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We consider a design for a cyclic microrefrigerator using a superconducting flux qubit. Adiabatic modulation of the flux combined with thermalization can be used to transfer energy from a lower temperature normal metal thin film resistor to another one at higher temperature. The frequency selectivity of photonic heat conduction is achieved by including the hot resistor as part of a high frequency LC resonator and the cold one as part of a low-frequency oscillator while keeping both circuits in the underdamped regime. We discuss the performance of the device in an experimentally realistic setting. This device illustrates the complementarity of information and thermodynamic entropy as the erasure of the quantum bit directly relates to the cooling of the resistor.

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I. INTRODUCTION

For the purpose of quantum computing, the coherence properties of superconducting quantum bits (qubits) should be optimized by decoupling them from all noise sources as well as possible. However, many interesting experiments can also be envisioned when the decoupling is far from perfect. One such experiment closely related to coherence optimization is using a qubit as a spectrometer for the environmental noise by monitoring the effect of the environment on the quantum two-level system. Here, we focus on the opposite phenomenon, i.e., the effect of a qubit on the environment. Recently, a superconducting flux qubit with a quite small tunneling energy from the point of view of quantum computing was cooled using sideband cooling and a third resistor from about 400 down to 3 mK. Motivated by this experiment, we consider the possibility of using a single quantum bit as a cyclic refrigerator for environmental degrees of freedom. The utilized heat conduction mechanism is photonic, which was also recently studied in the experiment. Besides the possible practical uses, the device is interesting physically as it directly illustrates the connection between information entropy and thermodynamical entropy. For related superconducting coolers, see, e.g., Refs. 9–11.

II. FLUX-QUBIT COOLER AND THE THERMODYNAMIC CYCLE

We study a flux qubit coupled inductively to two different loops, as shown in Fig. 1(a). In loop $j$ ($j=1,2$), we have a resistor $R_j$ in series with an inductor $L_j$ and a capacitor $C_j$. These form two damped harmonic oscillators. The resistors are in general at different temperatures $T_1$ and $T_2$. The coupling of the qubit to the admittances of the two loops, $Y_1$ and $Y_2$, is assumed to be sufficiently large to dominate the relaxation of the qubit. This assumption can be easily validated by, e.g., increasing the mutual inductance. The flux qubit is an otherwise superconducting loop except for three or four Josephson junctions with suitably chosen parameters. In particular, one of the junctions is made smaller than the others to form a two-level system. When biased close to half of the flux quantum $\Phi_0=h/2e$, the qubit can be described (in persistent current basis) by the Hamiltonian

$$H/h = -\frac{1}{2}(\Delta \sigma_x + e\sigma_z),$$

where $\sigma_x$ and $\sigma_z$ are Pauli matrices, $h\epsilon = 2\Gamma P (\Phi-\Phi_0)/2$ is the flux-tunable energy bias, and $\Phi$ is the controllable flux threading the qubit loop. Away from $\Phi=\Phi_0/2$, the eigenstates have the persistent currents $\pm i P$ circulating in the loop. The tunneling energy $h\Delta$ results in an anticrossing at $\Phi = \Phi_0/2$ and there the expectation value of current of the energy eigenstates is zero. The transition angular frequency of the qubit is $\omega = \sqrt{\epsilon^2 + \Delta^2}$.

Consider the ideal cycle shown in Figs. 1(b) and 1(c) where the bias of the flux qubit is swept slowly (at a frequency $\omega$). In the example shown, two loops are used, and the cycle in the qubit temperature-entropy plane is shown in Fig. 1(c). The temperature of the hot resistor is assumed to be sufficiently large to dominate the relaxation of the qubit.
frequency $f$ slower than $\Delta/2\pi$) between two extreme values $\epsilon_1$ and $\epsilon_2$ corresponding to two different energy level separations $\hbar \epsilon_1$ and $\hbar \epsilon_2$. Let us further assume that $\omega_{12} = \omega_{LC} + Q_1$ and $Q_2 \gg 1$, where $\omega_{LC} = \sqrt{L_j/C_j}$ and $Q_1 = L_j/C_j/R_1$. This choice guarantees that the qubit mainly couples to resistor $R_1$ ($R_2$) at bias point 1 (2). We emphasize that although both resistors are considered baths from the point of view of qubit relaxation, only resistor 2 is strictly assumed to be a heat reservoir, whereas resistor 1 has to be “small” in order to cool down. The cooling cycle consists of steps O, P, Q, and R. First, in step O, the qubit has the angular frequency $\omega_2$ and is allowed to thermalize. Because of the bandwidth limitations imposed by the reactive elements, the qubit tends to thermalize with resistor $R_2$ to temperature $T_2$. However, ideally, the qubit splitting is large enough such that the thermal population is small. In the next step P, the flux bias is ideally pure quantum state of the qubit gets erased and the entropy of resistor 1 decreases such that one can say that some information is “stored” in the resistor as it thermalizes. The golden rule transition rates between the instantaneous $\epsilon$-dependent eigenstates due to resistor $j$ are given by

$$\Gamma_j^{\pm} = \frac{2\pi}{\hbar^2} |\langle 0; \epsilon | dH/d\Phi | 1; \epsilon \rangle|^2 M_j^2 S_j(\pm \omega_j),$$

where the positive sign corresponds to relaxation. The total thermalization rate is $\Gamma_j = \Gamma_j^+ + \Gamma_j^-$. Here, the unsymmetrized noise spectrum is given by

$$S_j(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega t} (\delta I_1(0) \delta I_1(t)) dt$$

$$= \frac{1}{2\pi} \left[ \frac{2\hbar \omega \text{Re} Y_j(\omega)}{1 - \exp(-\beta \hbar \omega)} \right],$$

where $\text{Re} Y_j(\omega) = R_j^{-1} \left[ 1 + Q_j^2 \left( \frac{\omega}{\omega_{LC} - \frac{\omega_{LC}}{\omega}} \right)^2 \right]$ is the real part of admittance of circuit $j$. The total relaxation rate is thus

$$\Gamma_{\text{th}} = \frac{2(I_f \Delta M_j)^2 \coth \left( \frac{\beta \hbar \omega}{2} \right)}{R \hbar \omega \left[ 1 + Q_j^2 \left( \frac{\omega}{\omega_{LC} - \frac{\omega_{LC}}{\omega}} \right)^2 \right].}$$

To model the behavior of the device, we utilize the Bloch master equation given in our case by
\[
\dot{\mathbf{M}} = -\mathbf{B} \times \mathbf{M} - \Gamma_{1i} \mathbf{M}_{1i} + \mathbf{M}_{T1} - \Gamma_{2i} \mathbf{M}_{1i} - \mathbf{M}_{T2} - \Gamma_{2i} \mathbf{M}_z,
\]

where \(\mathbf{M} = \text{Tr}(\hat{\sigma}_d)\) is the “magnetization” of the qubit and \(\mathbf{B} = \Delta \mathbf{e} + \epsilon \mathbf{z}\) is the fictitious magnetic field. Note, however, that the \(z\) component of \(\mathbf{B}\) and \(\mathbf{M}\) do correspond to real magnetic field and magnetization, respectively. In Eq. (7), \(M_i\) and \(M_{\perp}\) are the components of the magnetization parallel and perpendicular to \(B\), respectively. These are explicitly

\[
\dot{M}_i = \frac{1}{\omega^2} (\Delta M_x + \epsilon M_z) (\Delta \mathbf{e} + \epsilon \mathbf{z}),
\]

and \(\Gamma_{z} = (\Gamma_{1i} + \Gamma_{2i})/2\) is the dephasing rate. We neglect pure dephasing due to the intentionally large dominating thermalization rate. Equation (7) describes the dynamics of a quantum two-level system coupled to two dissipative baths. The baths tend to relax the qubit toward instantaneous equilibrium with two competing rates. Equations of this type are often used in the stationary case, but they also yield very good predictions for strong driving as, for instance, in the case of the Landau-Zener interference. \(^{16}\) As is obvious from Eq. (7), the qubit actually tends to relax toward an effective \(\epsilon\)-dependent equilibrium magnetization

\[
\dot{M}_T = \left( \frac{\Delta \mathbf{e} + \epsilon \mathbf{z}}{\omega} \right) \sinh \left( \frac{\beta \mathbf{H}}{2} \right),
\]

and \(\Gamma_{z} = (\Gamma_{1i} + \Gamma_{2i})/2\) is the dephasing rate. The den-

IV. SIMULATION RESULTS

To illustrate the practical potential of the device, we show in Fig. 2 the simulated cooling power with sinusoidal driving of \(\epsilon(t)\) compared to the ideal case along with the actual loop in the entropy temperature plane. The heat flow \(P_j\) from resistor \(j\) to the qubit is simply obtained by integrating the product of the thermalization rate and the energy deficit, i.e.,

\[
P_j = \int_{0}^{\infty} d\omega \{ [\rho(\omega)] [\beta_j, \epsilon(t)] H] - \text{Tr}[\rho(\omega) H] \}.
\]

The density matrix \(\rho(t) = 1/2 \mathbf{M}(t) \mathbf{\sigma}\) is solved numerically using the Bloch equation (system is followed over a few periods until it has converged to the limit cycle). We see that the actual simulated behavior does not significantly deviate at low \(f\) from the ideal behavior and that cooling powers on the order of femtowatts can be achieved with reasonable sample parameters. The oscillatory behavior at high \(f\) is interpreted as the Landau-Zener interference. \(^{16,17}\) The ideal operation frequency for cooling would be the highest frequency where the cycle is still fairly adiabatic, i.e., about 1 GHz is the optimal frequency in the present example.

\[
\int_{0}^{\infty} \frac{d\omega}{2\pi} [4\hbar \omega^3 M^2 \text{Re} Y_1(\omega) \text{Re} Y_2(\omega)(n_1(\omega) - n_1(\omega))] = P_{\gamma}
\]

where \(n_1(\omega) = [\exp(\beta \hbar \omega) - 1]^{-1}\) are the boson occupation factors and \(M\) is the mutual inductance between the loops. For detuned well underdamped resonators, the photonic heat conduction turns out to be quite negligible. For instance, for the values of Fig. 3 with \(M = 5\) pH and \(R_1 = R_2 = 1\) \(\Omega\), we get only \(P_{\gamma} = 2 \times 10^{-18}\) W even if \(T_1 = 0\) K and \(T_2 = 300\) mK. Figure 2 illustrates the calculated equilibrium temperature versus operation frequency obtained numerically by finding the
balance between the dominating phononic heat conduction and the integrated cooling power. We see that almost a factor of 2 reduction of $T_1$ is possible with realistic parameters.

In practice, the drop of $T_1$ can be measured, e.g., using a SINIS (superconductor-insulator-normal metal-insulator-superconductor) thermometer,\textsuperscript{19,20} in which resistor 1 will serve as the normal metal $N$. Its reading is sensitive to the electronic temperature of $N$ only, and self-heating can be made very small. The resistors should be made out of thin film normal metals such as copper or gold with typically sub-1-$\Omega$ sq resistance. Volume can be picked freely. To get the resonant frequencies and quality factor as above, we need $L_1=320$ pH, $C_1=3.2$ pF, $L_2=80$ pH, and $C_2=0.8$ pF, which are also realistic. For the inductor, one may use either the Josephson or the kinetic inductance of superconducting wire while the capacitance values are similar to those in typical flux qubits.\textsuperscript{2} To satisfy the conditions of the above numerical example, we need quite large mutual inductances which, however, can be easily achieved using, e.g., kinetic inductance.\textsuperscript{21} The strong driving also requires rather large inductance between the microwave line and the qubit, which should not result in uncontrolled relaxation. For instance, $M_{mw}=5$ pH coupling (which is realistic) to the control line is acceptable as it would result in at most $3\times10^7$ s$^{-1}$ relaxation rate assuming a $50$ $\Omega$ environment at 0.3 K. This choice will not degrade the performance of the device significantly since driving is much faster. Yet, sufficiently strong driving corresponding to the example in Fig. 2 can be achieved with a modest 3 $\mu$A ac current. Fabrication process most likely will require three lithography steps.

\section{V. Conclusions}

In conclusion, we have described a method of using a superconducting flux qubit driven strongly, yet adiabatically, at microwave frequency to cool an external metal resistor. Here we considered $LC$ resonators to achieve the required frequency selectivity but a coplanar waveguide resonator or a mechanical oscillator could be used in principle, too. We demonstrated by a numerical example that it is possible to observe the associated temperature decrease experimentally. This effect is directly related to the loss of information and thus to the increase of entropy of the quantum bit.

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\begin{figure}[h]
\includegraphics[width=0.5\textwidth]{fig3}
\caption{Equilibrium temperature as a function of pump frequency for three different phonon bath temperatures. The temperature of resistor 1 (volume, $10^{-21}$ m$^3$) is shown with a dashed line, while the temperature of resistor 2 (volume, $10^{-19}$ m$^3$) is shown with a solid line. The bath temperatures $T_{ph}$ are made very small. The resistors should be made out of thin film normal metal such as copper or gold with typically sub-1-$\Omega$ sq resistance. Volume can be picked freely. To get the resonant frequencies and quality factor as above, we need $L_1=320$ pH, $C_1=3.2$ pF, $L_2=80$ pH, and $C_2=0.8$ pF, which are also realistic. For the inductor, one may use either the Josephson or the kinetic inductance of superconducting wire while the capacitance values are similar to those in typical flux qubits.\textsuperscript{2} To satisfy the conditions of the above numerical example, we need quite large mutual inductances which, however, can be easily achieved using, e.g., kinetic inductance.\textsuperscript{21} The strong driving also requires rather large inductance between the microwave line and the qubit, which should not result in uncontrolled relaxation. For instance, $M_{mw}=5$ pH coupling (which is realistic) to the control line is acceptable as it would result in at most $3\times10^7$ s$^{-1}$ relaxation rate assuming a $50$ $\Omega$ environment at 0.3 K. This choice will not degrade the performance of the device significantly since driving is much faster. Yet, sufficiently strong driving corresponding to the example in Fig. 2 can be achieved with a modest 3 $\mu$A ac current. Fabrication process most likely will require three lithography steps.

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