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Published in:
Physical Review Letters

DOI:
10.1103/PhysRevLett.105.030401

Published: 12/07/2010

Document Version
Publisher's PDF, also known as Version of record

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Decoherence in Adiabatic Quantum Evolution: Application to Cooper Pair Pumping

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Accurate control of quantum systems has been one of the greatest challenges in physics for the past decades. Adiabatic temporal evolution [1] has attracted a lot of attention [2–5] in this respect, since it provides robustness against timing errors and typically utilizes evolution in the ground state of the system. Such evolution has been argued to be robust against relaxation and environmental noise [6–8].

The combined effect of adiabatic evolution and dissipation were considered by many authors using various techniques and with different aims and assumptions; see, e.g., Refs. [6,9–13]. We derive in this Letter a unique master equation that treats the combined effect of noise and adiabatic driving consistently and, thus, provides a pioneering tool for studying the effects of decoherence in quantum control protocols employing adiabaticity [3,4]. We find that adiabatic evolution should not be treated in the secular approximation [14]. Furthermore, the master equation incorporates new terms ensuring relaxation into the correct time-dependent control fields. The total Hamiltonian of the system and its environment, $H(t)$, is the sum of three terms: $H(t) = H_S(t) + H_E + V$, where $H_S(t)$ denotes the time-dependent system Hamiltonian, $H_E$ is the bath Hamiltonian, and $V$ is the system-bath coupling. Assuming that the driving does not directly affect the coupling term between the system and the environment, we can write $V = X \otimes Y$, where $X$ is a bath operator and $Y$ is a system operator. In the case of weak system-noise coupling and slow driving, a convenient basis to describe the dynamics of the system is the instantaneous energy eigenstate basis, also called the adiabatic basis. Defined by $H_S(t)\vert \psi_n(t) \rangle = E_n(t)\vert \psi_n(t) \rangle$, the states $\vert \psi_n(t) \rangle$ are assumed to be normalized and nondegenerate. We denote by $D(t)$ the transformation from a given fixed basis to the adiabatic one. The evolution of the transformed density matrix is governed by the effective Hamiltonian

$$\tilde{H}^{(1)}(t) = \tilde{H}_S(t) + \hbar w(t) + \tilde{V}(t) + H_E,$$

where $\tilde{H}_S(t) = D^\dagger(t)H_S(t)D(t)$, $\tilde{V}(t) = D^\dagger(t)VD(t) = X \otimes \tilde{Y}(t)$, and $w = -iD^\dagger(t)D$. We note that there are a few possible strategies for treating the dissipation. The usual one is to disregard $w$ in the calculation of the dissipative rates [6]. Then, the zero-temperature environment tends to relax the system to the ground state of $H_S(t)$, while the rotation $w$ tries to excite the system. The resulting state is different from both the adiabatic ground state (ground state of $H_S$) and from the ground state of $\tilde{H}_S + \hbar w$. The second strategy is to perform a series of transformations to the superadiabatic bases [13,28] and then treat the dissipation. The first step would be to diagonalize $\tilde{H}_S + \hbar w$ with a unitary transformation $D_1$ and get a much smaller nonadiabatic correction $\omega_1 = -iD_1^\dagger D_1$. Here the dissipation (treated in Markov approximation) takes us to the ground state of $\tilde{H}_S + \hbar w$. Although not exact, the second strategy allows...
one to treat the combined effect of noise and driving consistently. Here we adopt this strategy to calculate the lowest order correction to the adiabatic dissipative dynamics of a two-level system. As we will show, up to higher order corrections, this treatment correctly accounts for the relaxation to the ground state of the superadiabatic Hamiltonian $\hat{H}_S + \hbar w$. By using standard methods explained, e.g., in Ref. [14], we arrive at the following master equation for the reduced system density matrix $\hat{\rho}_I(t)$ in the interaction picture [29]:

$$\frac{d\hat{\rho}_I(t)}{dt} = i[\hat{\rho}_I(t), w_I(t)]$$

$$- \frac{1}{\hbar^2} \text{Tr}_E \left[ \int_0^t dt' \left[ \hat{\rho}_I(t') \otimes \rho_E, \hat{V}_I(t') \right], \hat{V}_I(t) \right],$$

$$+ \frac{i}{\hbar^2} \text{Tr}_E \left[ \int_0^t dt' \int_0^t dt'' \left[ \hat{\rho}_I(t'), \rho_E, [w_I(t'), \hat{V}_I(t'')], \hat{V}_I(t) \right], \right] \rho_E,$$

where $\text{Tr}_E$ indicates trace over the environmental degrees of freedom and $\rho_E$ is the stationary density operator of the environment. To obtain Eq. (2) we have to take consistently into account corrections up to the order $w V V$, resulting in a nonstandard commutator expression. The interaction picture operators are defined as $\hat{O}_I(t) = e^{iH_S t/\hbar} \hat{U}_I(t, 0) \hat{O}(t) \hat{U}_S(t, 0) e^{-iH_S t/\hbar}$, where $\hat{U}_S(t, 0) = e^{-i\int_0^t \hat{H}_S(\tau) d\tau/\hbar}$ is the system time-evolution operator. In Eq. (2), the first contribution on the right-hand side is of order $\alpha = \hbar/\Delta T_p$, where $\Delta$ is the minimum gap in the spectrum of $H_S$ and $T_p$ is the period on which the Hamiltonian is varied [30]. The second term is as in the standard Bloch-Redfield theory. The third one is a cross term of the drive and dissipation ensuring relaxation to the proper ground state [13].

We now focus on the case of a general two-state system, with the instantaneous eigenstates $|g\rangle$ (ground state) and $|e\rangle$ (excited state). In this case, returning to the Schrödinger picture, we can recast Eq. (2) into

$$\dot{\rho}_{gg} = -2\Im \langle w_{ge}\rho_{ge} \rangle - (\Gamma_g + \Gamma_{eg}) \rho_{gg} + \Gamma_{eg} + \tilde{\Gamma}_0 \text{Re}(\rho_{ge}) + \frac{\text{Re}(w_{ge})}{\omega_0} \left( (2\tilde{\Gamma}_g - \tilde{\Gamma}_0)(1 - \rho_{gg}) - (2\tilde{\Gamma}_e - \tilde{\Gamma}_0)\rho_{ge} \right)$$

$$+ 2 \frac{\text{Re}(w_{ge}) \text{Re}(\rho_{ge})}{\omega_0} (\Gamma_g + \Gamma_{eg} - \Gamma_{gg}),$$

and

$$\dot{\rho}_{ge} = i w_{ge} (2\rho_{ge} - 1) + i(w_{ge} - w_{ge}) \rho_{ge} + i \omega_0 \rho_{ge} - i(\Gamma_g + \Gamma_{eg}) \Im \rho_{ge} - \Gamma_{eg} \rho_{ge} + (\tilde{\Gamma}_g + \tilde{\Gamma}_e) \rho_{gg} - \Gamma_{gg}$$

$$+ \left[ \frac{w_{ge}}{\omega_0} (2\Gamma_g - \Gamma_{eg}) - i \frac{\Im \rho_{ge}}{\omega_0} (\Gamma_g - \Gamma_{eg}) \right] \rho_{ge} - \left[ \frac{w_{ge}}{\omega_0} (2\Gamma_e - \Gamma_{eg}) + i \frac{\Im \rho_{ge}}{\omega_0} (\Gamma_{eg} - \Gamma_{ge}) \right] (1 - \rho_{gg})$$

$$+ 2 \left[ \frac{w_{ge}}{\omega_0} \text{Re}(\rho_{ge}) + 2i \frac{\text{Im}(w_{ge})}{\omega_0} \Im \rho_{ge} \right] (\tilde{\Gamma}_0 - \tilde{\Gamma}_e - \tilde{\Gamma}_g).$$

By $O_{ij}$ we denote the matrix elements $\langle m|O|n\rangle$ of a general operator $O$, with $m, n = e, g$, except $w_{mn} = -i\langle m|\hat{n}\rangle$. We have defined the rates $\Gamma_g = \frac{\gamma_g}{\hbar^2} S(-\omega_0)$ (excitation), $\Gamma_{eg} = \frac{\gamma_{eg}}{\hbar^2} S(\omega_0)$ (relaxation), and $\Gamma_{ge} = 2 \frac{\gamma_{eg}}{\hbar^2} S(0)$ (dephasing) and the less common transition terms $\Gamma_{xs} = \frac{\gamma_{xs}}{\hbar^2} S(\pm \omega_0)$, $\tilde{\Gamma}_0 = 2 \frac{\gamma_{xs}}{\hbar^2} S(0)$, $\Gamma_x = \frac{\gamma_x}{\hbar^2} S(\pm \omega_0)$, and $\tilde{\Gamma}_0 = 2 \frac{\gamma_x}{\hbar^2} S(0)$. Here the matrix elements of $Y$ obey $Y_{gg}(t) = -Y_{eg}(t)$ and $Y_{ge}(t) = Y_{ge}(t)$ [31]. The energy separation between the two states is $\hbar \omega_0$, which varies along the pumping trajectory. The power spectrum of the noise is defined through $S(\omega) = \int_{-\infty}^{\infty} \langle X(t) X(t) \rangle e^{i\omega \tau} d\tau$.

Throughout, we have used Markov approximation; i.e., we neglect the variation of $\hat{\rho}_I(t)$ between $t$ and $t + \tau_e$, assuming that the correlation time of the bath, $\tau_e$, is much shorter than the typical relaxation time of the system, $1/\Gamma$. Furthermore, we made the approximation of adiabatic rates; i.e., in the calculation of the rates we neglect the slow variation of $\omega_0$, $Y$, and $w$, assuming the bath correlation time to be much shorter than the driving period $\tau_e \ll T_p$. On the other hand, Eqs. (3) and (4) include all the nonsecular terms traditionally neglected [14]. They introduce cross-dependence between $\rho_{gg}$ and $\rho_{ge}$ in the dissipative terms, and, in our problem, omitting them would lead to unphysical results, such as violation of charge conservation.

We are interested in the quasistationary limit that the system reaches when the evolution is adiabatic and it is initially in the ground state. We thus look for the solutions of $\dot{\rho}_{gg} = 0$ and $\dot{\rho}_{ge} = 0$ for $\alpha \ll 1$. Since $w_{mn} = O(\alpha)$, in the absence of dissipation, we find that $\rho_{gg} \approx 1 + O(\alpha^2)$ and $\rho_{ge} \approx -w_{ge}/\omega_0 + O(\alpha^2)$ are the desired solutions. In the zero-temperature limit $S(-\omega_0) = 0$, to the first order in $\alpha$, Eqs. (3) and (4) yield, again, $\rho_{gg} = 1 + O(\alpha^2)$ and the following equation for the off-diagonal element up to order $\alpha$: $i \omega_0 \Omega_{ge} - \Gamma_g \Omega_{gg} - i \Gamma_{eg} \Im(\Omega_{ge}) = 0$, with $\Omega_{gg} = \rho_{gg} + w_{ge}/\omega_0$. The solution of this equation is exactly the same as for the closed system: $\rho_{ge} = -w_{ge}/\omega_0$. Therefore, the ground state evolution is not influenced by coupling to a zero-temperature Markovian environment in
the adiabatic limit. Note that including the imaginary part of the rates, e.g., the Lamb shift, does not change this result.

The vanishing of the effects of dissipation is consistent with the following simple argument. In the zero-temperature limit, and to first order in $\alpha$, the effect of dissipation is to bring the system to the instantaneous ground state of the effective Hamiltonian $\tilde{H}_{I} = \tilde{H}_{S} + h\nu$, which means that in the eigenbasis of $\tilde{H}_{I}$ spanned by the eigenvectors, $|\tilde{\psi}_{n}^{(1)}\rangle$, the density matrix has the form $\rho_{\tilde{\psi}^{(1)}} = \langle \tilde{\psi}_{n}^{(1)} | \rho | \tilde{\psi}_{n}^{(1)} \rangle = \delta_{n\mu} \delta_{n\nu} + O(\alpha^{2})$ independent of the dissipative rates. Thus, within our approximations, the ground state evolution is robust against zero-temperature environmental noise and the expectation value of any operator in the quasistationary evolution does not depend on the specific properties of the environment. If, instead, we neglect the nonsecular terms, we obtain the same solution for $\rho_{gg}$, but the evolution of $\rho_{ge}$ is influenced by the noise as $\rho_{ge} = -w_{ge}C_{0}(\omega_{0} + i\Gamma/2)$, where $\Gamma$ represents a combination of the dissipative rates. This leads to different expectation values of physical observables that depend on $\rho_{ge}$ and to the loss of robustness of the ground state dynamics. Therefore, in general, the nonsecular terms cannot be neglected: They give a leading order contribution in $\Gamma\alpha/\Delta$ to the dynamics.

To test our theory on a concrete example, we discuss a superconducting Cooper pair pump. It consists of an array of Josephson junctions coupled to two superconducting leads, being subject to time-dependent external fields. As discussed by various authors (see, e.g., Ref. [19]), the transferred charge is the sum of a dynamic and a geometric contribution: $Q = Q^{D} + Q^{G}$. The first one corresponds to the average supercurrent and the second one to pumping. Assuming that only two levels are involved, the two contributions to the charge transferred through junction $i$ in a pumping cycle can be written as

$$Q_{i}^{D} = \int_{0}^{T_{p}} (\rho_{gg}I_{gg} + \rho_{ge}I_{ge}) dt,$$

$$Q_{i}^{G} = \int_{0}^{T_{p}} 2|\mathrm{Re}(\rho_{ge}I_{ge})| dt,$$

where $I_{i}$ is the current operator through junction $i$. Here wefocus on the pumped charge, i.e., $Q_{i}^{G} = \int_{0}^{T_{p}} I_{i}^{G} dt$ [32]. By substituting $\rho_{ge} = -w_{ge}/\omega_{0}$ in Eq. (6) we arrive at the well-known formula for the adiabatically pumped current in a closed system [18] $I_{i}^{G} = -e/\omega_{0} |\mathrm{Re}(w_{ge}I_{ge})|$. As discussed above, this is also the limit of the adiabatic evolution in the presence of environmental noise.

In particular, we consider the Cooper pair sluice [27] of Fig. 1. It consists of a single superconducting island, coupled to two superconducting leads via two SQUIDs, i.e., Josephson junctions whose critical currents can be tuned by magnetic fluxes. The electrostatic potential on the island can be controlled by a gate voltage $V_{g}$, and there is a constant superconducting phase difference $\varphi = \varphi_{L} - \varphi_{R}$ between the two leads. In the absence of noise, the Hamiltonian of the sluice can be written as

$$H_{S} = E_{C}(n - n_{g})^{2} - J_{L} \cos(\varphi_{L} - \theta) - J_{R} \cos(\theta - \varphi_{R}).$$

Here $\theta$ and $n$ are the operators for the superconducting phase of the island and the number of excess Cooper pairs on it, respectively. The Josephson couplings to the left and right lead are denoted as $J_{L}$ and $J_{R}$, respectively, $n_{g} = C_{g}V_{g}/2e$ is the normalized gate charge, and $E_{C} = 2e^{2}/C_{\Sigma}$ is the charging energy of the sluice; $C_{g}$ is the gate capacitance and $C_{\Sigma}$ the total capacitance of the island. The current operators of the left and right junctions read $I_{L} = \frac{\pi}{2} J_{L} \sin(\varphi_{L} - \theta)$ and $I_{R} = \frac{\pi}{2} J_{R} \sin(\theta - \varphi_{R})$, respectively. For $E_{C} \gg \max(J_{L}, J_{R})$ and $n_{g} \approx 1/2$ only two charge states, $|1\rangle$ and $|0\rangle$, i.e., one or no extra Cooper pairs on the island, are relevant. Dissipation is then mostly due to gate voltage fluctuations. Other noise sources, not considered here, are determined by fluctuations of the fluxes in the SQUIDs or in $\varphi$ [19]. In the two-level approximation, the coupling between sluice and charge noise has the form $V = -g_{\sigma} \otimes \delta V_{g}(t)$, where $g = eC_{g}/C_{\Sigma}$ is the coupling constant, $\sigma_{z} = |0\rangle\langle 0| - |1\rangle\langle 1|$, and $\delta V_{g}(t)$ is the gate voltage fluctuation. In the absence of dissipation, for the cycle of Fig. 1, with $J_{L} \in [J_{\min}, J_{\max}]$, $n_{g} \in [n_{g_{\min}}, n_{g_{\max}}]$, and for $E_{C} \ll J_{\max}$, one obtains the pumped charge in the adiabatic limit according to Eq. (6) as

$$Q_{i}^{G} = 2e \left(1 - 2\frac{J_{\min}}{J_{\max}} \cos \varphi \right)$$

for both junctions [27]. Thus the transported charge depends on $\varphi$, the average being one Cooper pair per cycle. In the presence of dissipation, Eqs. (3) and (4) were integrated numerically to obtain the temporal evolution of the density matrix along a pumping trajectory of Fig. 1. Figure 2 shows
adiabatic pumping at finite frequencies.

of the master equation suggests that dissipation can resume
by the zero-temperature environment. Numerical solution

demonstrated that the pumped charge is not influenced
include the nonsecular terms. As an example, we analyzed
combined effect of drive and relaxation. We found it important
for adiabatic evolution and weak coupling.

that, upon increasing the system-environment coupling at
finite frequencies \( f = T_p^{-1} \), the pumped charge approaches
the analytic result of Eq. (8) for adiabatic pumping. The solid
lines are from the numerical calculations based on Eqs. (3) and
(4) for \( f = T_p^{-1} = 100 \text{ MHz} \), with \( C_g/C_x = 0.015, 0.0175, \)
0.02, 0.025, and 0.3 from bottom to top. (b) Coupling depen-
dence of the pumped charge at \( \varphi = \pi/2 \). The dashed line shows
the analytic result as in (a). The solid lines are for \( f = 10, 100, \)
150, 200, and 300 MHz from top to bottom. The other parame-
ters are \( J_{\text{max}}/E_C = 0.1, I_{\text{min}}/I_{\text{max}} = 0.03, n_g \text{ max} = 0.8, n_g \text{ min} = \)
0.2, \( E_C/k_B = 1 \text{ K} (E_C/2\pi \hbar = 21 \text{ GHz}) \), \( R = 300 \text{ k}\Omega \), environment
temperature \( T = 0 \), \( S(\omega_0) = 2\hbar \omega_0 R, S(-\omega_0) = 0 \), and
\( S(0) = 2k_BT_0 R \), with \( T_0 = 0.1 \text{ K} \).

We thank R. Fazio for many very useful discussions. We
have received funding from the European Community’s
Seventh Framework Programme under Grant No. 238345
(GEOMDISS). M.M. acknowledges the Academy of
Finland and Emil Aaltonen Foundation for financial
support.

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30. Momentarily, adiabaticity is governed by \[ \|\omega_0\|/\omega_0 \text{, with } \|\omega_0\| \]
denoting the norm of \( \omega_0 \), \( \hbar \omega_0 \) the level separation.
31. We assume \( Y_g \) to be real, which applies for the gate
voltage fluctuations analyzed in Refs. [21,23].

FIG. 2 (color online). Pumped charge of the sluice under gate
charge noise. (a) Phase dependence of the pumped charge \( Q_p \)
in a fully symmetric pumping cycle with respect to \( J_L \) and \( J_R \) of
Fig. 1 as a function of the phase bias \( \varphi \). The dashed line shows
the analytic result of Eq. (8) for adiabatic pumping. The solid
lines are from the numerical calculations based on Eqs. (3) and
(4) for \( f = T_p^{-1} = 100 \text{ MHz} \), with \( C_g/C_x = 0.015, 0.0175, \)
0.02, 0.025, and 0.3 from bottom to top. (b) Coupling depen-
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0.2, \( E_C/k_B = 1 \text{ K} (E_C/2\pi \hbar = 21 \text{ GHz}) \), \( R = 300 \text{ k}\Omega \), environment
temperature \( T = 0 \), \( S(\omega_0) = 2\hbar \omega_0 R, S(-\omega_0) = 0 \), and
\( S(0) = 2k_BT_0 R \), with \( T_0 = 0.1 \text{ K} \).