Aalto University

OPEN ACCESS

This is an electronic reprint of the original article.
This reprint may differ from the original in pagination and typographic detail.

Pekola, Jukka; Brosco, V.; Möttönen, Mikko; Solinas, Paolo; Shnirman, A.

Decoherence in adiabatic quantum evolution: application to Cooper pair pumping

Published in:
Physical Review Letters

DOI:
10.1103/PhysRevLett.105.030401

Published: 12/07/2010

Document Version
Publisher's PDF, also known as Version of record

Please cite the original version:
Pekola, J., Brosco, V., Möttönen, M., Solinas, P., & Shnirman, A. (2010). Decoherence in adiabatic quantum evolution: application to Cooper pair pumping. Physical Review Letters, 105(3), 1-4. [030401]. https://doi.org/10.1103/PhysRevLett.105.030401

This material is protected by copyright and other intellectual property rights, and duplication or sale of all or part of any of the repository collections is not permitted, except that material may be duplicated by you for your research use or educational purposes in electronic or print form. You must obtain permission for any other use. Electronic or print copies may not be offered, whether for sale or otherwise to anyone who is not an authorised user.

Decoherence in Adiabatic Quantum Evolution: Application to Cooper Pair Pumping

J. P. Pekola,[1] V. Brosco,[2,3,4] M. Möttönen,[1,5,6] P. Solinas,[5] and A. Shnirman[7,8]

[1]*Low Temperature Laboratory, Aalto University, P.O. Box 13500, FI-00076 Aalto, Finland*
[2]*Dipartimento di Fisica, Universita' di Roma "La Sapienza," Piazzale A. Moro, 2 00185 Roma, Italy*
[3]*Institut für Theoretische Festkörperphysik, Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany*
[4]*ISC-CNR, Via dei Taurini, 19 00185 Roma, Italy*
[5]*Department of Applied Physics/COMP, Aalto University, P.O. Box 15100, FI-00076 Aalto, Finland*
[6]*Australian Research Council Centre of Excellence for Quantum Computer Technology, School of Electrical Engineering and Telecommunications, University of New South Wales, Sydney, New South Wales 2052, Australia*
[7]*Institut für Theorie der Kondensierten Materie, Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany*
[8]*DFG Center for Functional Nanostructures (CFN), Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany*

(Received 19 November 2009; revised manuscript received 3 May 2010; published 12 July 2010)

One of the challenges of adiabatic control theory is the proper inclusion of the effects of dissipation. Here we study the adiabatic dynamics of an open two-level quantum system deriving a generalized master equation to consistently account for the combined action of the driving and dissipation. We demonstrate that in the zero-temperature limit the ground state dynamics is not affected by environment. As an example, we apply our theory to Cooper pair pumping, which demonstrates the robustness of ground state adiabatic evolution.

DOI: 10.1103/PhysRevLett.105.030401 PACS numbers: 05.30.−d, 03.65.Vf, 03.65.Yz, 85.25.Cp

Accurate control of quantum systems has been one of the greatest challenges in physics for the past decades. Adiabatic temporal evolution [1] has attracted a lot of attention [2–5] in this respect, since it provides robustness against timing errors and typically utilizes evolution in the ground state of the system. Such evolution has been argued to be robust against relaxation and environmental noise [6–8].

The combined effect of adiabatic evolution and dissipation were considered by many authors using various techniques and with different aims and assumptions; see, e.g., Refs. [6,9–13]. We derive in this Letter a unique master equation that treats the combined effect of noise and adiabatic driving consistently and, thus, provides a pioneering tool for studying the effects of decoherence in quantum control protocols employing adiabaticity [3,4]. We find that adiabatic evolution should not be treated in the secular approximation [14]. Furthermore, the master equation incorporates new terms ensuring relaxation into the correct time-dependent ground state. When these issues are properly addressed, the expectation values of physical observables in the adiabatically steered ground state are not influenced by zero-temperature dissipation. We apply our theory to adiabatic charge transport in superconducting circuits in the presence of noise. In spite of its long history [15–18], this problem has recently attracted revived theoretical [19–22] and experimental [23,24] interest due to its fundamental relation to geometric [25] and topological [26] phases and to its potential applications in metrology [24,27].

We consider an open quantum system subject to external time-dependent control fields. The total Hamiltonian of the system and its environment, $H(t)$, is the sum of three terms: $H(t) = H_S(t) + H_E + V$, where $H_S(t)$ denotes the time-dependent system Hamiltonian, H_E is the bath Hamiltonian, and V is the system-bath coupling. Assuming that the driving does not directly affect the coupling term between the system and the environment, we can write $V = X \otimes Y$, where X is a bath operator and Y is a system operator. In the case of weak system-noise coupling and slow driving, a convenient basis to describe the dynamics of the system is the instantaneous energy eigenstate basis, also called the adiabatic basis, defined by $H_S(t)|\psi_n(t)\rangle = E_n(t)|\psi_n(t)\rangle$. The states $|\psi_n(t)\rangle$ are assumed to be normalized and nondegenerate. We denote by $D(t)$ the transformation from a given fixed basis to the adiabatic one. The evolution of the transformed density matrix is governed by the effective Hamiltonian

$$\tilde{H}^{(1)}(t) = \tilde{H}_S(t) + \hbar w(t) + \tilde{V}(t) + H_E, \tag{1}$$

where $\tilde{H}_S(t) = D^\dagger(t) H_S(t) D(t)$, $\tilde{V}(t) = D^\dagger(t) V D(t) = X \otimes \tilde{Y}(t)$, and $w = -iD^\dagger \dot{D}$.

We note that there are a few possible strategies for treating the dissipation. The usual one is to disregard w in the calculation of the dissipative rates [6]. Then, the zero-temperature environment tends to relax the system to the ground state of $H_S(t)$, while the rotation w tries to excite the system. The resulting state is different from both the adiabatic ground state (ground state of H_S) and from the ground state of $\tilde{H}_S + \hbar w$. The second strategy is to first perform a series of transformations to the superadiabatic bases [13,28] and then treat the dissipation. The first step would be to diagonalize $\tilde{H}_S + \hbar w$ with a unitary transformation D_1 and get a much smaller nonadiabatic correction $w_1 = -iD_1^\dagger \dot{D}_1$. Here the dissipation (treated in Markov approximation) takes us to the ground state of $\tilde{H}_S + \hbar w$. Although not exact, the second strategy allows

 © 2010 The American Physical Society

one to treat the combined effect of noise and driving consistently. Here we adopt this strategy to calculate the lowest order correction to the adiabatic dissipative dynamics of a two-level system. As we will show, up to higher order corrections, this treatment correctly accounts for the relaxation to the ground state of the superadiabatic Hamiltonian $\tilde{H}_S + \hbar w$. By using standard methods explained, e.g., in Ref. [14], we arrive at the following master equation for the reduced system density matrix $\tilde{\rho}_I(t)$ in the interaction picture [29]:

$$\frac{d\tilde{\rho}_I(t)}{dt} = i[\tilde{\rho}_I(t), w_I(t)] - \frac{1}{\hbar^2}\mathrm{Tr}_E\left\{\int_0^t dt'[[\tilde{\rho}_I(t)\otimes\rho_E, \tilde{V}_I(t')], \tilde{V}_I(t)]\right\} + \frac{i}{\hbar^2}\mathrm{Tr}_E\left\{\int_0^t dt'\int_0^{t'} dt''[[\tilde{\rho}_I(t) \otimes \rho_E, [w_I(t'), \tilde{V}_I(t'')]], \tilde{V}_I(t)]\right\}, \tag{2}$$

where Tr_E indicates trace over the environmental degrees of freedom and ρ_E is the stationary density operator of the environment. To obtain Eq. (2) we have to take consistently into account corrections up to the order $w\,V\,V$, resulting in a nonstandard commutator expression. The interaction picture operators are defined as $\tilde{O}_I(t) = e^{iH_Et/\hbar}U_S^\dagger(t,0)\tilde{O}(t)U_S(t,0)e^{-iH_Et/\hbar}$, where $U_S(t,0) = e^{-i\int_0^t \tilde{H}_S(\tau)d\tau/\hbar}$ is the system time-evolution operator. In Eq. (2), the first contribution on the right-hand side is of order $\alpha = \hbar/(\Delta T_p)$, where Δ is the minimum gap in the spectrum of H_S and T_p is the period on which the Hamiltonian is varied [30]. The second term is as in the standard Bloch-Redfield theory. The third one is a cross term of the drive and dissipation ensuring relaxation to the proper ground state [13].

We now focus on the case of a general two-state system, with the instantaneous eigenstates $|g\rangle$ (ground state) and $|e\rangle$ (excited state). In this case, returning to the Schrödinger picture, we can recast Eq. (2) into

$$\dot{\rho}_{gg} = -2\Im\mathrm{m}(w_{ge}^*\rho_{ge}) - (\Gamma_{ge}+\Gamma_{eg})\rho_{gg} + \Gamma_{eg} + \tilde{\Gamma}_0\Re\mathrm{e}(\rho_{ge}) + \frac{\Re\mathrm{e}(w_{ge})}{\omega_0}[(2\tilde{\Gamma}_+ - \tilde{\Gamma}_0)(1-\rho_{gg}) - (2\tilde{\Gamma}_- - \tilde{\Gamma}_0)\rho_{gg}] + 2\frac{\Re\mathrm{e}(w_{ge})\Re\mathrm{e}(\rho_{ge})}{\omega_0}(\Gamma_{ge}+\Gamma_{eg}-\Gamma_0), \tag{3}$$

and

$$\dot{\rho}_{ge} = iw_{ge}(2\rho_{gg}-1) + i(w_{ee}-w_{gg})\rho_{ge} + i\omega_0\rho_{ge} - i(\Gamma_{ge}+\Gamma_{eg})\Im\mathrm{m}(\rho_{ge}) - \Gamma_\varphi\rho_{ge} + (\tilde{\Gamma}_+ + \tilde{\Gamma}_-)\rho_{gg} - \tilde{\Gamma}_+ + \left[\frac{w_{ge}}{\omega_0}(2\Gamma_- - \Gamma_\varphi) - i\frac{\Im\mathrm{m}(w_{ge})}{\omega_0}(\Gamma_{eg}-\Gamma_{ge})\right]\rho_{gg} - \left[\frac{w_{ge}}{\omega_0}(2\Gamma_+ - \Gamma_\varphi) + i\frac{\Im\mathrm{m}(w_{ge})}{\omega_0}(\Gamma_{eg}-\Gamma_{ge})\right](1-\rho_{gg}) + 2\left[\frac{w_{ge}^*}{\omega_0}\Re\mathrm{e}(\rho_{ge}) + 2i\frac{\Re\mathrm{e}(w_{ge})}{\omega_0}\Im\mathrm{m}(\rho_{ge})\right](\tilde{\Gamma}_0 - \tilde{\Gamma}_+ - \tilde{\Gamma}_-). \tag{4}$$

By O_{kl} we denote the matrix elements $\langle m|O|n\rangle$ of a general operator O, with $m, n = e, g$, except $w_{mn} = -i\langle m|\dot{n}\rangle$. We have defined the rates $\Gamma_{ge} = \frac{Y_{ge}^2}{\hbar^2}S(-\omega_0)$ (excitation), $\Gamma_{eg} = \frac{Y_{ge}^2}{\hbar^2}S(+\omega_0)$ (relaxation), and $\Gamma_\varphi = 2\frac{Y_{gg}^2}{\hbar^2}S(0)$ (dephasing) and the less common transition terms $\tilde{\Gamma}_\pm = \frac{Y_{gg}Y_{ge}}{\hbar^2}S(\pm\omega_0)$, $\tilde{\Gamma}_0 = 2\frac{Y_{gg}Y_{ge}}{\hbar^2}S(0)$, $\Gamma_\pm = \frac{Y_{gg}^2}{\hbar^2}S(\pm\omega_0)$, and $\Gamma_0 = 2\frac{Y_{ge}^2}{\hbar^2}S(0)$. Here the matrix elements of Y obey $Y_{gg}(t) = -Y_{ee}(t)$ and $Y_{eg}(t) = Y_{ge}(t)$ [31]. The energy separation between the two states is $\hbar\omega_0$, which varies along the pumping trajectory. The power spectrum of the noise is defined through $S(\omega) = \int_{-\infty}^{\infty}\langle X_I(\tau)X_I(0)\rangle e^{i\omega\tau}d\tau$.

Throughout, we have used Markov approximation; i.e., we neglect the variation of $\tilde{\rho}_I(t)$ between t and $t+\tau_c$, assuming that the correlation time of the bath, τ_c, is much shorter than the typical relaxation time of the system, $1/\Gamma$. Furthermore, we made the approximation of adiabatic rates; i.e., in the calculation of the rates we neglect the slow variation of ω_0, Y, and w, assuming the bath correlation time to be much shorter than the driving period $\tau_c \ll T_p$. On the other hand, Eqs. (3) and (4) include all the nonsecular terms traditionally neglected [14]. They introduce cross-dependence between ρ_{gg} and ρ_{ge} in the dissipative terms, and, in our problem, omitting them would lead to unphysical results, such as violation of charge conservation.

We are interested in the quasistationary limit that the system reaches when the evolution is adiabatic and it is initially in the ground state. We thus look for the solutions of $\dot{\rho}_{gg} = 0$ and $\dot{\rho}_{ge} = 0$ for $\alpha \ll 1$. Since $w_{mn} = O(\alpha)$, in the absence of dissipation, we find that $\rho_{gg} \simeq 1 + O(\alpha^2)$ and $\rho_{ge} \simeq -w_{ge}/\omega_0 + O(\alpha^2)$ are the desired solutions. In the zero-temperature limit $S(-\omega_0) = 0$, to the first order in α, Eqs. (3) and (4) yield, again, $\rho_{gg} = 1 + O(\alpha^2)$ and the following equation for the off-diagonal element up to order α: $i\omega_0\Omega_{ge} - \Gamma_\varphi\Omega_{ge} - i\Gamma_{eg}\Im\mathrm{m}(\Omega_{ge}) = 0$, with $\Omega_{ge} \equiv \rho_{ge} + w_{ge}/\omega_0$. The solution of this equation is exactly the same as for the closed system: $\rho_{ge} = -w_{ge}/\omega_0$. Therefore, the ground state evolution is not influenced by coupling to a zero-temperature Markovian environment in the adiabatic limit. Note that including the imaginary part of the rates, e.g., the Lamb shift, does not change this result.

The vanishing of the effects of dissipation is consistent with the following simple argument. In the zero-temperature limit, and to first order in α, the effect of dissipation is to bring the system to the instantaneous ground state of the effective Hamiltonian $\tilde{H}_1 = \tilde{H}_S + \hbar w$, which means that in the eigenbasis of $\tilde{H}_1$ spanned by the eigenvectors, $|\tilde{\psi}_n^{(1)}\rangle$, the density matrix has the form $\tilde{\rho}_{mn}^{(1)} = \langle\tilde{\psi}_m^{(1)}|\rho|\tilde{\psi}_n^{(1)}\rangle = \delta_{mg}\delta_{ng} + O(\alpha^2)$ independent of the dissipative rates. Thus, within our approximations, the ground state evolution is robust against zero-temperature environmental noise and the expectation value of any operator in the quasistationary evolution does not depend on the specific properties of the environment. If, instead, we neglect the nonsecular terms, we obtain the same solution for ρ_{gg}, but the evolution of ρ_{ge} is influenced by the noise as $\rho_{ge} = -w_{ge}/(\omega_0 + i\Gamma/2)$, where Γ represents a combination of the dissipative rates. This leads to different expectation values of physical observables that depend on ρ_{ge} and to the loss of robustness of the ground state dynamics. Therefore, in general, the nonsecular terms cannot be neglected: They give a leading order contribution in $\Gamma\alpha/\Delta$ to the dynamics.

To test our theory on a concrete example, we discuss a superconducting Cooper pair pump. It consists of an array of Josephson junctions coupled to two superconducting leads, being subject to time-dependent external fields. As discussed by various authors (see, e.g., Ref. [19]), the transferred charge is the sum of a dynamic and a geometric contribution: $Q = Q^D + Q^G$. The first one corresponds to the average supercurrent and the second one to pumping. Assuming that only two levels are involved, the two contributions to the charge transferred through junction i in a pumping cycle can be written as

$$Q_i^D = \int_0^{T_p} (\rho_{gg} I_{i,gg} + \rho_{ee} I_{i,ee}) dt, \tag{5}$$

$$Q_i^G = \int_0^{T_p} 2\Re\mathrm{e}(\rho_{ge} I_{i,eg}) dt, \tag{6}$$

where $\hat{I}_i$ is the current operator through junction i. Here we focus on the pumped charge, i.e., $Q_i^G \equiv \int_0^{T_p} I_i^G dt$ [32]. By substituting $\rho_{ge} = -w_{ge}/\omega_0$ in Eq. (6) we arrive at the well-known formula for the adiabatically pumped current in a closed system [18] $I_i^G = -\frac{2}{\omega_0}\Re\mathrm{e}(w_{ge} I_{i,eg})$. As discussed above, this is also the limit of the adiabatic evolution in the presence of environmental noise.

In particular, we consider the Cooper pair sluice [27] of Fig. 1. It consists of a single superconducting island, coupled to two superconducting leads via two SQUIDs, i.e., Josephson junctions whose critical currents can be tuned by magnetic fluxes. The electrostatic potential on the island can be controlled by a gate voltage V_g, and there is a constant superconducting phase difference $\varphi = \varphi_L - \varphi_R$ between the two leads. In the absence of noise, the Hamiltonian of the sluice can be written as

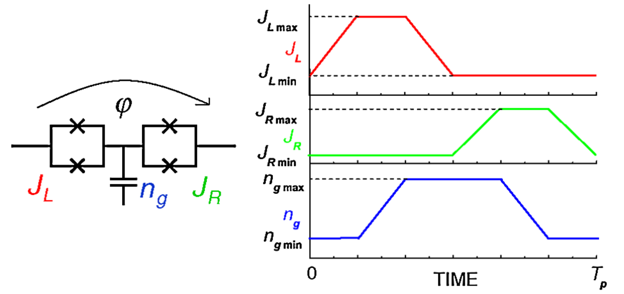

FIG. 1 (color online). An example of a quantum pump, the Cooper pair sluice, is shown on the left. A pumping cycle is sketched on the right. The time-dependent classical control parameters are the magnetic fluxes tuning the Josephson tunnel couplings J_L and J_R and the gate voltage controlling the offset charge n_g of the island. They vary in time with period T_p, whereas the phase difference across the device, φ, is stationary.

$$H_S = E_C(n - n_g)^2 - J_L\cos(\varphi_L - \theta) - J_R\cos(\theta - \varphi_R). \tag{7}$$

Here θ and n are the operators for the superconducting phase of the island and the number of excess Cooper pairs on it, respectively. The Josephson couplings to the left and right lead are denoted as J_L and J_R, respectively, $n_g = C_g V_g/2e$ is the normalized gate charge, and $E_C = 2e^2/C_\Sigma$ is the charging energy of the sluice; C_g is the gate capacitance and C_Σ the total capacitance of the island. The current operators of the left and right junctions read $I_L = \frac{2e}{\hbar} J_L \sin(\varphi_L - \theta)$ and $I_R = \frac{2e}{\hbar} J_R \sin(\theta - \varphi_R)$, respectively. For $E_C \gg \max\{J_L, J_R\}$ and $n_g \simeq 1/2$ only two charge states, $|1\rangle$ and $|0\rangle$, i.e., one or no extra Cooper pairs on the island, are relevant. Dissipation is then mostly due to gate voltage fluctuations. Other noise sources, not considered here, are determined by fluctuations of the fluxes in the SQUIDs or in φ [19]. In the two-level approximation, the coupling between sluice and charge noise has the form $V = -g\sigma_z \otimes \delta V_g(t)$, where $g = eC_g/C_\Sigma$ is the coupling constant, $\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$, and $\delta V_g(t)$ is the gate voltage fluctuation. In the absence of dissipation, for the cycle of Fig. 1, with $J_i \in [J_{\min}, J_{\max}]$, $n_g \in [n_{g\,\min}, n_{g\,\max}]$, and for $J_{\max} \ll E_C$, one obtains the pumped charge in the adiabatic limit according to Eq. (6) as

$$Q_i^G = 2e\left(1 - 2\frac{J_{\min}}{J_{\max}}\cos\varphi\right) \tag{8}$$

for both junctions [27]. Thus the transported charge depends on φ, the average being one Cooper pair per cycle. In the presence of dissipation, Eqs. (3) and (4) were integrated numerically to obtain the temporal evolution of the density matrix along a pumping trajectory of Fig. 1. Figure 2 shows

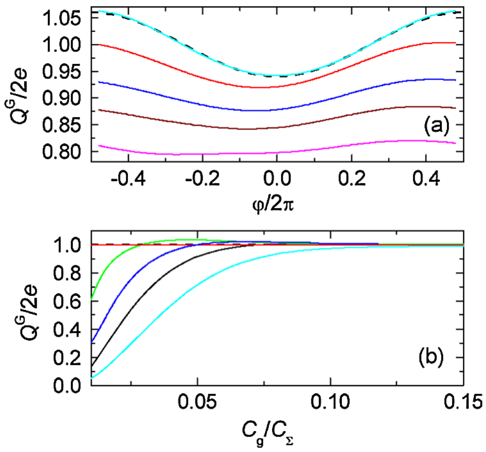

FIG. 2 (color online). Pumped charge of the sluice under gate charge noise. (a) Phase dependence of the pumped charge Q^G in a fully symmetric pumping cycle with respect to J_L and J_R of Fig. 1 as a function of the phase bias φ. The dashed line shows the analytic result of Eq. (8) for adiabatic pumping. The solid lines are from the numerical calculations based on Eqs. (3) and (4) for $f \equiv T_p^{-1} = 100$ MHz, with $C_g/C_\Sigma = 0.015$, 0.0175, 0.02, 0.025, and 0.3 from bottom to top. (b) Coupling dependence of the pumped charge at $\varphi = \pi/2$. The dashed line shows the analytic result as in (a). The solid lines are for $f = 10$, 100, 150, 200, and 300 MHz from top to bottom. The other parameters are $J_{\max}/E_C = 0.1$, $J_{\min}/J_{\max} = 0.03$, $n_{g\max} = 0.8$, $n_{g\min} = 0.2$, $E_C/k_B = 1$ K ($E_C/2\pi\hbar = 21$ GHz), $R = 300$ kΩ, environment temperature $T = 0$, $S(\omega_0) = 2\hbar\omega_0 R$, $S(-\omega_0) = 0$, and $S(0) = 2k_B T_0 R$, with $T_0 = 0.1$ K.

that, upon increasing the system-environment coupling at finite frequencies $f \equiv T_p^{-1}$, the pumped charge approaches the analytic result of Eq. (8) for adiabatic pumping at all values of φ; see Fig. 2(a). Figure 2(b) shows the coupling dependence of the pumped charge at various frequencies for $\varphi = \pi/2$. On lowering the frequency, all the data collapse towards the horizontal dashed line, which is again the result of Eq. (8). For $f = 10$ MHz, the numerical and analytic results are indistinguishable on this scale. Thus coupling to the zero-temperature Markovian environment seems to be useful for adiabatic ground state pumping. We note, however, that Eqs. (3) and (4) are strictly valid only for adiabatic evolution and weak coupling.

In conclusion, we derived a master equation for an adiabatically driven two-level system including the combined effect of drive and relaxation. We found it important to account for the time dependence of the Hamiltonian of the system in determining the dissipative rates and to include the nonsecular terms. As an example, we analyzed adiabatic Cooper pair pumping in the ground state and demonstrated that the pumped charge is not influenced by the zero-temperature environment. Numerical solution of the master equation suggests that dissipation can resume adiabatic pumping at finite frequencies.

We thank R. Fazio for many very useful discussions. We have received funding from the European Community's Seventh Framework Programme under Grant No. 238345 (GEOMDISS). M. M. acknowledges the Academy of Finland and Emil Aaltonen Foundation for financial support.

[1] M. V. Berry, Proc. R. Soc. A **392**, 45 (1984).
[2] G. Falci *et al.*, Nature (London) **407**, 355 (2000).
[3] E. Farhi *et al.*, Science **292**, 472 (2001).
[4] D. Bacon and S. T. Flammia, Phys. Rev. Lett. **103**, 120504 (2009).
[5] A. T. Rezakhani *et al.*, Phys. Rev. Lett. **103**, 080502 (2009).
[6] A. M. Childs, E. Farhi, and J. Preskill, Phys. Rev. A **65**, 012322 (2001).
[7] G. Florio *et al.*, Phys. Rev. A **73**, 022327 (2006).
[8] A. Fubini, G. Falci, and A. Osterloh, New J. Phys. **9**, 134 (2007).
[9] M. S. Sarandy and D. A. Lidar, Phys. Rev. A **71**, 012331 (2005).
[10] P. Thunström, J. Åberg, and E. Sjöqvist, Phys. Rev. A **72**, 022328 (2005).
[11] M. Wubs *et al.*, Phys. Rev. Lett. **97**, 200404 (2006).
[12] A. Carollo *et al.*, Phys. Rev. Lett. **90**, 160402 (2003).
[13] R. S. Whitney *et al.*, Phys. Rev. Lett. **94**, 070407 (2005).
[14] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions* (Wiley, New York, 1992).
[15] D. J. Thouless, Phys. Rev. B **27**, 6083 (1983).
[16] L. J. Geerligs *et al.*, Z. Phys. B **85**, 349 (1991).
[17] D. V. Averin, Solid State Commun. **105**, 659 (1998).
[18] J. P. Pekola, *et al.*, Phys. Rev. B **60**, R9931 (1999).
[19] R. Fazio, F. W. J. Hekking and J. P. Pekola, Phys. Rev. B **68**, 054510 (2003).
[20] M. Governale *et al.*, Phys. Rev. Lett. **95**, 256801 (2005).
[21] M. Möttönen *et al.*, Phys. Rev. B **73**, 214523 (2006).
[22] V. Brosco *et al.*, Phys. Rev. Lett. **100**, 027002 (2008).
[23] M. Möttönen, J. J. Vartiainen, and J. P. Pekola, Phys. Rev. Lett. **100**, 177201 (2008).
[24] J. J. Vartiainen, M. Möttönen, and J. P. Pekola, Appl. Phys. Lett. **90**, 082102 (2007).
[25] M. Aunola and J. J. Toppari, Phys. Rev. B **68**, 020502(R) (2003).
[26] R. Leone, L. P. Levy, and P. Lafarge, Phys. Rev. Lett. **100**, 117001 (2008).
[27] A. O. Niskanen, J. P. Pekola, and H. Seppä, Phys. Rev. Lett. **91**, 177003 (2003).
[28] M. V. Berry, Proc. R. Soc. A **414**, 31 (1987).
[29] Details of the derivation are given in arXiv:0911.3750.
[30] Momentarily, adiabaticity is governed by $|w|/\omega_0$, with $|w|$ denoting the norm of w, and $\hbar\omega_0$ the level separation.
[31] We assume Y_{ge} to be real, which applies for the gate voltage fluctuations analyzed here.
[32] The dynamical contributions can be experimentally separated from the geometric ones as discussed in Refs. [21,23].