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Search for the ac Josephson effect in superfluid $^3$He

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Experiments testing for the existence of the ac Josephson effect in superfluid $^3$He, analogous to phenomena observed in superconducting microbridges, have been performed. Small holes were employed as the weak link between two reservoirs filled with $^3$He; several different orifice geometries were tried. Simple model calculations suggest that steps in the flow characteristics should be observable with our resolution when an ac pressure modulation is applied across the weak link. We found that such effects do not exist for the parameter values used in our experiments.

In superconductors the ac Josephson effect has been observed both in the case of a thin insulating layer between two superconductors and in a narrow superconducting bridge. A similar phenomenon is expected to be present in superfluid $^3$He and $^4$He. An experiment searching for this effect is aimed at testing the fundamental macroscopic quantum coherence of a superfluid. The Josephson equation and the associated concept of phase slippage are the most basic and exact manifestations of our present understanding of superfluidity.

The Josephson effect is a consequence of phase coherence: In a bulk superconductor, when vortices are absent, the phase $\theta$ of the order parameter, $\psi = \psi_0 e^{i \theta}$, is constant or varies smoothly over macroscopic distances. The possibility of dissipationless currents prevents temporal variations of $\theta$. However, when two superconductors, 1 and 2, are coupled by means of a weak link only, the coherence may be broken. A change of the phase difference, $\Delta \theta = \theta_1 - \theta_2$, with time then leads to interference effects under suitable conditions.

In superfluid helium the experimentally feasible weak link is a small orifice connecting two liquid reservoirs. Many experiments have been performed to find the ac Josephson effect in He II. During several of these measurements, flow between two reservoirs, connected by a small orifice, showed a step structure in the presence of ultrasonic modulation. There exists, however, rather convincing evidence that at least most of the reported phenomena can be explained by standing acoustic waves. A recent experiment on $^3$He II showed no signs of the Josephson effect. It must therefore be concluded that Josephson phenomena have not been found experimentally in He II.

We describe in this Brief Report the first attempt to observe the ac Josephson effect in superfluid $^3$He. Our work was encouraged by the much longer coherence length $\xi_0$ in $^3$He (80 nm at pressure $P = 0$) than in He II (0.2 nm). In experiments on He II the dimensions of the orifice were three to five orders of magnitude larger than $\xi_0$. The coupling through a hole of this size may be too strong for sufficient suppression of phase coherence; in superconductors the width of the microbridges is typically a few times the coherence length. In the case of $^3$He, preparation of channels with dimensions comparable with $\xi_0$ is possible.

The orifice, depending on its size, may act as a weak link in two different ways. When its diameter $d \gg \xi_0$ the superfluid path is not broken inside the channel but the phase difference may change with time, for example, owing to emission of vortex rings from the orifice. When $d < \xi_0$, $\psi_0$ is reduced from its value in the bulk liquid and, in principle, a tunneling superflow can exist.

The length of the channel $L$ is also of crucial importance. The usual criterion is that $L$ must be comparable with $\xi_0$ for the ac Josephson effect to be observable. This follows from the requirement that $\Delta \theta \sim 2\pi$ and from the fact that $\Delta \theta \sim L/\xi$ if the flow velocity is near the value at which Cooper pairs of $^3$He atoms break; here $\xi = \xi_0 (1 - T/T_c)^{1/2}$ is the temperature-dependent coherence length. When $\Delta \theta \gg 2\pi$ the reduction in the superflow due to one phase slippage of $2\pi$ is small, leading to a low amplitude of the Josephson current.

Our experimental apparatus is shown schematically in Fig. 1. The mechanical construction of the $^3$He chamber is essentially that described in Ref. 9. The flexible wall between the two $^3$He volumes was made of aluminized Mylar, and it forms a sensitive capacitive pressure gauge. The displacement $x$ of this diaphragm was measured using a capacitance bridge operating at 9 kHz with 3-V amplitude; the resolution in $x$ was about 0.01 nm. The other side of the double capacitor was employed for producing a pressure

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{diagram.png}
\caption{Schematic illustration of our experimental $^3$He chambers (Ref. 9). The liquid volume $V = 2$ cm$^3$ in each compartment and the surface area of the circular diaphragm $A = 2.7$ cm$^2$.}
\end{figure}

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difference $\Delta P_{dc}$ by applying a dc bias voltage $U_{dc}$, which was either kept constant or swept from zero so that $U_{dc}^2$ and thus also $\Delta P_{dc}$ increased linearly with time.

A simple capacitive transducer, placed at a distance of less than 1 mm from the flow orifice, was used to produce an ac pressure wave. Because the wavelength of first sound at a typical frequency of 50 kHz is about 4 mm, the ac pressure amplitude $\Delta P_{ac}$ is significantly attenuated between the transducer and the orifice. At low frequencies, $\Delta P_{ac} \approx \epsilon_0 s^2 U_0 U_{ac}$, assuming that the elastic restoring force of the membrane is small; here $U_0$ is the dc bias voltage of the transducer, $U_{ac} \sin 2\pi ft$ is the applied ac voltage, $s \approx 0.1$ mm is the distance between the capacitor plates, and $\epsilon_0$ is the dielectric constant. Above the resonance frequency of the transducer ($f_0 = 1$ kHz) the response decreases approximately as $1/f^2$.

In our experiments (cf. Table I) we used two basically different weak link geometries. The first was made of Nuclepore filter, which is a polycarbonate membrane containing a large number of etched particle-track holes. The nominal channel diameter varied from 30 to 80 mm; the thickness of the membrane was either 5 or 10 $\mu$m. The second type of weak link was a single hole, prepared in a Mylar foil with use of a laser beam.

The behavior of our apparatus under the bias voltage $U_{dc}$ on one side of the double capacitor is described by the equation of motion for the diaphragm

$$\eta \frac{d^2 x}{dt^2} = \alpha U_{dc}^2 - \lambda x - \Delta P_{dc},$$

where $\eta$ is the mass per unit area of the diaphragm, $x$ is its displacement from equilibrium, $\alpha U_{dc}^2$ is the electrostatic force, $\lambda x$ is the elastic restoring force caused by the tension of the diaphragm, and $\Delta P_{dc}$ is the pressure difference between the two compartments. We have neglected in Eq. (1) the effect of the ac modulation, of the order of $(v/V) \Delta P_{ac}$, because the volume $V$ between the transducer and the orifice is about 1% of the total volume $V$ of each compartment. Conservation of mass requires that the supercurrent

$$J_a = \rho (A/a) \frac{dx}{dt},$$

where $\rho$ is the density of the liquid, $A$ is the area of the diaphragm, and $a$ is the total cross section of the holes in the weak link. In Eq. (2) we have omitted a small term caused by the compressibility of the liquid.

Assuming an idealized current-phase relationship we can write

$$J_a = J_0 \sin \Delta \theta + J_b,$$

where $\Delta \theta$ satisfies the Josephson equation

$$\frac{d^2 \Delta \theta}{dt^2} = \frac{2m_3}{\rho} + \left[ \Delta P_{dc} \Delta P_{ac} \sin(2\pi ft) \right];$$

$2m_3$ is the mass of a Cooper pair. In Eq. (3) we have added a “background” supercurrent $J_b$. In this form Eq. (3) approximates the case of a long channel, where the current-phase relationship is multivalued and $J_a$ oscillates around a nonzero $J_b$.

The numerical solution of Eqs. (1)–(4) for the measurable quantity $x$ as a function of time shows steps when the Josephson condition

$$\Delta P_{dc} = (h \rho/2m_3) n f$$

is satisfied; $n = 1, 2, \ldots$. The rate of change of $x$ in the vicinity of the steps is determined by $J_0$ and by $dU_{dc}/dt$. The height and shape of the steps is a function of $J_0$, $J_b$, $\Delta P_{ac}$, $f$, $\lambda$, $A$, and $dU_{dc}/dt$. The inertia of the diaphragm, entering as $\eta (d^2 x/dt^2)$, is unimportant since the quantities on the right-hand side of Eq. (1) vary slowly in comparison with 3 kHz, the natural frequency of the diaphragm.

The heights $\Delta x$ of the calculated first and second steps are indicated in Fig. 2. The parameter values used are given in the figure caption. In multihele geometries $J_s << J_a$. Therefore $J_0$ was set equal to $J_a = 0.5$ kg/m$^2$s; this value was calculated from the theoretical$^{11}$ depairing current

$$J_0 = 3.06(1 - T/T_c)^{3/2} \text{ kg/m}^2\text{s}$$

in $^3$He-B at $P = 0$ and $T/T_c = 0.7$. The dependence of $\Delta x$ on the Josephson current follows closely the relation $\Delta x \propto (J_0 f)^{1/2}$, and, as a function of $\Delta P_{ac}$ and $f$, it has the Bessel function behavior

$$\Delta x \propto \left[J_a \left(2m_3/\rho \Delta P_{ac} h f^{1/2} \right)^{1/2}\right].$$

The first step [cf. Eq. (5)] in Fig. 2 is seen at $\Delta P_{dc} = 0.54$ Pa; its structure, however, is limited to a narrow interval which is not resolvable on our time scale.

The aim of our experiment was to find anomalies in the flow pattern, as suggested by the simple model, and to see

<table>
<thead>
<tr>
<th>Hole diameter $d$ ($\mu$m)</th>
<th>Hole length $L$ ($\mu$m)</th>
<th>Number of holes</th>
<th>Pressure $P$ (bar) $0.1 \text{ MPa}$</th>
<th>Frequency $f$ (kHz)</th>
<th>$\Delta P_{ac}$ (Pa)</th>
<th>Resolution in $x$ (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>10</td>
<td>$3 \times 10^5$</td>
<td>0.35, 25</td>
<td>1–1000</td>
<td>0.01–3</td>
<td>0.5</td>
</tr>
<tr>
<td>0.05</td>
<td>5</td>
<td>$3 \times 10^3$</td>
<td>0</td>
<td>1–200</td>
<td>0.01–10</td>
<td>0.01</td>
</tr>
<tr>
<td>0.03</td>
<td>5</td>
<td>$1.8 \times 10^3$</td>
<td>0</td>
<td>1–500</td>
<td>0.01–10</td>
<td>0.01</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>1</td>
<td>0</td>
<td>1–1000</td>
<td>0.01–10</td>
<td>0.01</td>
</tr>
</tbody>
</table>

TABLE I. Parameters of Josephson measurements (in all experiments $0.6 < T/T_c < 1$, $\Delta P_{dc} = 0$–5 Pa, sweep time of $U_{dc}$ from 0 to 100 V: 0–670 s).
FIG. 2. The displacement x of the diaphragm vs time in an experiment where \( U_{dc} \) was swept from 0 to 100 V in 67 s. The sizes of the first two steps according to our model calculations [cf. Eqs. (1)–(6)] are shown in the figure and correspond to the following parameter values: \( \rho = 82 \frac{kg}{m^3}, \alpha = 4.9 \times 10^{-4} \frac{Pa}{V^2}, \lambda = 1.8 \times 10^5 \frac{Pa}{m}, A_1/a = 4.5 \times 10^4, U_0 = 205 \, V, \, U_{dc} = 3 \, V, \, \Delta P_{dc} = 0.54 \, Pa, J_s = 0.5 \frac{kg}{m^2s}, \) and \( f = 100 \, kHz \). Values of \( \Delta P_{dc} \) are shown on the upper scale. By chance, \( \Delta P_{dc} \approx \Delta P_{dc} \) at the calculated location of the first step.

whether they satisfy the Josephson condition of Eq. (5). Several types of experiments were performed. In the simplest case \( f \) was kept constant while \( \Delta P_{dc} \) was changed by sweeping \( U_{dc} \). Measurements during which \( f \) was swept with approximately constant \( \Delta P_{dc} \) were also performed. Signals were recorded digitally and thus averaging over many measurements was possible. Experimental parameters were varied within broad limits; this may be seen from Table I.

No sign of the behavior expected for a Josephson effect was observed in any of our measurements. Moreover, no effect whatsoever of the high-frequency transducer on the flow was found; proper operation of the transducer was verified before and after each cooldown. Figure 2 shows the displacement of the diaphragm as a function of time in a measurement where \( U_{dc} \) was swept, in 67 s, from 0 to 100 V. With our resolution the steps predicted by the model calculations should be observable.

Most of our measurements (cf. Table I) were performed under zero pressure in the B phase at \( 0.6 < T/T_c < 1 \). A few runs were made in the A phase at 25 bars pressure with similar negative results.

The crucial problem in a Josephson experiment is the weak link; one does not know a priori the most favorable geometry. There may be reasons why the Nuclepore filters are not suitable; with many long channels in parallel, phase slippage in each channel separately may smear out the possible step structure. A single small hole is a more ideal weak link, but in this case the resolution becomes the limiting factor because the total current is proportional to the area of the hole. In a large orifice, on the other hand, the background current \( J_0 \) is high owing to strong coupling across the channel and, consequently, \( J_0 \) is small, which leads to a small step.

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