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Decoherence in Adiabatic Quantum Evolution: Application to Cooper Pair Pumping

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One of the challenges of adiabatic control theory is the proper inclusion of the effects of dissipation. Here we study the adiabatic dynamics of an open quantum system subject to external time-dependent control fields. The total Hamiltonian of the system and its environment, $H(t)$, is the sum of three terms: $H(t) = H_S(t) + H_E + V$, where $H_S(t)$ denotes the time-dependent system Hamiltonian, $H_E$ is the bath Hamiltonian, and $V$ is the system-bath coupling. Assuming that the driving does not directly affect the coupling term between the system and the environment, we can write $V = X \otimes Y$, where $X$ is a bath operator and $Y$ is a system operator. In the case of weak system-noise coupling and slow driving, a convenient basis to describe the dynamics of the system is the instantaneous energy eigenstate basis, also called the adiabatic basis, defined by $H_S(t)|\psi_n(t)\rangle = E_n(t)|\psi_n(t)\rangle$. The states $|\psi_n(t)\rangle$ are assumed to be normalized and nondegenerate. We denote by $D(t)$ the transformation from a given fixed basis to the adiabatic one. The evolution of the transformed density matrix is governed by the effective Hamiltonian

$$\tilde{H}^{(1)}(t) = \tilde{H}_S(t) + \hbar w(t) + \tilde{V}(t) + \hbar E,$$

where $\tilde{H}_S(t) = D^\dagger(t)H_S(t)D(t)$, $\tilde{V}(t) = D^\dagger(t)VD(t) = X \otimes \tilde{Y}(t)$, and $w = -iD^\dagger \tilde{D}$. We note that there are a few possible strategies for treating the dissipation. The usual one is to disregard $w$ in the calculation of the dissipative rates [6]. Then, the zero-temperature environment tends to relax the system to the ground state of $H_S(t)$, while the rotation $w$ tries to excite the system. The resulting state is different from both the adiabatic ground state (ground state of $H_S$) and from the ground state of $\tilde{H}_S + \hbar w$. The second strategy is to first perform a series of transformations to the superadiabatic bases [13,28] and then treat the dissipation. The first step would be to diagonalize $\tilde{H}_E + \hbar w$ with a unitary transformation $D_1$ and get a much smaller nonadiabatic correction $w_1 = -iD_1^\dagger \tilde{D}_1$. Here the dissipation (treated in Markov approximation) takes us to the ground state of $\tilde{H}_S + \hbar w$. Although not exact, the second strategy allows...
one to treat the combined effect of noise and driving consistently. Here we adopt this strategy to calculate the lowest order correction to the adiabatic dissipative dynamics of a two-level system. As we will show, up to higher order corrections, this treatment correctly accounts for the relaxation to the ground state of the superadiabatic Hamiltonian $\tilde{H}_S + h\omega$. By using standard methods explained, e.g., in Ref. [14], we arrive at the following master equation for the reduced system density matrix $\tilde{\rho}_f(t)$ in the interaction picture [29]:

$$\frac{d\tilde{\rho}_f(t)}{dt} = i[\tilde{\rho}_f(t), w_f(t)] - \frac{1}{h^2} Tr_E \left\{ \int_0^t dt' \left[ [\tilde{\rho}_f(t'), \tilde{\rho}_E, \tilde{V}_f(t')], \tilde{V}_f(t) \right] \right\} + \frac{i}{h^2} Tr_E \left\{ \int_0^t dt' \int_0^{t'} dt'' \left[ [\tilde{\rho}_f(t'), \tilde{\rho}_E, [w_f(t'), \tilde{V}_f(t')]], \tilde{V}_f(t) \right] \right\}$$

$$+ \frac{i}{h^2} \frac{\partial}{\partial t} \left[ \int_0^t dt' \left[ [\tilde{\rho}_f(t'), \tilde{\rho}_E, \tilde{V}_f(t')], \tilde{V}_f(t) \right] \right] + \frac{i}{h^2} \frac{\partial}{\partial t} \left[ \int_0^t dt' \int_0^{t'} dt'' \left[ [\tilde{\rho}_f(t'), \tilde{\rho}_E, [w_f(t'), \tilde{V}_f(t')]], \tilde{V}_f(t) \right] \right]$$

$$\equiv \left\langle \left\{ \tilde{\rho}_f(t), w_f(t) \right\} \right\rangle - \frac{1}{h^2} \left\langle \left\{ \tilde{\rho}_f(t), \tilde{\rho}_E, \tilde{V}_f(t') \right\} \right\rangle$$

One can express this equation in terms of the reduced density matrix $\rho_{\text{reduced}}(t)$, where $\rho_{\text{reduced}}(t)$ is the density matrix of the system alone.

By $O_{ij}$ we denote the matrix elements $\langle m|O|n\rangle$ of a general operator $O$, with $m, n = e, g$, except $w_{mn} = -i\langle m|n\rangle$. We have defined the rates $\Gamma_{ge} = \frac{\hbar}{\Delta \omega} S(-\omega_0)$ (excitation), $\Gamma_{eg} = \frac{\hbar}{\Delta \omega} S(\omega_0)$ (relaxation), and $\Gamma_{ge} = 2\frac{\hbar}{\Delta \omega} S(0)$ (dephasing) and the less common transition terms $\Gamma_{\text{x}} = \frac{\hbar}{\Delta \omega} S(\pm \omega_0)$, $\Gamma_{\text{y}} = \frac{\hbar}{\Delta \omega} S(0)$, $\Gamma_{\text{z}} = \frac{\hbar}{\Delta \omega} S(\pm \omega_0)$, and $\Gamma_{\text{z}} = \frac{\hbar}{\Delta \omega} S(0)$. Here the matrix elements of $Y$ obey $Y_{gg}(t) = -Y_{ee}(t)$ and $Y_{ee}(t) = Y_{gg}(t)$ [31]. The energy separation between the two states is $\hbar \omega_0$, which varies along the pumping trajectory. The power spectrum of the noise is defined through $S(\omega) = 2 \pi \int_{-\infty}^{\infty} X_f(t) X_f(0) e^{i\omega \tau} d\tau$.

Throughout, we have used Markov approximation; i.e., we neglect the variation of $\tilde{\rho}_f(t)$ between $t$ and $t + \tau_c$, assuming that the correlation time of the bath, $\tau_c$, is much shorter than the typical relaxation time of the system, $1/\Gamma$. Furthermore, we made the approximation of adiabatic rates; i.e., in the calculation of the rates we neglect the slow variation of $\omega_0$, $Y$, and $w$, assuming the bath correlation time to be much shorter than the driving period $\tau_c \ll T_p$. On the other hand, Eqs. (3) and (4) include all the nonsecular terms traditionally neglected [14]. They introduce cross-dependence between $\rho_{gg}$ and $\rho_{ge}$ in the dissipative terms, and, in our problem, omitting them would lead to unphysical results, such as violation of charge conservation.

We are interested in the quasiadiabatic limit that the system reaches when the evolution is adiabatic and it is initially in the ground state. We thus look for the solutions of $\dot{\rho}_{gg} = 0$ and $\dot{\rho}_{ge} = 0$ for $\alpha \ll 1$. Since $w_{mn} = O(\alpha)$, in the absence of dissipation, we find that $\rho_{gg} \approx 1 + O(\alpha^2)$ and $\rho_{ge} \approx -w_{ge}/\omega_0 + O(\alpha^2)$ are the desired solutions. In the zero-temperature limit $S(-\omega_0) = 0$, to the first order in $\alpha$, Eqs. (3) and (4) yield, again, $\rho_{gg} = 1 + O(\alpha^2)$ and the following equation for the off-diagonal element up to order $\alpha$: $i\omega_0 \Omega_{ge} - \Gamma_{ge} \Omega_{ge} - i\Gamma_{eg} \Delta m(\Omega_{ge}) = 0$, with $\Omega_{ge} = \rho_{ge} + w_{ge}/\omega_0$. The solution of this equation is exactly the same as for the closed system: $\rho_{ge} = -w_{ge}/\omega_0$. Therefore, the ground state evolution is not influenced by coupling to a zero-temperature Markovian environment in
the adiabatic limit. Note that including the imaginary part of the rates, e.g., the Lamb shift, does not change this result.

The vanishing of the effects of dissipation is consistent with the following simple argument. In the zero-temperature limit, and to first order in $\alpha$, the effect of dissipation is to bring the system to the instantaneous ground state of the effective Hamiltonian $\tilde{H}_1 = \tilde{H}_S + \hbar \omega$, which means that in the eigenbasis of $\tilde{H}_1$ spanned by the eigenvectors, $|\tilde{\psi}^{(1)}_m\rangle$, the density matrix has the form $ho^{(1)}_{nm} = \langle \tilde{\psi}^{(1)}_m | \rho | \tilde{\psi}^{(1)}_n \rangle = \delta_{mg}\delta_{ng} + O(\alpha^2)$ independent of the dissipative rates. Thus, within our approximations, the ground state evolution is robust against zero-temperature environmental noise and the expectation value of any operator in the quasi-stationary evolution does not depend on the specific properties of the environment. If, instead, we neglect the nonsecular terms, we obtain the same solution for $\rho_{gg}$, but the evolution of $\rho_{ee}$ is influenced by the noise as $\rho_{ee} = -w_{ee}/(\omega_0 + i\Gamma/2)$, where $\Gamma$ represents a combination of the dissipative rates. This leads to different expectation values of physical observables that depend on $\rho_{ee}$ and to the loss of robustness of the ground state dynamics. Therefore, in general, the nonsecular terms cannot be neglected: They give a leading order contribution in $\Gamma \alpha/\Delta$ to the dynamics.

To test our theory on a concrete example, we discuss a superconducting Cooper pair pump. It consists of an array of Josephson junctions coupled to two superconducting leads, being subject to time-dependent external fields. As discussed by various authors (see, e.g., Ref. [19]), the transferred charge is the sum of a dynamic and a geometric contribution: $Q = Q^D + Q^G$. The first one corresponds to the average supercurrent and the second one to pumping. Assuming that only two levels are involved, the two contributions to the charge transferred through junction $i$ in a pumping cycle can be written as

$$Q^D_i = \int_0^{T_p} \rho_{gg} \tilde{I}_{gg} + \rho_{ee} \tilde{I}_{ee} dt,$$

$$Q^G_i = \int_0^{T_p} 2\hbar \text{e}(\rho_{gg} \tilde{I}_{gg}),$$

where $\tilde{I}_i$ is the current operator through junction $i$. Here we focus on the pumped charge, i.e., $Q^G_i = \int_0^{T_p} I_i^G dt$ [32]. By substituting $\rho_{ee} = -w_{ee}/\omega_0$ in Eq. (6) we arrive at the well-known formula for the adiabatically pumped current in a closed system [18] $I^G_i = -\frac{\hbar}{2\omega_0} \text{e}(w_{gg} \tilde{I}_{gg})$. As discussed above, this is also the limit of the adiabatic evolution in the presence of environmental noise.

In particular, we consider the Cooper pair sluice [27] of Fig. 1. It consists of a single superconducting island, coupled to two superconducting leads via two SQUIDs, i.e., Josephson junctions whose critical currents can be tuned by magnetic fluxes. The electrostatic potential on the island can be controlled by a gate voltage $V_g$, and there is a constant superconducting phase difference $\varphi = \varphi_L - \varphi_R$ between the two leads. In the absence of noise, the Hamiltonian of the sluice can be written as

$$H_S = E_C(n - n_g)^2 - J_L \cos(\varphi_L - \theta) - J_R \cos(\varphi_R) - J_{\text{SL}}(\sin(\varphi_L - \theta) - \sin(\varphi_R)), \quad \text{(7)}$$

Here $\theta$ and $n$ are the operators for the superconducting phase of the island and the number of excess Cooper pairs on it, respectively. The Josephson couplings to the left and right lead are denoted as $J_L$ and $J_R$, respectively, $n_g = C_g V_g / 2e$ is the normalized gate charge, and $E_C = 2e^2 / C_\Sigma$ is the charging energy of the sluice; $C_g$ is the gate capacitance and $C_\Sigma$ the total capacitance of the island. The current operators of the left and right junctions read $I_L = \frac{\hbar}{2e} J_L \sin(\varphi_L - \theta)$ and $I_R = \frac{\hbar}{2e} J_R \sin(\varphi_R)$, respectively. For $E_C \gg \max(J_L, J_R)$ and $n_g \approx 1/2$ only two charge states, $|1\rangle$ and $|0\rangle$, i.e., one or no extra Cooper pairs on the island, are relevant. Dissipation is then mostly due to gate voltage fluctuations. Other noise sources, not considered here, are determined by fluctuations of the fluxes in the SQUIDs or in $\varphi$ [19]. In the two-level approximation, the coupling between sluice and charge noise has the form $V = -g_{\varphi} \sigma_{\varphi} \delta V_g(t)$, where $g = eC_g / C_\Sigma$ is the coupling constant, $\sigma_{\varphi} = |0\rangle\langle 0| - |1\rangle\langle 1|$, and $\delta V_g(t)$ is the gate voltage fluctuation. In the absence of dissipation, for the cycle of Fig. 1, with $J_i \in [J_{\text{min}}, J_{\text{max}}]$, $n_g \in [n_{g_{\text{min}}}, n_{g_{\text{max}}}]$, and for $J_{\text{max}} \ll E_C$, one obtains the pumped charge in the adiabatic limit according to Eq. (6) as

$$Q^G_i = 2e \left(1 - 2 \frac{J_{\text{min}}}{J_{\text{max}}} \cos \varphi\right), \quad \text{(8)}$$

for both junctions [27]. Thus the transported charge depends on $\varphi$, the average being one Cooper pair per cycle. In the presence of dissipation, Eqs. (3) and (4) were integrated numerically to obtain the temporal evolution of the density matrix along a pumping trajectory of Fig. 1. Figure 2 shows

![FIG. 1 (color online). An example of a quantum pump, the Cooper pair sluice, is shown on the left. A cycling cycle is sketched on the right. The time-dependent classical control parameters are the magnetic fluxes tuning the Josephson tunnel couplings $J_L$ and $J_R$ and the gate voltage controlling the offset charge $n_g$ of the island. They vary in time with period $T_p$, whereas the phase difference across the device, $\varphi$, is stationary.](image)
adiabatic pumping at finite frequencies.

of the master equation suggests that dissipation can resume demonstrated that the pumped charge is not influenced include the nonsecular terms. As an example, we analyzed the system in determining the dissipative rates and to combined effect of drive and relaxation. We found it important adiabatically driven two-level system including the com-

that, upon increasing the system-environment coupling at finite frequencies \( f = T_p^{-1} \), the pumped charge approaches the analytic result of Eq. (8) for adiabatic pumping. The solid lines are from the numerical calculations based on Eqs. (3) and (4) for \( f = T_p^{-1} = 100 \text{ MHz} \), with \( C_x/C_Z = 0.015, 0.0175, 0.02, 0.025, \) and 0.3 from bottom to top. (b) Coupling dependence of the pumped charge at \( \varphi = \pi/2 \). The dashed line shows the analytic result as in (a). The solid lines are for \( f = 10, 100, 150, 200, \) and 300 MHz from top to bottom. The other parameters are \( J_{\text{max}}/E_C = 0.1, J_{\text{min}}/J_{\text{max}} = 0.03, n_g_{\text{max}} = 0.8, n_e_{\text{min}} = 0.2, E_C/k_B = 1 \text{ K} (E_C/2\pi h = 21 \text{ GHz}), R = 300 \text{ k}\Omega, \) environment temperature \( T = 0 \), \( S(\omega_0) = 2\hbar\omega_0 R, S(-\omega_0) = 0 \), and \( S(0) = 2k_B T_0 R, \) with \( T_0 = 0.1 \text{ K} \).

In conclusion, we derived a master equation for an adiabatically driven two-level system including the combined effect of drive and relaxation. We found it important to account for the time dependence of the Hamiltonian of the system in determining the dissipative rates and to include the nonsecicular terms. As an example, we analyzed adiabatic Cooper pair pumping in the ground state and demonstrated that the pumped charge is not influenced by the zero-temperature environment. Numerical solution of the master equation suggests that dissipation can resume adiabatic pumping at finite frequencies.

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[29] Details of the derivation are given in arXiv:0911.3750.
[30] Momentarily, adiabaticity is governed by \( |w|/\omega_0 \), with \( |w| \) denoting the norm of \( w \), and \( \hbar \omega_0 \) the level separation.
[31] We assume \( Y_e \) to be real, which applies for the gate voltage fluctuations analyzed in Refs. [21,23].

FIG. 2 (color online). Pumped charge of the sluice under gate charge noise. (a) Phase dependence of the pumped charge \( Q/G \) in a fully symmetric pumping cycle with respect to \( J_L \) and \( J_R \) of Fig. 1 as a function of the phase bias \( \varphi \). The dashed line shows the analytic result of Eq. (8) for adiabatic pumping. The solid lines are from the numerical calculations based on Eqs. (3) and (4) for \( f = T_p^{-1} = 100 \text{ MHz} \), with \( C_x/C_Z = 0.015, 0.0175, 0.02, 0.025, \) and 0.3 from bottom to top. (b) Coupling dependence of the pumped charge at \( \varphi = \pi/2 \). The dashed line shows the analytic result as in (a). The solid lines are for \( f = 10, 100, 150, 200, \) and 300 MHz from top to bottom. The other parameters are \( J_{\text{max}}/E_C = 0.1, J_{\text{min}}/J_{\text{max}} = 0.03, n_g_{\text{max}} = 0.8, n_e_{\text{min}} = 0.2, E_C/k_B = 1 \text{ K} (E_C/2\pi h = 21 \text{ GHz}), R = 300 \text{ k}\Omega, \) environment temperature \( T = 0 \), \( S(\omega_0) = 2\hbar\omega_0 R, S(-\omega_0) = 0 \), and \( S(0) = 2k_B T_0 R, \) with \( T_0 = 0.1 \text{ K} \).