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Nongalvanic thermometry for ultracold two-dimensional electron domains

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Measuring the temperature of a two-dimensional electron gas at temperatures of a few mK is a challenging issue, which standard thermometry schemes may fail to tackle. We propose and analyze a nongalvanic thermometer, based on a quantum point contact and quantum dot, which delivers virtually no power to the electron system to be measured.

The availability of high-mobility two-dimensional electron gases (2DEGs), combined with the ability to cool them down to low temperatures, has led to the discovery of outstanding physical phenomena, such as the quantum Hall effect. Refrigeration schemes are currently under investigation to cool the 2DEG below the conventional operating temperature of a dilution fridge (around 20 mK), down to 1 mK or below.2,3 This achievement would open the way to a range of experiments of fundamental relevance and to a number of applications: electron interferometry,4 study of correlated phases5 and exotic effects,6 charge pumping,7 quantum computing,8,9 and so on.

Several different types of electron thermometers have been proposed and realized.10 However, as the temperature of electrons gets down to the mK range and below, finding a proper way to measure it in a non-invasive way becomes a critical issue. With the coupling between electrons and phonons becoming weaker and weaker, the power load that a micrometer-sized electron domain can sustain without overheating shrinks down to a few aW or less. In this regime, detection schemes based on transport measurements, such as the “conventional” quantum dot thermometer (QDT), become impractical as they inject high-energy quasiparticles which heat the system up, when not bringing it out of thermal equilibrium.

In this letter, we propose nongalvanic thermometry for 2DEGs. Here, nongalvanic refers to the absence of current transport between the measured electron system and the thermometer leads. We start with a quick review of the QDT. Then, we introduce its nongalvanic counterpart, whose building blocks are a quantum dot (QD) and a quantum point contact (QPC). This device delivers virtually no power to the electron domain to be measured. We model its operation with standard theory and analyze its performance by choosing realistic parameters. Finally, we discuss the problem of measurement backaction.

An implementation of the QDT is shown in Fig. 1(a). The QD, typically defined by split-gate confinement, is connected by tunnel barriers to two distinct 2DEG regions, one of which is the electron domain to be measured. At zero bias, every time a resonant level of the dot crosses the Fermi energy of the leads, the conductance displays a Coulomb-blockade peak.11 If the two leads share the same temperature, the latter is simply determined from the peak linewidth, to which it is proportional.12 On the other hand, when the temperature of the source and drain leads are different, one can still detect the two temperatures independently by applying a voltage bias much greater than the thermal energy of the hotter lead, or even with a single zero-bias measurement, provided the temperature difference is large enough.13

Based on a transport measurement, this scheme unavoidably brings in dissipation. Of the total power dissipated during the operation of the thermometer, let us estimate the fraction $Q_R$ that goes to the domain. This is associated to the tunneling of hot quasiparticles, contributing a heat flow $Q_R = \Gamma f(E)$, where $\Gamma$ is the coupling strength between the resonant level of the dot and the domain (assumed to be energy and temperature-independent), $E$ is the energy of the resonant level (with respect to the Fermi energy of the domain), and $f$ is the electron distribution function in the

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FIG. 1. Galvanic (a) versus nongalvanic (b) QDT. In (a), temperature is determined by the linewidth of Coulomb-blockade peaks, obtained from a transport measurement. In (b), from the average occupation of the dot, read out in a nongalvanic fashion by a QPC placed nearby.
domain. We shall assume that a quasiequilibrium regime holds, so that \( f(E) = [1 + \exp(E/k_BT_e)]^{-1} \), \( T_e \) being the temperature of the domain.

To perform the readout, we must vary \( E \) in a range wide enough to characterize the spread of the Fermi distribution. For definiteness, we set this range to \([-3k_BT_e, 3k_BT_e]\), so that \( f \) takes values between 0.05 and 0.95. Averaging over such a sweep, we obtain \( \langle Q_R \rangle \approx 0.55 \Gamma k_BT_e \). Now, a lower bound for \( \Gamma \) comes from the need for adequate signal-to-noise ratio, the current at resonance being of the order of \( e^2R \), Coulomb-blockade thermometry requires thermal broadening of the peak to dominate above intrinsic (Lorentzian) broadening. This condition, which must hold regardless of dissipation, reads \( h\Gamma \ll k_BT_e \); for \( T_e = 10 \text{ mK} \), it gives \( \Gamma \ll 200 \text{ MHz} \).

In the following, we will assume \( \Gamma = 10 \text{ MHz} \), which according to our estimate corresponds to \( Q_R/T_e \approx 80 \text{ aW/K} \). This figure must be compared to the cooling power provided by all relevant heat-relaxation channels. For definiteness, let us take as the electron a portion of a GaAs/AlGaAs 2DEG of representative density and mobility. At subkelvin temperatures, the heat flow from electrons into phonons is given (for GaAs-based 2DEGs) by the expression \( Q_{e-ph} = \Sigma A (T_e^5 - T_{ph}^5) \), where \( T_{ph} \) is the temperature of the phonon bath, \( A \) is the area of the domain, and \( \Sigma \) is a constant of the order of \( 30 \text{ fW} \mu \text{m}^{-2}\text{K}^{-5} \).

In Figure 2, we plot the steady-state \( T_e \) for 1 and 100 \( \mu \text{m}^2 \)-sized domains, versus \( T_{ph}, T_e \) is determined from a power balance equation of the form \( \sum_i Q_i/T_e = 0 \), with \( Q_i \) denoting the heat flow into the domain due to the \( i \)th channel.

Each curve refers to a different configuration, to be discussed below. The straight line marked \( T_e = T_{ph} \) is plotted for reference, and stands for the case where no additional heat load is put on the domain. As soon as the QDT is introduced, the situation changes dramatically: \( T_e \) follows \( T_{ph} \) only down to about 100 mK, below which a saturation occurs. This is due to the weakening of electron-phonon interaction, which is no longer able to carry the dissipated heat away. Furthermore, as \( Q_{e-ph} \) scales with the domain area, the smaller domain saturates at a higher \( T_e \). The ineffectiveness of the electron-phonon coupling at these temperatures has recently motivated the development of electronic coolers. We take this possibility into account by considering the case where a quantum-dot refrigerator (QDR) \( Q_{QDR} = CT_e^2 \), with \( C = 0.31 \text{ pW/K}^2 \). Thanks to the QDR, the curves with QDT + QDR now saturate at much lower temperatures, of the order of 1 mK or below. Notice that the saturating \( T_e \) no longer depends on the domain area; this is because at such low temperatures, the competition is between the QDT and the QDR, with the phonon bath playing little or no role. For simplicity, in the discussion above, we have included no other sources of heat besides the QDT. In reality, the electronic temperature is eventually limited by parasitic heat sources, such as radiation from higher-temperature stages and noise in the electrical lines. Likewise, the performance assumed for the QDR must be taken as an idealization: a recent experiment pointed out deviations from the ideal behavior already at 110 mK, possibly due to nonequilibrium effects.

The nongalvanic device that we propose is shown in Fig. 1(b). As in the QDT discussed above, the strongly nonlinear density of states of a QD is exploited to probe the energy distribution of the domain. All the difference lies in the way this information is read out: instead of performing a transport measurement across the dot, we measure its average occupation in a nongalvanic fashion with the help of a QPC placed nearby. If the gate sweep is performed adiabatically, the heat flow into the domain is minimal, making the nongalvanic thermometer a candidate device for temperature measurements of ultracold electron domains. In the following, we will describe its operation with a quantitative model.

Let us start from the QD. The latter is preferably operated in the “quantum” Coulomb blockade regime, meaning that both its charging energy and orbital level spacing are much greater than the thermal energy. As a result, electron tunneling only takes place between the dot and a single energy level. As for the galvanic QDT, we further require \( \hbar\Gamma \ll k_BT_e \), so that we can neglect intrinsic broadening effects. The mean occupation of the level is then given by

\[
\langle n_{dot} \rangle = f(E_{0}^{DD} - e\alpha V_G),
\]

where \( E_0^{DD} \) is a reference energy for the level and \( \alpha \) the lever arm of the gate on the dot.

Our next question is how the change in \( \langle n_{dot} \rangle \) affects the current \( I \) through the QPC, in the presence of a voltage bias \( V_b \). In the Landauer-Büttiker formalism, \( I = \frac{2e}{h} \int_{-\infty}^{\infty} dET(E,E^{OPC})[f(E-eV_b,T_L)-f(E,T_L)] \), where \( T_L \) is the temperature of the QPC leads (in general, \( T_e \neq T_L \)) and \( T(E,E^{OPC}) \) is the energy-dependent transmission coefficient of the QPC. Assuming a single ballistic channel and using a saddle potential, \( T(E,E^{OPC}) = \{1 + \exp[-2\pi(E-E^{OPC})/\hbar\omega_s]\}^{-1} \), where \( \omega_s \) is a characteristic energy of the confinement and \( E^{OPC} \) denotes the bottom of the potential for the one-dimensional electron channel defined by the QPC. Upon
changing $V_g$, the potential landscape at the QPC changes due to the capacitive couplings QPC-QD and QPC-gate. As these couplings are small, we regard them as perturbations and model their effect by a shift of the potential $E_{QPC}$ with respect to a reference value $E_0^{QPC}$. The latter is tuned by the gates defining the constriction and defines the working point of the QPC. We shall further denote by $\beta$ the lever arm of the dot on the QPC, and by $\gamma$ that of the gate. In general, we expect $\gamma \ll \beta$. Then we write $E_{QPC}$ as

$$E_{QPC} = \beta e^2 \langle n_{dot} \rangle / C_\Sigma - e\gamma V_G - E_0^{QPC}. \quad (2)$$

In the limit $eV_b, k_B T_L \ll h\omega_m$, $T$ is approximately constant in the range where the electron distributions of the leads vary. The expression for $T$ then simplifies as $I = \frac{2}{\hbar} T(0, E_{QPC}) V_b$. Notice that $T_L$ no longer appears in this expression. By contrast, $T_c$ determines $\langle n_{dot} \rangle$, which affects $E_{QPC}$ and hence $T_c$.

In Fig. 3(a), we plot $I$ versus $V_g$ for different values of $T_c$. As $V_g$ is made more negative, $I$ steadily decreases due to the spurious coupling between the gate and the QPC. Yet as the resonant level crosses the Fermi energy of the domain from above, $\langle n_{dot} \rangle$ sharply decreases by one, leading to a step-like increase in $I$. This gives rise to a sawtooth pattern at zero temperature, which gets progressively smeared as $T_c$ is increased.

Besides $I$, a relevant quantity for thermometry is the gate-to-QPC transconductance $G_{tr} = dI/dV_g$, which can be directly measured using a lock-in amplifier. By direct calculation, we find

$$G_{tr} = \frac{2e^2}{h} eV_b \frac{dT}{dE_{QPC}} \left( \gamma + 2\beta \frac{e^2}{C_\Sigma} \frac{df}{dE} \right). \quad (3)$$

As a function of $V_g$, a series of dips appear on top of a positive baseline [see Fig. 3(b)]. The dips are proportional to the derivative of the Fermi function, and their FWHM $\Delta V_g$ to the domain temperature $T_c$. Explicitly,

$$T_c = \frac{e\gamma}{2\log(3+2\sqrt{2})k_B} \Delta V_g. \quad (4)$$

The constant relating $\Delta V_g$ to $T_c$ is a simple combination of fundamental constants and the lever arm $\gamma$, which can be determined experimentally from a measurement of the QD charging energy and the cross-capacitance between the gate and the QD. This fact makes of the nongalvanic QDT a primary thermometer, i.e., a thermometer that can measure absolute temperatures without relying on other thermometers (e.g., for calibration).10

We conclude this discussion by giving a figure of merit for each measurement mode. If we choose to measure $I$, such a figure may well be the current gain $A_I = \delta I / \delta T_c$, which is eventually limited when $T_c < 100 \mu K$. Since $A_I$ scales as the inverse of $T_c$, the lower the temperature, the higher the gain. Yet, the sharpness of the sawtooth also increases at lower temperature, so that the measurement becomes more and more sensitive to the dot potential. Fluctuations of $V_g$ of the order of $1 \mu V$, included in the model, are responsible for the bending of the curves below $50 \mu K$. As for $G_{tr}$, we can proceed in the same way and define a transconductance gain $A_{G} = \delta G / \delta T_c$. $A_{G}$ is plotted in the inset of Fig. 3(b).

Similarly to $A_I$, $A_G$ is also maximized when $T_c = 100 \mu K$. $A_G \approx 10 \mu S / mK$. The dependence on $T_c$ is the same as for $A_I$. At very low $T_c$, $A_G$ is eventually limited by the amplitude of the lock-in modulation applied to $V_g$.

So far, we have implicitly assumed that the state of the QD is not influenced by our readout procedure; that is, we have neglected any measurement backaction. In the following, we shall take it into account and show that its effects are indeed negligible in a suitable range of parameters. In doing so, we are led to consider two different mechanisms: current fluctuations through the QPC (that is, shot noise) and charge fluctuations in the QPC (Refs. 29 and 30). The nature of these two is very different. In particular, the way current fluctuations...
fluctuations couple to the dot depends on the specific measurement circuit. By contrast, the backaction due to charge fluctuations is fundamentally unavoidable. Indeed, it is related to the Heisenberg backaction of the detector (QPC) on the quantum system whose state we are measuring (QD).

We shall describe both mechanisms using the theory of photon-assisted tunneling (PAT). \(^{31}\) Let \(S_T(\omega)\) be the spectrum of voltage fluctuations on the dot; the probability of PAT with energy \(E\) is then \(P(E) = \frac{1}{\pi} \int_{-\infty}^{\infty} \text{Re} (J(\omega) + i E \frac{l}{\hbar} f(\omega)) d\omega\), where the phase-phase correlation function \(J(\omega)\) is related to \(S_T(\omega)\) by \(J(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{S_T(\omega')}{\omega'^2} (e^{-i\omega \tau} - 1) d\omega'.\) The modified \(\langle n_{dot}\rangle\), accounting for PAT, is given by

\[
\langle n_{dot} \rangle = \int_{-\infty}^{\infty} f(E - E_0^{\text{QD}} - eZ\nu) P(E) dE, \tag{5}
\]

which is a convolution of the distribution function of the domain with the \(P(E)\) function. Even in the presence of PAT, our previous analysis is still correct provided \(P(E) \approx \delta(E)\) and recover the unperturbed result.

Let us consider current noise first. Given its spectral density \(S_C(\omega)\), the spectrum of voltage fluctuations in the dot is obtained by \(S_V(\omega) = \left| Z(\omega) \right|^2 S_C(\omega)\), where we have introduced a transimpedance \(Z\) as in Ref. 32. As a first approximation, we may write \(Z(\omega) \approx Z(0) = \tau R_0\), where \(R_0\) is the resistance of the QPC leads and \(\tau\) is a lever arm describing the asymmetric coupling between QD and QPC leads. The behavior of \(P(E)\) at finite energies is then given by

\[
P(E) = \frac{\tau^2 R_0^2}{\hbar^2 \bar{Z}(E)} \delta(E)\tag{6}
\]

where \(\bar{Z}(E) = \pi R_0^2 \tau^2\). Now, shot noise in the QPC has the spectrum \(S_E = (e^2/\hbar) R_0^2\). We take \(R_0 = 0.1 R_K\), \(\tau = 0.1\), \(E = 1/2\) and \(V_b = 2.5 \mu V\) (so that \(I = 100 \mu A\)), we get \(E/k_B = 3 \mu K\). As revealed by this analysis, the backaction due to current noise can be made negligible by a combination of low-resistance leads and small \(V_b\).

Let us now turn to charge noise. The spectrum of charge fluctuations on the dot, induced by the QPC, is given to the first order in \(V_b\) and \(\omega\) by the expression \(S_Q(\omega) = 2C_0^2 R_b e V_b^2\), where \(R_b\) is the nonequilibrium charge relaxation resistance defined in Ref. 29, and \(C_0\) the electrochemical capacitance of the QPC “to” the dot. Charge fluctuations are related to voltage fluctuations by the total capacitance of the dot: \(S_V = (1/C_0^2) S_Q\), so that \(S_V = 2(C_0^2/C_S^2) e V_b R_b\). As for current noise, we have \(P(E) = \frac{2\tau R_0 S_E(E)}{\hbar^2 \bar{Z}(E)}\). The energy spread for this \(P(E)\) is given by \(E = (C_0^2/C_S^2) (R_b/R_K) e V_b\).

In conclusion, we have addressed the problem of measuring the temperature of 2DEG microdomains cooled down to the base temperature of state-of-art dilution refrigerators and possibly below. Already at 100 mK, conventional schemes based on transport are inadequate, due to overheating. We have argued that nongalvanic thermometry may overcome this limitation. Our results suggest that a nongalvanic thermometer such as that considered may be conveniently employed at temperatures ranging from tens of mK down to tens of \(\mu K\).

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