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Magnetic resonance within vortex cores in the B phase of superfluid ^{3}He

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We investigate a magnetic susceptibility of vortices in the B phase of multicomponent triplet superfluid ^{3}He focusing on a contribution of bound fermionic states localized within vortex cores. Several order-parameter configurations relevant to different types of quantized vortices in ^{3}He B are considered. It is shown quite generally that an ac magnetic susceptibility has a sharp peak at the frequency corresponding to the energy of interlevel spacing in the spectrum of bound fermions. We suggest that measuring of a magnetic resonance within vortex cores can provide a direct probe of a discrete spectrum of bound vortex-core excitations.

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I. INTRODUCTION

Probing of a quasiparticle spectrum in Fermi superfluids containing quantized vortices has been a challenging problem since the pioneering work of Caroli *et al.*[1] They predicted theoretically that the internal electronic structure of quantized vortices in superconductors should consist of low-energy fermionic excitations localized within the vortex cores with characteristic interlevel spacing defined as $\Delta_0^2/E_F \ll \Delta_0$, where Δ_0 is the energy gap far from the vortex line and E_F is the Fermi energy. For conventional s-wave superconductors with axially symmetric vortex lines the low-energy ($|\varepsilon| \ll \Delta_0$) part of the quasiparticle spectrum has the following form:

$$\varepsilon(\mu,k_z) \simeq -\hbar\omega_v\mu. \tag{1}$$

Here μ and k_z are the projections of quasiparticle angular and linear momenta onto the vortex axis and

$$\hbar\omega_v \approx \frac{\Delta_0}{\sqrt{k_F^2 - k_z^2}\,\xi}, \tag{2}$$

where ξ is a coherence length in superconductor or superfluid. Due to the quantization of angular motion the projection of angular momentum μ takes discrete values which are half integer for the s-wave superconductor and integer for the p-wave superconductors and superfluids such as ^{3}He (see Ref. 2). Thus the spectrum of bound vortex-core fermions [Eq. (1)] consists of a ladder of states which varies from Δ_0 at $\mu=-\infty$ to Δ_0 as $\mu=\infty$. An example of the Caroli-de Gennes-Matricon spectrum for the vortex in p-wave superfluid is shown in Fig. 1(a).

Bound fermions determine many of the physical properties of vortices in superconductors and Fermi superfluids such as ^{3}He. In particular, vortex dynamics is determined by the kinetics of vortex-core quasiparticles (see, for example, review Ref. 3 and also Refs. 4–6) and low-temperature thermodynamic properties are determined by the peculiarities of spectrum of bound fermions. Recently there has been much attention focused on the vortex-core states in chiral triplet superconductors in connection with the topologically protected zero energy states being a realization of self-conjugated Majorana fermions (see, for example, Ref. 7). Such objects have topological nature[2] and are the key ones for the realization of topological quantum computation which is presently generating much interest.[8]

One of the most striking demonstrations of the existence of such bound states have been scanning tunneling microscope (STM) experiments showing a peak of the local density of quasiparticle states for the scanning tip position at the vortex center.[9] However the spatial and energy resolution of STM techniques has been insufficient to investigate a discrete energy levels of vortex-core quasiparticles which were predicted by Caroli, de Gennes, and Matricon.

Another experimental tool to study the discreet nature of the vortex-core quasiparticle spectrum would be measuring of the dynamical responses of Fermi superfluids determined by the motion of vortices. In particular, the ac conductivity of superconductors in superclean regime should demonstrate a peak at the frequency determined by the interlevel transitions of quasiparticles localized within the vortex cores.[10] The same is also true for the drag viscosity coefficient in the B phase of ^{3}He (see Refs. 3 and 4). Of particular interest are the effects of fermion bound states in a vortex core on the rotational dynamics of vortices with spontaneously broken axisymmetry.[11] It has been predicted that a resonance absorption of an external rf field can occur at the external frequency comparable with the interlevel distance of localized states which is similar to the cyclotron Landau damping.

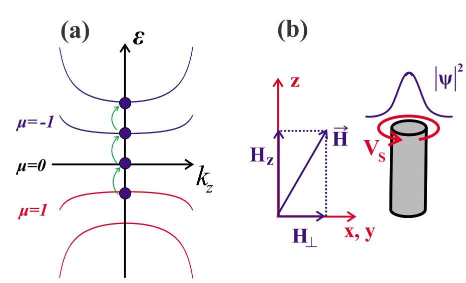

FIG. 1. (Color online) (a) Sample spectrum of Caroli-de Gennes-Matricon states for a vortex in a p-wave superfluid. (b) Sketch of the system consisting of vortex line directed along the z axis under the action of magnetic field. The time-dependent magnetic field $\mathbf{H}=\mathbf{H}(t)$ leads to the transitions in the ladder of Caroli-de Gennes-Matricon states. The interlevel transitions caused by the time-dependent magnetic field are shown schematically in the panel (a) by arrows.

However experimental realization of the mechanisms described above is still lacking. In superconductors the main obstacle is a rather restricting condition for the material purity, which is hardly obtainable in conventional superconductors. In superfluid ^{3}He the reason is an extremely high frequency of vortex motion which is required to excite the transitions between quantized energy levels inside vortex cores. Thus it seems that probing of the Caroli-de Gennes-Matricon theory with the help of existing to date experimental approaches is unattainable.

Our basic idea is to employ for this purpose the Zeeman interaction of quasiparticles in Fermi systems with external magnetic field. Quasiparticle spin degree of freedom has been paid little attention up to date in connection with the properties of the quasiparticle spectra of vortex cores. The reason for it is that in conventional superconductors with singlet pairing the spin is a completely independent variable and it does not affect the orbital motion of quasiparticles. On the other hand, the situation is completely different in multicomponent triplet Fermi superfluids. Indeed, in this case the Cooper pair wave function is a superposition of different spin states,[12]

$$\hat{\Delta} = -i(\hat{\boldsymbol{\sigma}} \cdot \mathbf{d})\hat{\sigma}_y, \tag{3}$$

where $\mathbf{d}$ is a vector in three-dimensional (3D) space and $\sigma_{x,y,z}$ are Pauli matrices in conventional spin space. In case if the direction of spin vector $\mathbf{d}$ is different at the different points of Fermi sphere, quasiparticles "see" the direction of Cooper pair spin which essentially depends on the direction of quasiparticle wave vector $\mathbf{k}$. If there is no other spin-dependent potentials then quasiparticle spin is determined only by the order parameter and it occurs to be dependent on the direction of quasiparticle propagation. Thus we may conclude that in multicomponent triplet superfluids quasiparticle spin and orbital degrees of freedom are effectively coupled.

The schematic configuration of the system that we propose to study is shown in Fig. 1(b). In general we consider vortices in the B phase of superfluid ^{3}He under the action of external magnetic field. We assume that the magnetic field applied to the vortex consists of a large constant component along the vortex axis H_z and a small time-varying component directed perpendicular to the vortex axis $H_\perp = H_\perp(t)$. As we will show below due to the effective spin-orbital coupling for quasiparticles in ^{3}He B the time-dependent magnetic field can excite the transitions in a ladder of Caroli-de Gennes-Matricon states through the Zeeman interaction with quasiparticle spins. These transitions are shown schematically by arrows in Fig. 1(a). In turn, such transitions lead to the resonant energy absorption at the frequency determined by the interlevel spacing which observation would be the demonstration of a discrete structure of spectrum of bound vortex-core states [Eq. (1)].

It is natural to expect that the interlevel energy spacing which in ^{3}He is of the order $\omega_v \sim 0.1$ MHz (see Ref. 4) would determine the resonant frequency of magnetic response of vortex cores. Note that such resonance is qualitatively different from the magnetic resonance of free nuclear spins (NMR) when the resonant frequency is determined by the constant component of applied magnetic field. Within the frequency domain and range of magnetic fields we are interested in such type of "usual" NMR can be neglected. Indeed, the gyromagnetic ratio of ^{3}He nucleus is of the order $\gamma_g \sim 10^4$ Hz/G therefore the Larmour frequency of $\omega_L = \gamma_g H \sim 0.1$ MHz corresponds to magnetic field of about $H = \omega_L / \gamma_g \sim 10$ G which is much less than typical magnetic fields of the order $H \sim 100$ G used in NMR experiments in superfluid ^{3}He. Under such conditions the frequencies that we will be interested in lie outside the region of usual NMR peak $\omega_v \ll \omega_L$ which means that we can completely neglect the Larmour spin precession.

A special remark should be made about a potential observability of the considered effect in state-of-the-art magnetic-resonance experiments. First of all we note that although the energy-level spacing between adjacent vortex-core levels in ^{3}He B is on the order of $\hbar\omega_v \sim \Delta_0/(k_F\xi) \sim 10^{-5}$ K the resonant transitions between these levels can be observed at much higher temperature $T \gg \hbar\omega_v$. Indeed the resonant frequency $\omega_v \sim 0.1$ MHz is much larger than the ^{3}He NMR resolution scale since in many experiments the resonance peaks of width much smaller than 1 KHz were resolved (see, for example, Ref. 13). Furthermore, as we will see below, the width of the considered resonance peak is determined by the quasiparticle decay time τ, which is determined by the density of the normal component of ^{3}He (Ref. 3). At low temperatures $T \ll T_c$, where T_c is a critical temperature the normal component freezes out and the decay time τ grows as e^{T/T_c} (see Refs. 3, 4, and 14). Thus at the temperature domain $\hbar\omega_v \ll T \ll T_c$ (practically starting from $0.5T_c$) the condition $\omega_v \tau \gg 1$ is satisfied which means that the magnetic resonance at frequency ω_v should be well resolved.

This paper is organized as follows. In Sec. II we give an overview of the theoretical framework, namely, the Bogoliubov–de Gennes theory which we use to analyze the spectra of bound fermions and kinetic theory to calculate a nonequilibrium magnetization of vortex cores in time-dependent magnetic field. The main results are presented in Sec. III, in particular, the transformation of the quasiparticle spectra is discussed in Secs. III A and III B. The nonequilibrium magnetization and paramagnetic susceptibility of vortex cores and dissipation losses are addressed in Sec. III C. We summarize our results in Sec. IV. Some of the details of our calculations are given in appendices.

II. BASIC EQUATIONS

A. Order parameter

In general order parameter of triplet superfluid has the form given by Eq. (3). Further assuming a p-wave pairing which is the most relevant case for the superfluid ^{3}He (see Ref. 15) we write the order parameter in the form

$$\hat{\Delta}_{\mathbf{k}} = A_{\alpha i}(i\hat{\boldsymbol{\sigma}}_\alpha \hat{\sigma}_y)(k_i/k_F). \tag{4}$$

The 3×3 matrix with complex coefficients $A_{\alpha i}$ in the expression above can be represented as an expansion

$$A_{\alpha i} = \Delta_0 \sum_{\mu\nu} a_{\mu\nu} \lambda^{\mu}_{\alpha} \lambda^{\nu}_{i}, \tag{5}$$

where $\lambda^{\pm}_{i,\alpha} = (\mathbf{x}_{i,\alpha} \pm i\mathbf{y}_{i,\alpha})$, $\lambda^{0}_{i,\alpha} = \mathbf{z}_{i,\alpha}$ are the eigenfunctions of orbital momentum $\hat{L}_z$ and spin $\hat{S}_z$ with eigenvalues μ and ν correspondingly.

Further in this paper we will deal with axially symmetric quantized vortices in superfluid ^{3}He described in Refs. 16–18. The axial symmetry of such objects is determined by a generator of rotations around the vortex axis $\hat{Q} = \hat{J}_z - M\hat{I}$, where $\hat{J}_z = \hat{L}_z + \hat{R}\hat{S}_z$ is the projection of the internal angular momentum of Cooper pairs onto the vortex axis and $\hat{I}$ is a generator of gauge rotations so that $\hat{I}A_{\alpha i} = A_{\alpha i}$. We introduce here the operator of 3D rotation $\hat{R}$ which transforms the coordinate axes in spin space into the ones in orbital space. Such rotation of coordinate axes leads to the transformation of the order parameter according to the following rule:

$$\tilde{A}_{\alpha i} = R_{\alpha\beta} A_{\beta i}. \tag{6}$$

As we will see further the relative rotation of spin and orbital coordinate axes does not lead to qualitatively new results. Therefore basically we will assume that the spin and orbital quantization axes coincide with the vortex axis z and will just briefly discuss the appropriate changes in the resulting formula which take into account the rotation [Eq. (6)].

The axially symmetric order-parameter components satisfying the equation $\hat{Q}A_{\alpha i} = 0$ are given by the expansion [Eq. (5)] with the following choice of coefficients:

$$a_{\mu\nu} = C_{\mu\nu}(r) e^{i(M-\mu-\nu)\varphi}, \tag{7}$$

where φ and r are polar coordinates with the origin at the vortex center.

The coefficient M in Eq. (7) is determined by the vorticity M_v and the internal angular momentum of Cooper pairs far from the vortex axis. For example, the B phase of ^{3}He is characterized by the zero internal angular momentum and therefore $M = M_v$. However in the A phase of ^{3}He as well as in the chiral state of triplet superconductor Sr_2RuO_4 in the homogeneous state,[19] we have $J_z = \pm 1$ and therefore $M = M_v \pm 1$.

Let us now consider the transformation of the vortex order parameter under the action of several discrete symmetries: time inversion T, space inversion P, and rotation by the angle π over the x axis U_2. Under the time inversion we get $TA_{\alpha i} = A^{*}_{\alpha i}$ therefore

$$T(C_{\mu\nu}) = C^{*}_{-\mu,-\nu}. \tag{8}$$

Under the spatial inversion we get

$$P(C_{\mu\nu}) = (-1)^{M-\mu-\nu} C_{\mu\nu} \tag{9}$$

and finally

$$U_2(C_{\mu\nu}) = (-1)^{\mu+\nu} C_{-\mu,-\nu}. \tag{10}$$

As we will see later it is very important that we can construct a general expression for the pseudoscalar from the amplitudes of the order-parameter components $C_{\mu\nu}$. Indeed, let us denote

$$\tilde{C}_n = \sum_{|\mu+\nu|=n} C_{\mu\nu}.$$

Note that $n = 0, 1, 2$ since $|\mu|, |\nu| \leq 1$ in ^{3}He. Then a general expression of the pseudoscalar which can be composed from the order-parameter components has the following gauge invariant form:

$$\alpha_p = \mathrm{Im}(\tilde{C}^{*}_0 \tilde{C}_1) + \mathrm{Im}(\tilde{C}^{*}_2 \tilde{C}_1). \tag{11}$$

From Eqs. (8)–(10) it is obvious that

$$T(\tilde{C}_n) = \tilde{C}^{*}_n,$$

$$P(\tilde{C}_n) = (-1)^{n+M} \tilde{C}_n,$$

$$U_2(\tilde{C}_n) = (-1)^n \tilde{C}_n.$$

Therefore assuming that the vortices are singly quantized $M = \pm 1$ from the above expressions we get that

$$P, T, U_2(\alpha_p) = -\alpha_p.$$

In this paper we will consider only the vortices in the B phase of superfluid ^{3}He. For singly quantized vortices $M = \pm 1$ there can exist five basic components of the order parameter. Among them are $C_{1,-1}$, $C_{-1,1}$, and C_{00} which correspond to the main B phase, $C_{0,1} = C_A$ and $C_{1,0} = C_\beta$ which correspond to the additional A and β phases localized inside vortex core. The additional A phase has a zero-spin projection ($\mu = 0$) and unit projection of orbital momentum ($\nu = 1$) on the z axis while β phase has $\mu = 1$ and $\nu = 0$. Far from the vortex core at $r \gg \xi_v$ only B superfluid phase exists so that $C_{1,-1} = C_{-1,1} = C_{00} = 1$ and $C_{A,\beta} = 0$. The vortex type is determined by the behavior of amplitudes $C_{\mu\nu}$ at smaller distances $r \sim \xi_v$ and there exist five types of vortices.[18]

Vortices of o and u types are singular so that only the superfluid components of B phase $C_{1,-1}$, $C_{-1,1}$, and C_{00} are nonzero. These amplitudes are real for the most symmetric o vortex which is invariant under the action of three basic discrete symmetry transformations: $P_1 = P$, $P_2 = PTU_2$, and $P_3 = TU_2$. The less symmetric u vortex with conserved parity $P_1 = P$ but broken $P_3 = TU_2$ discrete symmetry is characterized by the complex amplitudes of the order-parameter components.

Nonsingular v, w, and uvw vortices have superfluid cores with the inclusion of A and β phases. The functions $C_{A,\beta} = C_{A,\beta}(r)$ describing the spatial distributions of additional A and β phases inside vortex core are finite at $r = 0$ and vanish outside the core at $r \gg \xi_v$. The v and w vortices are characterized by real B phase amplitudes. If $C_{A,\beta}$ are also real then we have a v vortex with conserved $P_2 = PTU_2$ symmetry. The case when $\mathrm{Re}(C_{A,\beta}) = 0$, $\mathrm{Im}(C_{A,\beta}) \neq 0$ corresponds to w vortex with conserved $P_3 = TU_2$ symmetry. The less symmetric uvw vortex with all discrete symmetries P_1, P_2, and P_3 broken has complex amplitudes of B, A, and β phases.

Returning to the definition of the pseudoscalar [Eq. (11)] and applying it to the particular case of vortices in ^{3}He B it is easy to see that $\alpha_p = 0$ for the singular o and u vortices as well as for the nonsingular v vortex. At the same time for w and uvw vortices we get a nonzero pseudoscalar $\alpha_p \neq 0$. Assuming, for example, the model situation when $C_{1,-1}=C_{-1,1}=C_{00}=C_B(r)$ and $C_A\neq 0$ we obtain from Eq. (11) that for w and uvw vortices

$$\alpha_p = \mathrm{Im}(C_B^* C_A). \tag{12}$$

As we will see below such classification of vortices in terms of the pseudoscalar has a direct connection with qualitatively different modification of quasiparticle spectrum by an external magnetic field.

B. Quasiclassical Bogoulubov-de Gennes equations

Let us now turn to the spectrum of quasiparticles, which is described by the Bogoulubov-de Gennes (BdG) equations. The quasiclassical form of BdG equations was derived in Refs. 20 and 21 and reads as follows:

$$-i\frac{\hbar k_\perp}{m}\frac{\partial}{\partial s}U + \hat{\Delta}_{\mathbf{k}}V = (\varepsilon - \hat{P})U, \tag{13}$$

$$i\frac{\hbar k_\perp}{m}\frac{\partial}{\partial s}V + \hat{\Delta}_{\mathbf{k}}^{+}U = (\varepsilon + \hat{P}^*)V. \tag{14}$$

Here $k_\perp=\sqrt{k_F^2-k_z^2}$ is the component of Fermi momentum perpendicular to vortex axis $\mathbf{z}$, $\hat{P}=\mu_B(\mathbf{H}\cdot\hat{\boldsymbol{\sigma}})$ is the Zeeman term, and $\hat{\Delta}_{\mathbf{k}}$ is a gap operator [Eq. (4)]. We assume that the magnetic field $\mathbf{H}$ applied to the system enters the equations only through the Zeeman terms and in general the magnetic field dependence of gap function $\hat{\Delta}_{\mathbf{k}}=\hat{\Delta}_{\mathbf{k}}(\mathbf{H})$.

Within quasiclassical formalism we should express the real space coordinates through the coordinate

$$s = (\mathbf{n}_{\mathbf{k}}\cdot\mathbf{r}) \tag{15}$$

along the trajectory characterized by the direction of quasiparticle momentum in xy plane $\mathbf{n}_{\mathbf{k}}=\mathbf{k}_\perp/k_\perp=(\cos\theta_p,\sin\theta_p)$ and the impact parameter

$$b = (\mathbf{z}\cdot[\mathbf{n}_{\mathbf{k}}\times\mathbf{r}]). \tag{16}$$

The impact parameter is related to the angular momentum projection μ along the vortex axis through the usual classical mechanics formula $\mu=-k_\perp b$.

The energy spectrum of the quasiclassical BdG equations $\varepsilon=\varepsilon(\mu,\theta_p,k_z)$ depends on trajectory angle θ_p, angular momentum μ, and linear momentum k_z projections. Later we will use the following general symmetry of quasiclassical spectrum (see Appendix A):

$$\varepsilon(\mu,\theta_p,k_z) = -\varepsilon(-\mu,\theta_p+\pi,-k_z), \tag{17}$$

which holds for an arbitrary triplet order parameter. Also we will assume for simplicity that the order parameter is symmetric with respect to the sign reversal of magnetic field

$$\hat{\Delta}_{\mathbf{k}}(\mathbf{H}) = \hat{\Delta}_{\mathbf{k}}(-\mathbf{H}),$$

which is justified as long as we neglect the spontaneous magnetic moment of vortex cores.[16] Under such assumption the following symmetry of the spectrum is valid (see Appendix A):

$$\varepsilon(\mathbf{H}) = \varepsilon(-\mathbf{H}). \tag{18}$$

Let us now consider the Green's functions of BdG system of Eqs. (13) and (14)

$$\check{G}^{R(A)}(s_1,s_2) = \sum_n \frac{\Psi_n^+(s_1)\Psi_n(s_2)}{\epsilon - \varepsilon_n \pm i\delta}, \tag{19}$$

where we introduce the two component eigenfunctions $\Psi_n=(U_n,V_n)$ of BdG equations corresponding to the energy level ε_n. Further we will use a relation between the functions [Eq. (19)] and the quasiclassical Green's functions $\check{g}^{R(A)}=\check{g}^{R(A)}(\mathbf{k},\mathbf{r},\epsilon)$ having the following form:[22,23]

$$\check{g}^{R(A)}(s) = -i\frac{\hbar^2 k_\perp}{m}[\check{G}^{R(A)}(s,s+0) + \check{G}^{R(A)}(s,s-0)],$$

where the coordinate s along trajectory is related to the vectors $\mathbf{r}$ and $\mathbf{k}$ according to Eq. (15).

Then taking the derivatives of the both sides of BdG system and using the normalization condition $\int\Psi_n\Psi_n^+ ds=1$ it is easy to obtain the expression which we will use later,

$$\int \mathrm{Tr}\frac{\partial\check{H}}{\partial\mathbf{H}}(\check{g}^R - \check{g}^A)ds = 4\pi\frac{\hbar^2 k_\perp}{m}\sum_n \delta(\epsilon-\varepsilon_n)\frac{\partial\varepsilon_n}{\partial\mathbf{H}}. \tag{20}$$

In the equation above we use the following magnetic-field-dependent part of the Hamiltonian:

$$\check{H} = \begin{pmatrix} \mu_B(\hat{\boldsymbol{\sigma}}\cdot\mathbf{H}) & \hat{\Delta}_{\mathbf{k}} \\ \hat{\Delta}_{\mathbf{k}}^{+} & -\mu_B(\hat{\boldsymbol{\sigma}}^*\cdot\mathbf{H}) \end{pmatrix}, \tag{21}$$

where the diagonal terms define the interaction of nuclear spins with the external magnetic field and μ_B is a nuclear magneton. This Hamiltonian can be rewritten as $\check{H}=\mu_B(\check{\mathbf{S}}\cdot\mathbf{H})+\check{\Delta}_{\mathbf{k}}$, where we have introduced a matrix gap function

$$\check{\Delta}_{\mathbf{k}} = \begin{pmatrix} 0 & \hat{\Delta}_{\mathbf{k}} \\ \hat{\Delta}_{\mathbf{k}}^{+} & 0 \end{pmatrix}$$

and the operator of quasiparticle spin

$$\check{\mathbf{S}} = (\hat{\tau}_3\hat{\sigma}_x, \hat{\sigma}_y, \hat{\tau}_3\hat{\sigma}_z). \tag{22}$$

Here the Pauli matrices $\hat{\sigma}_{x,y,z}$ and $\hat{\tau}_{1,2,3}$ act in spin and Nambu spaces correspondingly. Note that Eq. (20) takes into account not only the energy shift due to the interaction of nuclear spin with magnetic field but also the field dependence of gap function $\hat{\Delta}_{\mathbf{k}}=\hat{\Delta}_{\mathbf{k}}(\mathbf{H})$. In the next section we will use relation (20) to derive the expression for magnetization of vortex cores.

C. Magnetization of vortex cores and energy dissipation

At first we need to derive an expression for the paramagnetic response of vortex cores in superfluid ^{3}He B. Our essential interest is in the dynamics of vortex-core magnetization driven by the time-dependent external magnetic field and the corresponding energy dissipation due to the interaction of quasiparticles with the heat bath.

We start with the exact expression of spin magnetic moment valid for the general nonequilibrium system,

$$\mathbf{M} = -\mu_B \nu_0 \int \frac{d\epsilon}{4}\frac{d\Omega_{\mathbf{k}}}{4\pi} d\mathbf{r}\, \mathrm{Tr}\, \check{\mathbf{S}}\check{g}^K, \tag{23}$$

where we introduce the quasiclassical Keldysh function $\check{g}^K = \check{g}^K(\mathbf{k},\mathbf{r},t,\epsilon)$ and the operator of quasiparticle spin is given by Eq. (22). The integration in momentum space is done over the Fermi sphere so that $(d\Omega_{\mathbf{k}})/(4\pi)=(dk_z d\theta_p)/(4\pi k_F)$ and ν_0 is a density of states at the Fermi level.

In general the average rate of dissipation losses in the system can be calculated as the work of the source of external magnetic field as follows:

$$Q_\varepsilon = -\langle \mathbf{M}\cdot\dot{\mathbf{H}}\rangle_t, \tag{24}$$

where the brackets denote the time average. However some parts of the total magnetization [Eq. (23)] of the system correspond to a reversible exchange of the energy between the source and the system and for the time periodical processes drop out from the expression for the work of external source.

To take into account such reversible energy exchange let us note at first that the average rate of energy loss does not change if we add to Eq. (24) the time averaged change in the system energy or to be more precise the thermodynamic potential Ω. Indeed, for the time periodical processes the average change in system thermodynamic potential is zero. Thus we can substitute the expression for the dissipation losses [Eq. (24)] by the following:

$$Q_\varepsilon = -\left\langle \frac{\delta E_{tot}}{\delta\mathbf{H}}\cdot\dot{\mathbf{H}}\right\rangle_t. \tag{25}$$

In the expression above we introduce the total energy which describes the interaction between the vortex and the source of external magnetic field $E_{tot}=\Omega+E_m$, where Ω is the thermodynamic potential of the system in the magnetic field and E_m is the part of magnetic energy which describes the work done by the external source $\delta E_m=-\delta R=\mathbf{M}\cdot\delta\mathbf{H}$. The variation in E_{tot} with respect to the magnetic field

$$\frac{\delta E_{tot}}{\delta\mathbf{H}} = \frac{\delta\Omega}{\delta\mathbf{H}} + \mathbf{M} \tag{26}$$

gives exactly the flow of the energy to the heat bath, i.e., the dissipation losses.

To evaluate the expression for the energy dissipation [Eq. (25)] we should calculate the variation in thermodynamic potential with the magnetic field $\delta\Omega/\delta\mathbf{H}$. For this purpose we use the standard expression for the variation in the thermodynamic potential (see, for example, Ref. 23),

$$\delta\Omega = \nu_0 \int \mathrm{Tr}[\delta\check{H}\check{g}^{(st)}(\mathbf{k},\mathbf{r})]\frac{d\epsilon}{4}\frac{d\Omega_{\mathbf{k}}}{4\pi}d\mathbf{r} - \int \mathrm{Tr}\frac{\delta(\hat{\Delta}_{\mathbf{k}}(\mathbf{r})\hat{\Delta}^*_{\mathbf{k}}(\mathbf{r}))}{\lambda}\frac{d\Omega_{\mathbf{k}}}{4\pi}d\mathbf{r}, \tag{27}$$

where λ is the weak-coupling constant and the variation in Hamiltonian (21) is given by

$$\delta\check{H} = \left[\mu_B\check{\mathbf{S}} + \frac{\partial\check{\Delta}_{\mathbf{k}}}{\partial\mathbf{H}}\right]\cdot\delta\mathbf{H}. \tag{28}$$

In Eq. (27) we have introduced the stationary part $\check{g}^{(st)}$ of the total Green's function $\check{g}^K$ which is time independent and can be expressed through the stationary retarded and advanced Green's functions $\check{g}^{R(A)}(\mathbf{k},\mathbf{r},\epsilon)$ and the equilibrium distribution function $f^{(0)}(\epsilon)=\tanh(\epsilon/2T)$ as follows:

$$\check{g}^{(st)} = (\check{g}^R - \check{g}^A)f^{(0)}(\epsilon).$$

When calculating the variation in thermodynamic potential we should take into account the self-consistent change in the gap function determined by the following equation:

$$\hat{\Delta}_{\mathbf{k}}(\mathbf{r},t) = \lambda\nu_0 \int \frac{d\epsilon}{4}\frac{d\Omega_{\mathbf{k}_1}}{4\pi}V(\mathbf{k},\mathbf{k_1})\hat{f}^K(\mathbf{k_1},\mathbf{r},t), \tag{29}$$

where $V(\mathbf{k},\mathbf{k_1})$ is a pairing interaction which for the p-wave superfluid has the form $V(\mathbf{k},\mathbf{k_1})=(\mathbf{k}\cdot\mathbf{k_1})$. Note that it is a total nonequilibrium Green's function $\hat{f}^K(\mathbf{k},\mathbf{r},t)$ which enters the self-consistency Eq. (29).

Substituting Eq. (29) into the expression for the variation in thermodynamic potential [Eq. (27)] we immediately obtain that

$$\frac{\delta\Omega}{\delta\mathbf{H}} = \mu_B\nu_0 \int \mathrm{Tr}[\check{\mathbf{S}}\check{g}^{(st)}(\mathbf{k},\mathbf{r})]\frac{d\epsilon}{4}\frac{d\Omega_{\mathbf{k}}}{4\pi}d\mathbf{r} - \nu_0\int \mathrm{Tr}\left[\frac{\partial\check{\Delta}_{\mathbf{k}}}{\delta\mathbf{H}}\check{g}^{(nst)}(\mathbf{k},\mathbf{r})\right]\frac{d\epsilon}{4}\frac{d\Omega_{\mathbf{k}}}{4\pi}d\mathbf{r}, \tag{30}$$

where we have introduced a nonstationary part of Green's function as

$$\check{g}^{(nst)} = \check{g}^K - \check{g}^{(st)}. \tag{31}$$

The next step is to get use of Eqs. (23) and (30) substituting them to Eq. (26). Then we obtain that $(\delta E_{tot})/(\delta\mathbf{H})=\mathbf{M}_{qp}$, where

$$\mathbf{M}_{qp} = -\nu_0\int\frac{d\epsilon}{4}\frac{d\Omega_{\mathbf{k}}}{4\pi}d\mathbf{r}\,\mathrm{Tr}\frac{\partial\check{H}}{\partial\mathbf{H}}\check{g}^{(nst)}. \tag{32}$$

The introduced quantity $\mathbf{M}_{qp}$ has the physical meaning of magnetic moment of the ensemble of quasiparticles which reside within vortex core. Indeed for an arbitrary system of noninteracting particles described by a single-particle Hamiltonian $\check{H}$ the operator of magnetic moment has the form[24] $\check{\mathbf{M}}_{qp}=-(\partial\check{H})/(\partial\mathbf{H})$ and expression (32) yields the nonstationary part of the thermodynamic average of $\check{\mathbf{M}}_{qp}$.

Further we will deal with the linear magnetic response and monochromatic processes so that it is convenient to introduce a paramagnetic susceptibility of quasiparticles in the frequency domain $\hat{\chi}_{qp}=\hat{\chi}_{qp}(\omega_{\mathrm{rf}})$ as follows:

$$\mathbf{M}_{qp}(\omega_{\mathrm{rf}}) = \hat{\chi}_{qp}\mathbf{H}(\omega_{\mathrm{rf}}),$$

where

$$\mathbf{H}(\omega_{\mathrm{rf}}) = \int_{-\infty}^{\infty}\mathbf{H}(t)e^{i\omega_{\mathrm{rf}}t}dt,$$

$$\mathbf{M}_{qp}(\omega_{\rm rf}) = \int_{-\infty}^{\infty} \mathbf{M}_{qp}(t) e^{i\omega_{\rm rf}t} dt.$$

Then for the monochromatic magnetic field the dissipation losses are determined by the standard expression

$$Q_\epsilon = \frac{\omega_{\rm rf}}{2} \mathbf{H}^* \hat{\chi}''_{qp} \mathbf{H}, \tag{33}$$

where $\hat{\chi}''_{qp} = \mathrm{Im}\, \hat{\chi}_{qp}$.

Expression (32) has an essential advantage since it allows to take into account the dependence of the energy gap on the magnetic field $\hat{\Delta}_{\mathbf{k}} = \hat{\Delta}_{\mathbf{k}}(\mathbf{H})$, which in general cannot be neglected. As we will see below the only thing we should know to calculate $\mathbf{M}_{qp}$, according to Eq. (32), is a dependence of quasiparticle energy on the magnetic field. The quasiparticle spectrum can be either calculated exactly for model situations with non-self-consistent gap $\hat{\Delta}_{\mathbf{k}}$ or it can be taken in some general form determined by the symmetry of the system. We will follow the latter way since it allows to take into account the magnetic field dependence of the gap function without extensive self-consistent calculations.

D. Distribution function and kinetic equation

To proceed further we assume that the deviations from equilibrium are small and the rate of magnetic field variation is much slower than the relaxation time of the gap function. Then we use the approximate expression for the nonstationary part of the Keldysh function valid for the slow variation in the system parameters,[23]

$$\check{g}^{(nst)} = (\check{g}^R - \check{g}^A) f_1, \tag{34}$$

where the spectral Green's functions $\check{g}^{R(A)}$ are taken for the stationary system and the function $f_1 = f_1(\mathbf{k}, \mathbf{r}, t)$ determines the nonequilibrium deviation of the symmetric part of generalized distribution function.

Our goal now is to rewrite Eq. (32) in terms of the spectrum of quasiclassical BdG equation. At first let us use the new coordinate system defined by relations (15) and (16). Then the integration over $d\mathbf{r}d\Omega_{\mathbf{k}}/(4\pi)$ in Eq. (32) transforms to the integration over $d\theta_p d\mu ds dk_z/(4\pi k_F k_\perp)$. Substituting expression (34) into Eq. (32) and using relation (20) we finally obtain the following equation for the nonstationary magnetic moment of vortex core quasiparticles:

$$\mathbf{M}_{qp} = -\frac{1}{8\pi^2} \sum_n \int d\mu d\theta_p dk_z \frac{\partial \varepsilon_n}{\partial \mathbf{H}} f_1. \tag{35}$$

Note that during the derivation of Eq. (35) we have neglected the Fermi-liquid corrections which are very important in the normal state of liquid ^{3}He (Ref. 15). Thus Eq. (35) cannot be applied to treat the normal component of ^{3}He. However it works well with the vortex-core quasiparticles. As we will see below the magnetic susceptibility of an ensemble of vortex-core excitations is much lower than that in the normal state. Therefore the corresponding molecular fields can be neglected as it is usually done when the contributions to different physical quantities of vortex-core quasiparticles are considered.[3]

Finally to calculate the magnetization with the help of Eq. (35) we only need to know the quasiparticle spectrum $\varepsilon_n = \varepsilon_n(\mu, \theta_p)$ which can be found solving the BdG equations [Eqs. (13) and (14)] and the distribution function $f(\mu, \theta_p, t) = f_0[\varepsilon_n(\mu, \theta_p)] + f_1$, which obeys the kinetic equation, derived in Refs. 22, 23, and 25,

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \theta_p} \dot{\theta}_p + \frac{\partial f}{\partial \mu} \dot{\mu} = St(f). \tag{36}$$

The collision integral on the right-hand size of Eq. (36) can be taken in the model relaxation-time approximation: $St(f) = (f - f_0)/\tau$, where f_0 is a equilibrium function which we take in the form: $f_0 = \tanh(\epsilon/2T)$.

Canonical variables μ, θ_p fulfill the Hamiltonian equations,

$$\dot{\theta}_p = \frac{\partial \varepsilon_n}{\partial \mu}; \quad \dot{\mu} = -\frac{\partial \varepsilon_n}{\partial \theta_p}. \tag{37}$$

With the help of Hamiltonian equations the kinetic equation [Eq. (36)] can be rewritten in the following:

$$\frac{\partial f}{\partial t} + \{\varepsilon_n, f\} = St(f), \tag{38}$$

where we use the Poisson bracket operator,

$$\{\varepsilon_n, f\} = \frac{\partial f}{\partial \theta_p} \frac{\partial \varepsilon_n}{\partial \mu} - \frac{\partial f}{\partial \mu} \frac{\partial \varepsilon_n}{\partial \theta_p}.$$

Using the Hamiltonian equations it is easy to see that $\{\varepsilon_n, f_0\} = 0$ as well as

$$\left\{ \varepsilon_n, \frac{df_0}{d\varepsilon} \right\} = 0.$$

Thus if $f = f_0$ the only term that survives in kinetic equation is

$$\frac{\partial f_0}{\partial t} = \frac{df_0}{d\varepsilon} \frac{\partial \varepsilon_n}{\partial t}.$$

Hence for the first-order correction to the distribution function $f = f_0 + f_1$ we obtain the equation

$$\frac{\partial f_1}{\partial t} + \{\varepsilon_n, f_1\} - \frac{f_1}{\tau} = -\frac{df_0}{d\varepsilon} \frac{\partial \varepsilon_n}{\partial t}. \tag{39}$$

This kinetic equation together with the expression for the dissipation losses [Eq. (33)] are the basic equations which we will use to analyze the paramagnetic response of vortex cores.

III. RESULTS

A. Quasiparticle spectrum

In zero magnetic field the BdG equations [Eqs. (13) and (14)] pertain the axial symmetry of wave functions which is described by the symmetry generator,

$$\hat{Q}_{qp} = \hat{J}_z - (M/2) \hat{I} \hat{\tau}_3. \tag{40}$$

Then the quantum number which characterizes quasiparticle spectrum and enters the Caroli-de Gennes-Matricon expres-

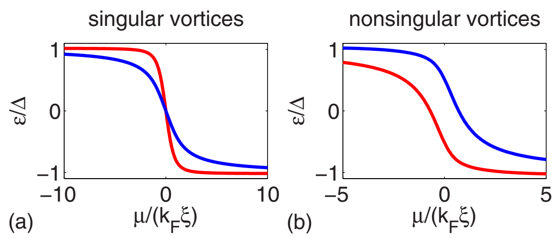

FIG. 2. (Color online) Anomalous energy branches $\varepsilon=\varepsilon(\mu)$ for (a) singular o and u vortices; and (b) nonsingular vortices.

sion is the eigenvalue of the symmetry generator $\hat{Q}_{qp}\Psi=\mu\Psi$.

In general due to the removed spin degeneracy the spectra of all singly quantized vortices in ^{3}He B consist of two different anomalous branches crossing the Fermi level.[26] The examples of anomalous branches for the singular and nonsingular vortices obtained by numerical solution of BdG equations [Eqs. (13) and (14)] are shown in Figs. 2(a) and 2(b) correspondingly.

For small energies near the Fermi level an analytical treatment of energy spectrum of Eqs. (13) and (14) is possible.[26] For singular o and u vortices the anomalous branches are similar to the standard Caroli-de Gennes-Matricon ones and intersect the Fermi level at zero angular momentum yet with different slopes corresponding to different spin states,

$$\varepsilon(\mu,\chi) = -\hbar\omega_\chi \mu, \tag{41}$$

$\hbar\omega_\chi \sim \Delta_0/(k_F\xi)$ and $\chi=\pm 1$ corresponds to the different spin states. The difference in slopes of anomalous branches is determined by the asymmetry of amplitudes $C_{1,-1}$, $C_{-1,1}$ and $C_{0,0}$ inside vortex core. Further we will assume that this asymmetry is small to neglect the spin degeneracy of anomalous branches for singular vortices and put $\omega_1=\omega_{-1}=\omega_v$.

On the contrary the spectral branches of nonsingular v, w, and uvw vortices intersect the Fermi level at finite angular momenta [see Fig. 2],

$$\varepsilon(\mu,q_z,\chi) = -\hbar\omega_\chi\mu + \gamma_1 q_z + \chi\gamma_2, \tag{42}$$

where $\gamma_{1,2}$ as well as ω_χ are even functions of $q_z=k_z/k_F$ and μ. Such requirements provide spectrum symmetry $\varepsilon(\mu,q_z,\chi)=-\varepsilon(-\mu,-q_z,-\chi)$ corresponding to the general invariance of BdG equations [Eq. (17)]. For the nonsingular vortices it is essential that the last term in Eq. (42) corresponding to the spin splitting of energy branches can be rather large $\gamma_2\sim\Delta_0$. Contrary to the case of singular vortices when calculating the spectrum transformation due to the applied magnetic field we will assume that these energy branches are weakly interacting, i.e., the splitting is much larger than the Zeeman terms in Eqs. (13) and (14).

Now let us suppose that there is a magnetic field applied to the system. In case when the magnetic field has a component $\mathbf{H}_\perp$ which is perpendicular to the vortex axis $\mathbf{z}$ the Zeeman term does not commute with the operator $\hat{Q}_{qp}$ given by Eq. (40) and its eigenvalue μ is no more a good quantum number. Therefore the quasiclassical spectrum of BdG equations [Eqs. (13) and (14)] should depend not only on μ but also on the conjugated angle variable which in our case coincides with θ_p such that $[\hat{Q}_{qp},\theta_p]=i$. Note that the angular dependence of the spectrum is determined by the component of the magnetic field $\mathbf{H}_\perp$ perpendicular to the vortex axis since the H_z component does not destroy the axial symmetry.

In general the BdG system [Eqs. (13) and (14)] can be solved only numerically. However the general properties of spectrum transformation in external magnetic field can be derived from the symmetry properties of the different types of vortices. We will consider the expansion of energy spectrum by powers of magnetic field $\mathbf{H}$ assuming that the Zeeman terms are much smaller than the spacing of quasiclassical levels determined by the energy scale Δ_0.

The spectrum should obey the symmetry relations (17) and (18) and be invariant under the simultaneous rotation of coordinate axes and magnetic field around the vortex axis $\mathbf{z}$. Then up to the first order in $H_\perp$ we can write the spectrum perturbed by the magnetic field,

$$\varepsilon(\mu,\theta_p,\chi) = \varepsilon_0 + \alpha_1(\mathbf{q}\cdot\mathbf{H}_\perp) + \alpha_2(\mathbf{z}\cdot[\mathbf{q}\times\mathbf{H}_\perp]), \tag{43}$$

where $\mathbf{q}=\mathbf{k}/k_F$. The first term in this expression describes the axially symmetric part of the spectrum. Although it can depend on magnetic field for our further consideration this dependence will be of no importance. Thus we assume that the first term in Eq. (43) corresponds to the spectrum without external magnetic field given by Eqs. (41) and (42). The remaining terms correspond to the spectrum perturbation due to the field introduction and the coefficients $\alpha_{1,2}$ do not depend on the angle θ_p. It is easy to check that this expression is invariant under the simultaneous rotation of coordinate axes and magnetic field around the vortex axis $\mathbf{z}$. The coefficients $\alpha_{1,2}$ should be expressed through the characteristics of the order-parameter distribution inside vortex core as well as the characteristics of the unperturbed quasiparticle wave function.

We start with the spectrum of singular vortices which in zero magnetic field is given by Eq. (41) with $\omega_1=\omega_{-1}=\omega_v$. We assume that this spectrum is degenerate by spin, therefore the transformation should be linear in Zeeman shift which lifts the spin degeneracy. The only possibility to fulfill the requirements above is to put $\alpha_i=\chi\beta_{si}$, where β_{si} are scalar coefficients, which do not depend on χ, θ_p, and are even in μ and q_z.

Now let us turn to the case of nonsingular vortices with lifted spin degeneracy even in nonperturbed spectrum [Eq. (42)]. It occurs that we can determine the general properties of the spectrum [Eq. (43)] from the symmetry requirements. Indeed, the spectrum should be invariant under the time T and spatial P inversions and also under the rotation of the coordinate system U_2. Also we note that the energy perturbation should be quadratic in magnetic field due to the spin splitting of the spectral branches [Eq. (42)] and the general spectrum symmetry [Eq. (18)] with respect to the total sign of magnetic field $\mathbf{H}$. Thus both the coefficients $\alpha_{1,2}$ should be proportional to the projection of the magnetic field on the vortex axis $H_z=(\mathbf{H}\cdot\mathbf{z})$.

At first let us assume that the coefficients $\alpha_{1,2}$ do not depend on spin. Then the only possibility to satisfy the symmetry requirements above is to choose

$$\alpha_{1,2} \sim \alpha_p H_z, \tag{44}$$

where α_p is a pseudoscalar given by Eq. (11),

$$P,T,U_2(\alpha_p) = -\alpha_p. \tag{45}$$

If we neglect the asymmetry of B phase components within the cores of nonsingular vortices and put $C_{-1,1} = C_{1,-1} = C_{0,0}$ then from the definition of the pseudoscalar [Eq. (12)] we immediately obtain that for the w and uvw vortices the symmetry allows that both the $\alpha_{1,2} \neq 0$.

As for the v vortex the pseudoscalar defined by Eq. (12) is obviously zero in this case. However $\alpha_{1,2}$ can be nonzero even for the v vortex if we assume that they can depend of quasiparticle spin quantum number χ. Indeed, the coefficient γ_2 in the spectrum [Eq. (42)] can be modified to include the magnetic-field-dependent terms as follows:

$$\tilde{\gamma}_2 = \gamma_2\{1 + \beta_1 q_z H_z(\mathbf{q}\cdot\mathbf{H}_\perp) + \beta_2 q_z H_z[\mathbf{z}\cdot(\mathbf{H}_\perp \times \mathbf{q})]\},$$

where $\beta_{1,2}$ are ordinary scalars. It is easy to check that the spectrum [Eq. (42)] with modified $\tilde{\gamma}_2$ satisfies the same discrete symmetries as the nonperturbed spectrum. Thus the coefficients in expansion [Eq. (43)] can be chosen as $\alpha_1 = \chi\gamma_2\beta_1 q_z H_z$ and $\alpha_2 = \chi\gamma_2\beta_2 q_z$.

We can conclude that the quasiparticle spectrum of singular and nonsingular vortices has the same form of Eq. (43) with $\alpha_{1,2} \neq 0$ for all types of vortices. However, the magnitude of coefficients $\alpha_{1,2} \neq 0$ is different for singular and nonsingular vortices. Indeed, in the former case the initial spectrum is assumed to be spin degenerated, therefore energy perturbation is of the first order in magnetic field, thus $\alpha_{1,2} \sim \mu_B$. On the other hand, the spin degeneracy of the spectrum of nonsingular vortices is lifted even at zero magnetic field. Therefore the energy perturbation is nonzero only in the second order of magnetic field and the coefficients in Eq. (43) are proportional to $\alpha_{1,2} \sim (\mu_B^2/\Delta_s)H_z$, where Δ_s denotes the energy of initial split of energy branches. In general, the splitting can be rather large $\Delta_s \sim \Delta_0$ (see Ref. 26) therefore we obtain that for nonsingular vortices the energy perturbation is smaller by the factor $\mu_B H_z/\Delta_0$ than for the singular vortices.

In general the spectrum [Eq. (43)] contains the terms which break angular symmetry and depend on the external magnetic field. Hence by changing the magnetic field $\mathbf{H}$ it is possible to excite the magnetic dipole transitions of quasiparticles between the neighboring Caroli-de Gennes-Matricon levels, which should lead to the resonant energy absorption for a definite frequency of magnetic field oscillations.

Finally in this section we should note that the possible rotation of spin-quantization axes with respect to the orbital ones given by Eq. (6) can be easily taken into account in the above argument. The only thing we need is to transform the quasiparticle wave functions as follows:

$$(\tilde{U},\hat{\sigma}_y\tilde{V}) = \exp[i(\sigma\cdot\mathbf{n})\varphi/2](U,\hat{\sigma}_y V), \tag{46}$$

where $\mathbf{n}$ is a rotation axis and φ is a rotation angle which parametrize the rotation matrix in Eq. (6),

$$(\hat{R})_{\alpha\beta} = \delta_{\alpha\beta} + (n_\alpha n_\beta - \delta_{\alpha\beta})(1-\cos\varphi) - e_{\alpha\beta\gamma} n_\gamma \sin\varphi,$$

where $\delta_{\alpha\beta}$ is a Kronecker delta and $e_{\alpha\beta\gamma}$ is an antisymmetric Levi-Civita tensor.

It is straightforward to check that such transformation [Eq. (46)] if applied to Eqs. (13) and (14) makes the spin axes of the order parameter coincide with the orbital ones. But simultaneously it leads to the effective rotation of magnetic field

$$\tilde{\mathbf{H}} = \hat{R}\mathbf{H}. \tag{47}$$

Note that the matrix $\hat{R}$ does not depend on the angle θ_p. Therefore the only change that should be done in the above consideration to take into account the rotation of spin axes is to replace everywhere the magnetic field by the rotated one [Eq. (47)].

B. Numerical solution of BdG equations

To confirm the general argument above we solve numerically the set of quasiclassical BdG equations [Eqs. (13) and (14)] to obtain the spectrum $\varepsilon = \varepsilon(\mu,\theta_p)$. We consider the model form of the vortex core such that the components corresponding to the B phase are equal $C_{1,-1} = C_{-1,1} = C_{0,0} = C_B$ and only an additional A phase component is present inside vortex core. Then the singular part of gap operator in (s,θ_p) representation is

$$\hat{\Delta}_B = C_B \frac{s - ib}{\sqrt{s^2+b^2}}\hat{D}, \tag{48}$$

where

$$\hat{D} = -q_z\hat{\sigma}_x e^{i\theta_p} + q_\perp \hat{\sigma}_z e^{i(1-\hat{\sigma}_z)\theta_p},$$

$\mathbf{q} = \mathbf{k_F}/k_F$ and $q_\perp = k_\perp/k_F$, $q_z = k_z/k_F$. The nonsingular part of gap function $\hat{\Delta}_A$ is given by

$$\hat{\Delta}_A = -C_A q_\perp e^{i\theta_p}\hat{\sigma}_x. \tag{49}$$

In Fig. 3 we show the isoenergetic lines on the plane μ, θ_p corresponding to the zero energy $\varepsilon = 0$ for several generic cases: (i) singular o and u vortices; (ii) nonsingular vortices for $H_z = 0$; (iii) nonsingular v vortex for $H_z \neq 0$; and (iv) w and uvw vortices for $H_z \neq 0$.

To understand the numerical results shown in Fig. 3 let us consider the expressions for isoenergetic lines $\mu = \mu(\theta_p,\varepsilon)$ which can be derived from the general expression for quasiparticle spectrum [Eq. (43)]. For singular vortices we obtain

$$\mu(\theta_p) = \chi\tilde{\beta}_1\cos(\theta_p - \theta_h) + \chi\tilde{\beta}_2\sin(\theta_p-\theta_h), \tag{50}$$

for nonsingular v vortices

$$\mu(\theta_p) = \mu_0 + \chi\tilde{\alpha}_{1v}\cos(\theta_p-\theta_h) + \chi\tilde{\alpha}_{2v}\sin(\theta_p-\theta_h), \tag{51}$$

and for nonsingular w and uvw vortices

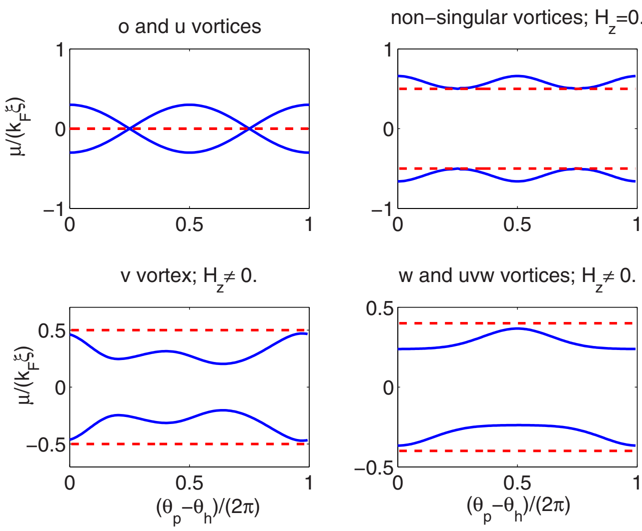

FIG. 3. (Color online) Isoenergetic lines $\mu(\theta_p)$ for zero energy $\varepsilon=0$ corresponding to the different types of vortices in He3-B. Dashed lines show $\mu(\theta_p)$ in zero magnetic field and solid lines correspond to the modification of $\mu(\theta_p)$ by the applied field.

$$\mu(\theta_p) = \mu_0 + \tilde{\alpha}_{1w}\cos(\theta_p-\theta_h) + \tilde{\alpha}_{2w}\sin(\theta_p-\theta_h), \quad (52)$$

where we introduce the angle θ_h characterizing the direction of perpendicular component of magnetic field $\mathbf{H}_\perp = H_\perp(\cos\theta_h,\sin\theta_h)$.

Then from Fig. 3 we can see that for the model vortex core [Eqs. (48) and (49)] the isoenergetic lines are indeed given by Eqs. (50)–(52) with $\tilde{\beta}_1 \neq 0$ and $\tilde{\beta}_2=0$ for singular vortices; $\tilde{\alpha}_{1v},\tilde{\alpha}_{2v}\neq 0$ for nonsingular v vortex; $\tilde{\alpha}_{2w}\neq 0$ and $\tilde{\alpha}_{1w}=0$ for nonsingular w and uvw vortices. In case if $H_z=0$ we have that all the coefficients are zero $\tilde{\alpha}_{1v},\tilde{\alpha}_{2v},\tilde{\alpha}_{1w},\tilde{\alpha}_{2w}=0$ and the spectrum transformation should be of the next order in $\mathbf{H}_\perp$, i.e., proportional to $(\mathbf{q}\cdot\mathbf{H}_\perp)^2 \sim H_\perp^2\cos[2(\theta_p-\theta_h)]$, which is also demonstrated in Fig. 3 (see the plot for nonsingular vortices; $H_z=0$).

In principle the numerical calculations described above allow to consider the spectrum transformation for the arbitrary values of Zeeman shift. However, the qualitative expression (43) have an advantage that they also take into account the self-consistent transformation of gap function in magnetic field $\hat{\Delta}_{\mathbf{k}}=\hat{\Delta}_{\mathbf{k}}(\mathbf{H})$. If this transformation is small it does not change the vortex symmetry therefore the general form of the energy spectrum conserves yet with modified coefficients $\alpha_{1,2}$ in Eq. (43). In our further considerations we will take these coefficients as phenomenological constants since their particular values do not affect the results qualitatively.

C. Magnetic susceptibility and energy dissipation

Now having in hand the expression for quasiparticle energy [Eq. (43)] it is then straightforward to calculate the nonstationary magnetization of vortex cores with the help of kinetic equation [Eq. (36)] and the general expression for the magnetization [Eq. (32)].

We will calculate the response magnetization of a single vortex using a unified expression for the spectrum [Eq. (43)] assuming that only the component of magnetic field perpendicular to the vortex line varies in time. Also without loss of generality we put $\alpha_1\neq 0$ and $\alpha_2=0$, which can always be done by rotating the coordinate frame around the vortex axis by an appropriate angle.

At first we will assume that an ac component of magnetic field is the one perpendicular to the vortex line. The component H_z is nonzero but time independent. Also for the beginning the perpendicular component is taken polarized along the $\mathbf{x}$ axis $\mathbf{H}_\perp = H_x e^{i\omega_{\mathrm{rf}}t}\mathbf{x}$. Then solving the kinetic equation (see Appendix B) and calculating the magnetization with the help of Eq. (35) we obtain the following form of the magnetic-susceptibility tensor $\chi_{xx}=\chi_{yy}=\chi_\parallel$ and $\chi_{xy}=-\chi_{yx}=\chi_\perp$, where

$$\chi_\parallel = \int_{-k_F}^{k_F}\frac{dk_z}{\omega_v}\frac{\alpha(\omega_{\mathrm{rf}}+i/\tau)\omega_{\mathrm{rf}}}{\omega_v^2-(\omega_{\mathrm{rf}}+i/\tau)^2}, \quad (53)$$

$$\chi_\perp = i\int_{-k_F}^{k_F}\frac{dk_z}{\omega_v}\frac{\alpha\omega_v\omega_{\mathrm{rf}}}{\omega_v^2-(\omega_{\mathrm{rf}}+i/\tau)^2}. \quad (54)$$

Here we have denoted $\omega_v=-\hbar^{-1}\partial\varepsilon_n/\partial\mu$ which is the frequency corresponding to the interlevel energy spacing. Further we will assume that this spacing is the same for all anomalous branches, i.e., ω_v does not depend on the index n. Here we have used the relation $(\hbar\omega_v)\partial f_0/\partial\varepsilon=-\partial f_0/\partial\mu$ and integrated over μ assuming for simplicity that ω_v and α do not depend on μ in order to perform the integration over μ. We have denoted $\alpha=N_v\alpha_1^2/(8\pi^2)$, where the overall factor N_v is a vortex density.

The magnetic susceptibility defined by Eqs. (53) and (54) has resonances at the frequencies $\omega_{\mathrm{rf}}=\pm\omega_v$. If the interlevel spacing ω_v depends on k_z [such as in Eq. (2)] the resonance peak transforms into the band with the absorption edge at $\omega_{\mathrm{rf}}=\omega_0=\min|\omega_v(k_z)|$. In this case for the large enough relaxation times $\tau\omega_0\gg 1$ the susceptibility is also peaked at $\omega_{\mathrm{rf}}=\pm\omega_0$ behaving as

$$\chi_\perp,\chi_\parallel \sim \frac{1}{\tau\sqrt{\omega_{\mathrm{rf}}^2-\omega_0^2}}.$$

The frequency-independent magnitude of susceptibility can be estimated as

$$\chi_\perp,\chi_\parallel \sim \chi_n\xi^2 N_v \quad (55)$$

for singular vortices and

$$\chi_\perp,\chi_\parallel \sim \chi_n\left(\frac{\mu_B H_z}{\Delta_s}\right)^2\xi^2 N_v \quad (56)$$

for nonsingular vortices, where $\chi_n\sim\nu_0\mu_B^2$ is a susceptibility of normal phase and Δ_s is the energy splitting of anomalous branches which can be taken on the order of bulk value of energy gap Δ_0. Note that the above estimations yield much lower values of the susceptibility as compared to the normal state. The main weakening factor in Eqs. (55) and (56) is $\xi^2N_v\ll 1$ which is in general small due to the spare distribution of vortices in typical experiments with ^{3}He B under rotation.

The resonant frequency of paramagnetic response where the absorption maximum takes place is selective to the polarization of external magnetic field. To demonstrate this let us consider two limiting cases: the linear polarization $\mathbf{H}=H_x e^{i\omega_{\rm rf}t}\mathbf{x}$ and the circular one $\mathbf{H}=H_\perp e^{i\omega_{\rm rf}t}(\mathbf{x}+iP\mathbf{y})$, where $P=\pm 1$ determines the direction of field rotation. In former case the dissipation determined by Eq. (33) has the form

$$Q_\varepsilon = \frac{\omega_{\rm rf}H_x^2}{2}\mathrm{Im}\,\chi_\parallel,$$

where

$$\mathrm{Im}\,\chi_\parallel = \int_{-k_F}^{k_F} dk_z \left[\frac{\alpha\omega_v\omega_{\rm rf}\tau}{(\omega_v-\omega_{\rm rf})^2\tau^2+1} - \frac{\alpha\omega_v\omega_{\rm rf}\tau}{(\omega_v+\omega_{\rm rf})^2\tau^2+1}\right].$$

So the resonant energy absorption takes place both at $\omega_{\rm rf}=\omega_v$ and $\omega_{\rm rf}=-\omega_v$.

In case of the circular polarization the dissipation rate is given by

$$Q_\varepsilon = \frac{\omega_{\rm rf}H_\perp^2}{2}\mathrm{Im}(\chi_\parallel + i\chi_\perp),$$

where

$$\mathrm{Im}(\chi_\parallel + i\chi_\perp) = \int_{-k_F}^{k_F} dk_z \frac{\alpha\omega_v\omega_{\rm rf}\tau}{(\omega_v+P\omega_{\rm rf})^2\tau^2+1}.$$

For the circular polarization the resonant frequency in the expression above depends on the direction of magnetic field rotation $\omega_{\rm rf}=P\omega_v$.

Thus we see that the paramagnetic susceptibility as well as the dissipation losses in the system have the resonances defined in general by the interlevel spacing in the spectrum of vortex-core fermion states [Eqs. (41) and (42)].

Interestingly, if we assume that the projection of magnetic field onto the vortex line is also time-dependent $H_z=H_z^0+\tilde{H}_z\cos(\omega_{\rm rf}t+\delta)$ then besides the described above resonance at the main frequency $\omega_{\rm rf}=\pm\hbar^{-1}\partial\varepsilon/\partial\mu$ there can also appear resonances at higher frequencies $\omega_{\rm rf}=n\omega_v$ and at fractional frequencies $\omega_{\rm rf}=\omega_v/n$, where n is an integer number. This situation can naturally be realized in the experiment since it is not the real magnetic field which determines magnetic response of vortices but a rotated one according to Eq. (47). In general the rotation matrix $\hat{R}$ is spatially dependent hence the effective field $\hat{R}\mathbf{H}$ has different directions at different points of the superfluid. To show the possibility of the additional resonances let us note that the expression for spectrum [Eq. (43)] is in general nonlinear in $\mathbf{H}$. In particular, for the nonsingular vortices the coefficients in Eq. (43) are proportional to H_z and therefore depend on time as $\alpha_{1,2}=\alpha_{1,2}^0+\tilde{\alpha}_{1,2}\cos(\omega_{\rm rf}t+\delta)$. This additional modulation of the coefficients leads to the appearance of $2\omega_{\rm rf}$ frequency terms in the energy spectrum [Eq. (43)]. Then solving the kinetic equation in standard way described in Appendix B yields the resonances at $\omega_{\rm rf}=\pm\omega_v/2$. In order to obtain the resonances at other frequencies it is necessary to consider the higher terms in spectrum expansion by the powers of the magnetic field $\mathbf{H}$.

IV. SUMMARY

To sum up, we have investigated the spectrum of bound fermion states on vortices in ^{3}He B modified by an external magnetic field. We have developed a general approach to study the spectrum perturbation based on symmetry grounds. It allowed us to determine qualitatively up to the constants of the order unity the form of bound states spectrum for different types of vortices in ^{3}He B. An important advantage of this phenomenological approach to the spectral problem is a possibility to take into account the modification of the order parameter by the external field without extensive numerical calculations.

We also consider the paramagnetic susceptibility of fermionic ensemble bound within vortex cores. We have shown quite generally that it is the fermionic magnetization which determines the energy losses in the ac external magnetic field driving the system out of equilibrium. It occurs that due to the coupling of orbital and spin quasiparticle degrees of freedom the ac magnetic field induces transitions of bound fermions between different energy levels in a ladder of Caroli-de Gennes-Matricon spectrum [Eq. (1)]. Consequently the paramagnetic susceptibility and energy absorption have resonance which occurs when the frequency of the ac external magnetic field equals the interlevel spacing. Due to the broken time inversion symmetry of vortex state the resonant behavior of energy dissipation depends on the polarization of the ac magnetic field. In particular, for a circularly polarized magnetic field rotating over the vortex axis the presence of resonance depends on the relation between vortex winding direction and the direction of field rotation.

Although the resonant absorption occurs at the same frequency for singular and nonsingular vortices the dissipation rate should be quite different in these two cases. Being proportional to $Q_\varepsilon \sim \omega_{\rm rf}\mu_B H_\perp^2$ for the o and u vortices it is much less for the nonsingular vortices when $Q_\varepsilon \sim \omega_{\rm rf}\mu_B H_\perp^2(\mu_B H/\Delta_0)^2$ since the magnetic field is assumed to be much smaller than the spin depairing one so that $\mu_B H \ll \Delta_0$.

Due to the resonant behavior of paramagnetic susceptibility at the frequency of interlevel transitions within the Caroli-de Gennes-Matricon spectrum we can conclude that measuring of a resonant magnetic response of vortex cores in ^{3}He B can provide a tool to study the discrete nature of bound fermions in vortex core. Also the difference in energy absorption rates for singular and nonsingular vortices can provide an evidence for the particular type of vortices realizing in ^{3}He under different experimental conditions.

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APPENDIX A: SYMMETRIES OF QUASIPARTICLE SPECTRUM

We are going to prove the general symmetries [Eqs. (17) and (18)] of the spectrum of BdG system [Eqs. (13) and (14)]. At first let us consider the symmetry [Eq. (17)]. Note that the coordinates in real space are related to the coordinates s,b as follows:

$$x = s\cos\theta_p + b\sin\theta_p,$$

$$y = s\sin\theta_p - b\cos\theta_p.$$

Thus the transformation of s,b,θ_p to $-s,-b,\theta_p+\pi$ does not change the coordinates x,y. It means that the coordinate part of function $\hat{\Delta}_{\mathbf{k}}$ remains intact. At the same time changing k_z,θ_p by $-k_z,\theta_p+\pi$ leads to the reverse of momentum direction. Then the total gap function changes its sign under the transformation $\hat{\Delta}_{\mathbf{k}}(s,\theta_p,k_z,b) = -\hat{\Delta}_{\mathbf{k}}(-s,\theta_p+\pi,-k_z,-b)$. Note that the matrix $\hat{\Delta}_k$ does not contain the Pauli matrix $\hat{\sigma}_y$ therefore $\hat{\Delta}_k^+ = \hat{\Delta}_k^*$. Then the complex conjugation of transformed BdG equations yields

$$-i\frac{\hbar k_\perp}{m}\frac{\partial}{\partial s}V^* + \hat{\Delta}_{\mathbf{k}}U^* = (-\varepsilon - \hat{P})V^*, \tag{A1}$$

$$i\frac{\hbar k_\perp}{m}\frac{\partial}{\partial s}U^* + \hat{\Delta}_{\mathbf{k}}^+V^* = (-\varepsilon + \hat{P}^*)U^*. \tag{A2}$$

Changing ε by $-\varepsilon$ we obtain the system coinciding with the initial set of Eqs. (13) and (14) which proves relation (17).

Now let us consider the symmetry [Eq. (18)]. We suppose here that $\hat{\Delta}_{\mathbf{k}}(\mathbf{H}) = \hat{\Delta}_{\mathbf{k}}(-\mathbf{H})$. Note that the matrix $\hat{\Delta}_{\mathbf{k}}$ does not contain the Pauli matrix $\hat{\sigma}_y$ therefore $\hat{\Delta}_{\mathbf{k}}^+ = \hat{\Delta}_{\mathbf{k}}^*$. Then the complex conjugated BdG equations has the form

$$-i\frac{\hbar k_\perp}{m}\frac{\partial}{\partial s}V^* + \hat{\Delta}_{\mathbf{k}}U^* = (\varepsilon + \hat{P})V^*, \tag{A3}$$

$$i\frac{\hbar k_\perp}{m}\frac{\partial}{\partial s}U^* + \hat{\Delta}_{\mathbf{k}}^+V^* = (\varepsilon - \hat{P}^*)U^*. \tag{A4}$$

Then changing $\mathbf{H}$ by $-\mathbf{H}$ we obtain initial set of Eqs. (13) and (14) which proves Eq. (18).

APPENDIX B: KINETIC EQUATION

In general the solution of kinetic equation [Eq. (39)] can be found in the following form:

$$f_1 = G(\theta_p,t)\frac{df_0}{d\varepsilon}. \tag{B1}$$

Then

$$\frac{\partial f_1}{\partial t} = \frac{\partial G}{\partial t}\frac{df_0}{d\varepsilon} + G\frac{d^2 f_0}{d\varepsilon^2}\dot{\varepsilon},$$

where the second term can be neglected. Also, $\{\varepsilon_n, G\} = -\omega_v(\partial G/\partial\theta_p)$. Therefore for the function $G(\theta_p,t)$ we obtain the following equation:

$$\frac{\partial G}{\partial t} - \omega_v\frac{\partial G}{\partial \theta_p} - \frac{G}{\tau} = -\frac{\partial \varepsilon_n}{\partial t}, \tag{B2}$$

where we denote $\hbar\omega_v = -(\partial\varepsilon_n/\partial\mu)$.

Further we will consider the solution of kinetic equation [Eq. (B2)] when the quasiparticle spectrum is given by Eq. (43) with $\alpha_1 \neq 0$, $\alpha_2 = 0$, and $\mathbf{H}_\perp = H_x e^{i\omega_{\mathrm{rf}}t}\mathbf{x}$. Then we have

$$\frac{\partial \varepsilon_n}{\partial t} = i\omega_{\mathrm{rf}}\alpha_1 q_x H_x e^{i\omega_{\mathrm{rf}}t}.$$

Let us search the solution of Eq. (B2) in the following form:

$$G = (A\cos\theta_p + B\sin\theta_p)e^{i\omega_{\mathrm{rf}}t}. \tag{B3}$$

Substituting this form to Eq. (B2) for the coefficients A and B we obtain

$$A = \omega_{\mathrm{rf}}\alpha_1\frac{(\omega_{\mathrm{rf}} + i/\tau)H_x}{\omega_v^2 - (\omega_{\mathrm{rf}} + i/\tau)^2}, \tag{B4}$$

$$B = i\omega_{\mathrm{rf}}\alpha_1\frac{\omega_v H_x}{\omega_v^2 - (\omega_{\mathrm{rf}} + i/\tau)^2}. \tag{B5}$$

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