamma\delta} u^\mu_\alpha u^\nu_\beta u^\rho_\gamma u^\sigma_\delta + \dots, \quad (59)$$

where the zeroth order term  $K$  corresponds to a “bare” cosmological constant. According to the imposed conditions, the tensors  $K_{\mu\nu}^{\alpha\beta}$  and  $K_{\mu\nu\rho\sigma}^{\alpha\beta\gamma\delta}$  depend only on  $g_{\mu\nu}$  or  $g^{\mu\nu}$  and the same holds for the other  $K$ -like tensors in the ellipsis of (59). In particular, the tensor  $K_{\mu\nu}^{\alpha\beta}$  of the quadratic term in (59) has the following form in the notation of Ref. [54]:

$$K_{\mu\nu}^{\alpha\beta} = c_1 g^{\alpha\beta} g_{\mu\nu} + c_2 \delta_\mu^\alpha \delta_\nu^\beta + c_3 \delta_\nu^\alpha \delta_\mu^\beta, \quad (60)$$

for real constants  $c_n$  with mass dimension 2. Distinct from the theory considered in Ref. [54], our tensor (60) does not contain a term  $c_4 u^\alpha u^\beta g_{\mu\nu}$ , as such a term would depend explicitly on  $u^\mu$  and contradict our assumptions (motivated by the postulated Lorentz invariance of the quantum vacuum).

The equation of motion for  $u^\mu$ ,

$$\nabla_\nu \frac{\partial \epsilon}{\partial u^\mu_\nu} = 0, \quad (61)$$

has the solution expected for a vacuum-variable  $q$ -type field in flat spacetime:

$$\bar{u}^\mu_\nu = u \delta^\mu_\nu, \quad u = \text{constant}, \quad (62)$$

where the specific solution of  $u^\mu_\nu$  is indicated by a bar. With this solution, the energy density in the action (58) is simply  $\epsilon(u)$  in terms of contracted coefficients  $K$ ,  $K_{\mu\nu}^{\alpha\beta}$ , and  $K_{\mu\nu\rho\sigma}^{\alpha\beta\gamma\delta}$  from (59). However, just as in Sec. V C, the energy-momentum tensor obtained by variation over  $g^{\mu\nu}$  and evaluated for solution (62) gives a *different* energy density denoted  $\tilde{\epsilon}_{\text{vac}}(u)$ :

$$\begin{aligned}\bar{T}_{\mu\nu} &= \frac{2}{\sqrt{-\bar{g}}} \frac{\delta S_{\text{vel}}}{\delta g^{\mu\nu}} \Big|_{u_\nu^\mu = \bar{u}_\nu^\mu, g_{\mu\nu} = \bar{g}_{\mu\nu}} \\ &= \bar{g}_{\mu\nu} \left( \epsilon(u) - u \frac{d\epsilon(u)}{du} \right) \equiv \bar{g}_{\mu\nu} \tilde{\epsilon}_{\text{vac}}(u),\end{aligned}\quad (63)$$

where  $\bar{g}_{\mu\nu}$  stands for the flat spacetime metric. This particular contribution to the energy-momentum tensor corresponds to the following cosmological constant in Einstein's gravitational field equations (21):

$$\Lambda = \epsilon(u) - u \frac{d\epsilon}{du} \equiv \tilde{\epsilon}_{\text{vac}}(u), \quad (64)$$

whose origin was discussed in Sec. III A on general grounds. In our formulation of the theory, different from the Einstein–aether theory considered by Jacobson [54], the nonzero vacuum value (62) of the field  $u_\nu^\mu$  does *not* violate Lorentz symmetry but leads to compensation of the bare cosmological constant  $K$  in the equilibrium vacuum.

However, the compensation of the bare cosmological constant occurs only for a vacuum solution with a nonzero equilibrium value  $u_0$ . The quantum vacuum corresponding to the solution  $u_0 = 0$  would have a cosmological constant  $\Lambda$  equal to the bare cosmological constant  $K \sim E_{\text{UV}}^4$ . This result illustrates an important difference between the following two kinds of vacua. The quantum vacuum with  $u_0 = 0$ , on the one hand, can exist only with external pressure  $P = -K$  and is not a self-sustained medium (see also footnote 1). By analogy with condensed-matter physics, this kind of quantum vacuum may be called “gas-like.” The quantum vacuum with nonzero  $u_0$ , on the other hand, can be stable at  $P = 0$ , provided that a nonzero solution  $u_0$  exists for  $P = 0$  and that it has positive compressibility,  $\chi(u_0) > 0$ . This kind of quantum vacuum may then be called “liquidlike” and a numerical example of such a vacuum (with the nonzero equilibrium vacuum variable  $q_0$  replacing the  $u_0$  from here) has been given in the last paragraph of the previous subsection.

### E. Possible spacetime origin of $q$ : Minimal volume

In this subsection, another suggestion for the possible origin of the vacuum variable  $q$  is made (mathematically, this suggestion is similar to the one of Sec. V C but, physically, not). The relevant variable  $q$  might, in fact, come from a fundamental antisymmetric field such as  $q_{\mu\nu\rho\sigma} = qe_{\mu\nu\rho\sigma}$ . This  $q$  would then be related to some kind of universal minimal 4-volume  $l^4$  (cf. Sec. 4.11 of Ref. [10]), so that  $q = \hat{q}l^4$  for a dimensionless number  $\hat{q}$ . This minimal volume  $l^4$  may be essential to the underlying microscopic theory [5], but it is the dimensionless vacuum variable  $\hat{q}$  which adjusts itself to zero pressure,  $P = 0$ , and this adjustment leads to a vanishing effective vacuum energy density,  $\tilde{\epsilon}_{\text{vac}} = -P = 0$ .

Let us end this section with a general remark. The separation of Secs. V B and V C in one group and

Secs. V D and V E in another group has been made in order to simplify the discussion. However, it may very well be that the origin of  $q$  lies in a fundamental theory which combines “dynamic fields” and “spacetime structure,” as alluded to in the first paragraph of the Introduction.

## VI. SUMMARY AND OUTLOOK

In this article, we have suggested that the value of the cosmological constant  $\Lambda$  is determined by a self-tuning thermodynamic variable  $q$  of the Lorentz-invariant self-sustained quantum vacuum. The natural value of the gravitating vacuum energy density  $\tilde{\epsilon}_{\text{vac}}(q_0)$ , identified with the cosmological constant  $\Lambda$ , would then be zero. For the perfect quantum vacuum, the equilibrium value  $q_0$  adjusts itself so that  $\tilde{\epsilon}_{\text{vac}}(q_0) = 0$ . In our considerations, a crucial role is played by the Lorentz invariance of the quantum vacuum, for which there is strong experimental support.

A small nonzero value of the cosmological constant (that is, small compared to Planck-scale energy densities) may result from perturbations of the Lorentz-invariant state of the universe, for example, by the presence of thermal matter. In fact, the presence of matter violates Lorentz invariance by introducing a preferred reference frame, where the matter is, on average, at rest. The resulting state of the universe is no longer Lorentz invariant and this allows for having a nonzero value of the vacuum energy density  $\tilde{\epsilon}_{\text{vac}}$ . However, the value of the vacuum energy density is small, as it is proportional to the perturbation which violates the original Lorentz invariance of the perfect (unperturbed) quantum vacuum.

The implication of our suggestion is that the so-called “dark energy” would be homogeneous, provided the matter is distributed homogeneously over large scales. But the precise interaction of gravitating matter (visible and “dark”) with the microscopic degree of freedom  $q$  from the quantum vacuum remains to be clarified. Perhaps, carefully planned astronomical observations can give a clue.

The vacuum energy density is governed by processes in the deep ultraviolet vacuum. Still, the approach followed in this article demonstrates that the thermodynamics of the vacuum energy can be described by a relatively minor extension of the low-energy (infrared) effective theory, namely, by the introduction of a *constant* vacuum variable  $q$ . This particular model expands on the idea that the cosmological constant problem belongs to the realm of infrared physics [11,58].

The introduction of an infrared thermodynamic variable  $q$ , describing certain properties of the deep vacuum and having the equilibrium value  $q_0 \equiv q_{\text{vac}}$ , can be considered as the first step in a bottom-up approach (trying to go from the effective low-energy theory to the fundamental microscopic theory). This simple approach already allowed us to introduce the thermodynamic notion of the vacuum com-

pressibility  $\chi_{\text{vac}}$  which is a new fundamental constant for the low-energy world and to estimate the thermodynamic back reaction on these constants ( $q_{\text{vac}}$ ,  $\chi_{\text{vac}}$ , and others) by low-energy phase transitions and the presence of thermal matter.

In this article, we have studied what may be called the “statics of dark energy.” It is to be hoped that the *dynamics* of the vacuum energy density, which describes the relaxation of the “cosmological constant” to its equilibrium

value, can be obtained by a further extension of our effective theory.

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