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Kinetic roughening in fiber deposition

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I. INTRODUCTION

Nonequilibrium properties of driven interfaces have attracted considerable interest recently due to theoretical reasons and the importance of growth processes on surfaces under molecular beam epitaxy conditions [1,2]. From the theoretical point of view, a particularly important property of driven interfaces is the ability to classify the growth dynamics according to the forms of the underlying stochastic differential equations that describe different physical systems. The simplest nonlinear growth equation of this type is the celebrated Kardar-Parisi-Zhang (KPZ) equation [3]

$$\frac{\partial \tilde{h}(\tilde{r},t)}{\partial t} = \nabla^2 \tilde{h}(\tilde{r},t) + \frac{\lambda}{2} |\nabla h(\tilde{r},t)|^2 + \eta(\tilde{r},t) + v_0,$$  \hspace{1cm} (1)

where $\tilde{h}(\tilde{r},t)$ is a single-valued height variable, $\eta(\tilde{r},t)$ is a Gaussian (white) noise term, and $v_0$ is a constant. The crucial ingredient in the KPZ equation is the nonlinear term proportional to $\lambda$, which manifests itself in the nontrivial tilt dependence of the velocity of the growing interface. One of the physically most interesting quantities associated with interface roughening is the average width, $w(L,t)$, which becomes a function of time and the system size $L$:

$$w^2(L,t) = \langle (h(\tilde{r},t) - \tilde{h}(t))^2 \rangle,$$  \hspace{1cm} (2)

where $\tilde{h}(t)$ denotes the spatial average of $h(\tilde{r},t)$ and the angular brackets denote an average over the noise. The width scales according to the Family-Vicsek scaling relation [4] as

$$w(L,t) = L^{\chi} f(t/L^z),$$  \hspace{1cm} (3)

where the scaling function $f(y)$ behaves as

$$f(y) \sim \begin{cases} y^{\beta} & \text{for } y \ll 1 \\ \text{const} & \text{for } y \gg 1. \end{cases} \hspace{1cm} (4)$$

The dynamical exponent $z$, the roughness exponent $\chi$, and the growth exponent $\beta$ are linked by $\chi = z\beta$ and the scaling relation $\chi + z = 2$. For a one-dimensional (1D) interface, the stationary probability distribution associated with Eq. (1) is known and the exponents are given by $\beta = 1/3$, $\chi = 1/2$, and $z = 3/2$.

Perhaps the easiest way to study the properties of the KPZ universality class is through simple lattice deposition models of interface growth, many of which are described asymptotically by Eq. (1). In some cases, the connection between such models and the KPZ equation can be made explicit and the parameter $\lambda$ calculated analytically [2]. Experimentally, however, the KPZ universality class has proved to be elusive, with the strongest evidence to date coming from slow combustion experiments of paper sheets [5].

In this paper we address the surface roughening of planar fiber networks, which is an interesting candidate for kinetic roughening. We employ a model for random deposition of flexible fibers on a lattice [6,7], which was originally introduced to describe the bulk properties of 3D random fiber networks, the prime example being ordinary paper. The bulk structures obtained from the model seem to describe well, e.g., the pore geometry of random fiber assemblies measured by creeping flow permeability [8] or light scattering properties of paper [9].

For surface roughening, the essential parameters in the model are the fiber length $L_f$ in lattice units and the flexibility $T_f$ of the deposited fibers, which induces overhangs in the bulk that constitute the eventual pore structure. It is known through studies of surface growth models that such bulk defects play an important role in the kinetic roughening of growing surfaces [2,10]. Thus it is of interest to study how this occurs in the case of fiber deposition as well.
One should note that the model does not reproduce the surface roughness properties of real paper sheets [11]. This is due to several simplifying assumptions. In the case of real paper, hydrodynamics is expected to play an important role. There are indeed nontrivial mass correlations [12] and clustering of fibers [13] present that have their origin in hydrodynamics during formation, similarly to the sedimentation problem [14,15]. Another factor missing from simple deposition models is due to the details of the paper manufacturing process. For instance, ordinary paper sheets are usually compressed mechanically. Such effects are hard to take into account as they depend on the actual initial spatial structure of the deposit itself [16].

In this paper our aim is to present a detailed study of the surface growth properties of the fiber deposition model described above and study the dependence of growth on the relevant parameters. Here we concentrate on the growth of 1D interfaces for simplicity. We start by briefly describing the deposition model in Sec. II. In Sec. III we present results of extensive numerical simulations of the kinetic roughening in the model. As expected, for any finite value of the flexibility parameter $T_f$, the kinetic roughening in the model is asymptotically described by the KPZ equation. We discuss in detail growth on tilted surfaces in the limits of small and large tilts and present analytic arguments for the scaling behavior of the parameter $\lambda$ in the KPZ equation. Finally, Sec. IV contains our conclusions and a discussion.

II. MODEL

In the model, fibers of length $L_f$ are deposited on a flat substrate of size $L$ with periodic boundary conditions. The original model is two dimensional, while here we will discuss the 1D version only. The “fibers” in the model are discretized to squares of size one in lattice units and form a chain of length $L_f$ (see Fig. 1). The flexibility of the fibers $T_f$ is defined to be the maximum vertical displacement allowed between two neighboring squares in a fiber in units of fiber thickness. Thus the larger $T_f$ is, the more flexible the fibers are. The limit $T_f = 0$ refers to completely stiff fibers.

The fibers are deposited one by one randomly onto the lattice. During deposition they are kept straight and parallel to the substrate. The part (or parts) of a fiber that first touches the underlying network stops moving and the rest of the fiber bends further down until either all parts touch the network beneath them or the maximum bending allowed by the deflection constraint $T_f$ is reached. Note that this rule conserves just the projection of the fiber on the lattice, but due to the allowed vertical displacements the fiber may stretch. Time is defined in terms of coverage, which is the amount of mass deposited per unit substrate length. Since the mass of a fiber is $L_f$, the deposition of $N$ fibers on a lattice of size $L$ takes $t = N(L_f/L)$ time steps.

The resulting network has different top and bottom surfaces because the fibers at the bottom are generally less deformed than the fibers at the top due to the closeness of the substrate. Here we examine only the behavior of the free top surface. It is defined by the set of local height variables $h(x,t)$ at each lattice site $x$. Thus the curve defined by $h(x,t)$ is a single-valued function so that overhangs in the surface structure are ignored. If the number of particles deposited in a unit time is kept constant, the spatial average of the surface height $\bar{h}(t) = \sum_{x=1}^{L} h(x,t)/L$ grows linearly in time.

Starting from a flat surface, the roughness of the emerging top surface develops in time in a way that clearly depends on $T_f$. The parameter $T_f$ controls the porosity of the network: high values of $T_f$ give a more dense network and low values a more porous one, as can be seen from Fig. 2.

III. RESULTS

A. Scaling exponents

To determine the scaling exponents associated with surface growth, we computed the surface width $w(L,t)$ for four different system sizes ($L=1000$, $1500$, $2500$, and $5000$) with the fiber flexibilities $T_f=0.1$ and $1.0$ and with various fiber lengths. The results shown here are for $T_f = 1.0$ and $L_f = 5$. At very early times, there is a regime where the fibers are uncorrelated corresponding to the random deposition (RD) case [1] (see the discussion at the end of this section). Following this, the data can be fitted to the form

$$w(L,t) = a(L)t^{\beta},$$

where $a(L)$ is a function of $L$. The error estimate was obtained by calculating the difference between the maximum

![FIG. 1. Deposition of a fiber with length $L_f=5$ and flexibility $T_f=3/4$.](image1)

![FIG. 2. Typical fiber networks with (a) $T_f=0.1$ and (b) $T_f=1.0$. The system size $L=500$ and the fiber length $L_f=5$. The white areas are pores and the gray areas indicate the fibers. The number of fibers is 5000 and each layer marked by either of the two shades of gray contains 500 fibers.](image2)
value of $\beta$ with which the curve (5) still remains within the
error bars of the data points (which were calculated from the
standard deviation of the data) and the growth exponent
given by a standard least-squares fit.

Figure 3 shows $\beta$ as a function of $1/L$ as obtained from
the data for $w(L, t)$. An extrapolation from the four data
points to $L \to \infty$ gives $\beta = 0.33 \pm 0.01$. In the inset, we also
show the scaling function $f(t/L^2)$ for a variety of system
sizes.

We also checked the consistency of the results by elimi-
nating any additive constants from $w(L, t)$ through [17]

$$\tilde{w}(L, t) = w(L, 2t) - w(L, t).$$

Although this method gave slightly lower values for $\beta$ than
the direct application of the least-squares method, a similar
extrapolation procedure as above gave again $\beta = 0.33 \pm 0.01$
as $L \to \infty$ for $T_f = 1.0$ and $L_f = 5$. Thus we take this to be our
best estimate for $\beta$.

Another way to determine the scaling exponents for a
given system is to study various height-height correlation
functions [1,17]. The general two-point correlation function
is defined as

$$C_g(r, t, t') = \langle [\delta h(x + r, t + t') - \delta h(x, t)]^2 \rangle,$$

where $\delta h(x, t) = h(x, t) - \bar{h}(t)$. The equal-time height-height
correlation function can now be defined as

$$G_h(r, t) = C_g(r, t, t' = 0),$$

which asymptotically behaves as

$$G_h(r, t) \sim \begin{cases} r^{2\chi} & \text{for } r \ll \xi^{1/\chi} \\ \xi^{2\beta} & \text{for } r \gg \xi^{1/\chi}. \end{cases}$$

Averaging $G_h(r, t)$ over $r$ for $r \gg \xi^{1/\chi}$, only the short-
wavelength components are lost and the corresponding function $\tilde{G}_h(t)$ behaves as

$$\tilde{G}_h(t) \sim \xi^{2\beta}.$$ (10)

We calculated the equal-time height-height correlation func-
tion of Eq. (8) for a system with $L = 5000$, $T_f = 1.0$, and
$L_f = 5$. Figure 4 shows the corresponding $\tilde{G}_h(t)$ on a log-log
scale. The error bars indicate the standard deviation of $\tilde{G}_h(t)$. A direct least-squares fitting to it gives $\beta = 0.338 
\pm 0.003$, in excellent agreement with results from the width.

The roughness exponent $\chi$ is obtained by applying Eq. (9)
to $G_h(r, t)$ in the $r \ll \xi^{1/\chi}$ regime. For a quick check, we simply used $G_h(r, t)$ at $t = 1000$. We define

$$\tilde{G}_h(r, t) = G_h(2r, t) - G_h(r, t)$$ (11)
to eliminate any additive constants. Figure 5 shows $\tilde{G}_h(r, t)$

![Figure 3](image3.png)

**FIG. 3.** $\beta$ as a function of $1/L$ for $T_f = 1.0$ and $L_f = 5$. The solid line is a least-squares fit to the four data points. Extrapolation as $L \to \infty$ gives $\beta = 0.33 \pm 0.01$. The inset shows the scaling function $f(t/L^2)$ and the dashed line has a slope of $1/3$.

![Figure 4](image4.png)

**FIG. 4.** $\tilde{G}_h(t)$ for a system with $L = 5000$, $T_f = 1.0$, and $L_f = 5$ on a log-log scale. A direct application of the least-squares fitting method gives $\beta = 0.338 \pm 0.003$. The error bars are calculated from the standard deviation of $\tilde{G}_h(t)$.

![Figure 5](image5.png)

**FIG. 5.** $\tilde{G}_h(r, t)$ as defined in Eq. (11) applied to the curve corresponding to $t = 1000$ (shown in the inset). The data points used are in the interval $r \in [6, 30]$ (the scaling regime actually extends up to $r \approx 100$). The slope of $\tilde{G}_h(r, t = 1000)$ in this interval gives $\chi = 0.49 \pm 0.02$. 

Both independent estimates obtained above for $\beta$ as well as the estimate for $\chi$ indicate unequivocally that the 1D fiber deposition model belongs to the KPZ universality class. The key factor determining this is the flexibility parameter $T_f$. In the limit where $T_f \to \infty$, the mass units comprising the fibers become unconnected at deposition and the model is equivalent to the RD. For $T_f < \infty$, however, the finite rigidity of the fibers leads to nontrivial correlations that manifest themselves as bulk defects (pores) in analogy to some other surface deposition models [2,10].

Finally, we would like to note that the crossover time $t_c$ from RD to the KPZ regime can be estimated as follows. Since each deposition event simultaneously fills $L_f$ sites, the width in the RD regime is

$$w_{RD}(t) \sim (t/L_f)^{1/2}.$$  

In the RD regime, the surface has no spatial correlations and hence $w_{RD}$ is a measure also of the nearest-neighbor height differences. The RD regime ends when these become of the order $T_f$, such that the finite flexibility of the fibers is “felt.” Setting $w_{RD}(t_c) = T_f$, we conclude that

$$t_c = T_f^2 L_f.$$  

Note in particular that for any nonzero flexibility an extended RD regime appears for sufficiently long fibers $L_f \gg 1/T_f^2$. Next we shall present detailed results for the influence of $T_f$ on growth rate and network density.

**B. Growth on tilted surfaces**

The average growth velocity in the vertical direction

$$\langle v \rangle = \left\langle \frac{\delta h}{\delta t} \right\rangle$$

depends according to the KPZ equation (1) on the local surface slopes $|\nabla h|$. Taking an average it can be written as

$$\langle v \rangle = v_0 + \frac{\lambda}{2} \langle (\nabla h)^2 \rangle,$$

where the slopes are assumed to be small. By introducing an overall tilt $m = \nabla h$ to the network the velocity can be written as

$$v(m) = v(0) + \frac{\lambda}{2} m^2.$$  

Thus, by measuring $v$ as a function of $m$ we can get an estimate for $\lambda$. Through the relation $v(m) = 1/\rho(m)$ this also provides information on the deposit density $\rho$ [10].

Figure 6 shows the typical behavior of $v(m)$ for both small and large values of $m$. It demonstrates that $v(m)$ has two regions. For small values of $m$, it follows Eq. (16), as shown by the dashed line, while for large values of $m$ the dependence becomes linear. We can define the crossover tilt $m_c$ between these two regions by the point of intersection of Eq. (16) and the linear fit, as shown in the figure. In Fig. 7 we plot $m_c$ as a function of the fiber flexibility $T_f$ for four different fiber lengths $L_f$. The plot shows that

$$m_c \approx T_f.$$  

In the following we study the two limits of $v(m)$ separately and develop analytic arguments to explain its behavior.

**I. Large tilts**

The linear regime for the growth velocity as a function of tilt evident in Fig. 6 is easy to obtain with the following arguments. Upon deposition, the fibers tend to conform along the surface according to the amount given by the stiffness parameter $T_f$. However, for increasing $m$ with $T_f < m$, each deposited fiber on the average touches the network only with one of its ends and the rest of the fiber hangs in midair. This situation is analogous to the case of columnar growth.
observed in some deposition models at oblique incidence [18]. The area between the fiber and the network (with the area of the fiber included) is

\[ A = L_f + \sum_{i=1}^{L_f-1} i(m-T_f) \]

\[ = L_f + \frac{1}{2} L_f (L_f-1) (m-T_f). \]  

The average increase in the surface height per unit time is \((A/L)(L/L_f)\). The growth velocity of the system can then be written as

\[ v(m) = \frac{A}{L} \frac{L}{L_f} v_\infty + v_\infty \]

\[ = \frac{1}{2} (L_f-1)(m-T_f)+2, \quad m>T_f, \]  

where \(v_\infty = v(T_f=\infty) = 1\) in our units. The validity of Eq. (19) was verified quantitatively for a system with \(L=2500\), \(T_f=1.0\), and \(L_f=11\) for which it gives \(v(m)=5.0m-3.0\) as \(m > 1.0\). This equation is indicated in Fig. 6 as a dot-dashed line and it shows perfect agreement with the data.

2. Small tilts

For small tilts and with \(m \ll T_f\), the growth rate \(v(m)\) is of the form of Eq. (16). The nonlinearity parameter \(\lambda\) can be then determined by least-squares fitting. Figure 8 shows how \(\lambda\) depends on the fiber flexibility \(T_f \) for four fiber lengths \(L_f \) when the system size is \(L=2500\). The original data (shown in the inset) collapse into a single scaling curve by multiplying \(T_f\) by \(L_f\) and dividing \(\lambda\) by \(\lambda_0 = \lambda(T_f=0)\) corresponding to a stiff fiber.

In the stiff fiber limit, \(\lambda_0\) was calculated separately as a function of \(L_f\) and the result is shown in Fig. 9. The least-squares fitting method suggests for it the quadratic function

\[ \lambda = c(L_f-1)^2, \]  

where \(c = 0.40 \pm 0.03\).

According to Fig. 8, in the case of small tilts \(\lambda\) has two regimes roughly separated by \(L_f/T_f \approx 1\). As \(L_f/T_f \approx 1\), \(\lambda \approx \lambda_0\) and it does not depend on \(T_f\). On the other hand, in the \(L_f/T_f \gg 1\) regime there is a clear inverse power law behavior as a function of \(T_f\). When the running exponent method was applied to \(\lambda/\lambda_0\) corresponding to the case of \(L_f = 27\), it gave the value of the exponent of this power law \(-2.8 \pm 0.4\), implying that \(\lambda/\lambda_0 \sim (L_f/T_f)^{-2.8}\). Thus we can summarize the scaling behavior of \(\lambda\) as

\[ \lambda \sim \begin{cases} \frac{(L_f-1)^2}{L_f} & \text{for } L_f/T_f \ll 1 \\ \frac{-L_f^{-2.8}}{T_f^{-2.8}} & \text{for } L_f/T_f \gg 1. \end{cases} \]  

We note that this form correctly satisfies the RD limits of the model where \(\lambda \to 0\) for either \(L_f=1\) or \(T_f \to \infty\).

To explain Eq. (21) we develop a simple scaling theory. In addition to the fiber length \(L_f\), the two characteristic slopes \(T_f\) and \(m\) define the horizontal scales \(a/m\) and \(a/T_f\), where \(a=1\) is the vertical lattice constant (the thickness of the fibers). For \(L_f \gg 1\), which corresponds to the physically interesting situation, the lattice length scale may be ignored and we can assume that the macroscopic features of the model depend only on the dimensionless numbers \(L_f/T_f\) and \(L_f/m\). Thus we can write

\[ v(m) = \Phi(L_f/L_f m, L_f/T_f). \]  

If Eq. (22) is a smooth function of \(m\), we obtain by the Taylor series expansion in the neighborhood of \(m=0\),

\[ v(m) \approx \Phi_0(L_f T_f)\left[1 + \frac{1}{2} C(L_f T_f) L_f^2 m^2\right], \]  

where

\[ C = \Phi(0,\beta)^{-1} \frac{\partial^2}{\partial \alpha^2} \Phi(\alpha,\beta) \]  

at \(\alpha = L_f m = 0\) and \(\beta = L_f T_f\). Thus, from Eq. (23),
\[ \lambda = L_f^2 C(L_f T_f). \]  

For \( T_f \to 0 \) one obtains the case of stiff fibers, which have a nonzero \( \lambda \). Hence the scaling function \( C \) in Eq. (24) becomes a constant and

\[ \lambda \sim L_f^2 \quad \text{for} \quad L_f T_f \ll 1. \]

For \( L_f \gg 1 \) this is in agreement with Eq. (21).

To explain the other scaling limit of \( \lambda \), we consider the surface width which for the KPZ equation obeys [19]

\[ w_{\text{KPZ}}(t) \sim (K^2 \lambda t)^{1/3} \]

in the asymptotic time-dependent growth regime. The constant \( K \) is defined through the stationary height difference correlation function [cf. Eq. (8)]

\[ \lim_{t \to \infty} G_h(r, t) \sim K|r|. \]

Since the stationary interface is essentially a random walk along \( h \), \( K \) can be interpreted as the square of the typical height difference between subsequent sites. Thus, for large \( T_f \) [20]

\[ K \sim T_f^2. \]

We now estimate the crossover time \( t_c \) from the RD to the KPZ regime by equating Eqs. (12) and (27), which yields, using Eqs. (25) and (29),

\[ t_c \sim L_f^3 (K^2 \lambda)^2 \sim L_f^7 T_f^8 C^2 (L_f T_f). \]

Comparing this to the previously derived expression (13) gives

\[ C(L_f T_f) \sim (L_f T_f)^{-3}, \]

which, when substituted into Eq. (25), leads to

\[ \lambda \sim \frac{1}{L_f T_f^3}, \quad L_f T_f \gg 1. \]

This agrees reasonably well with the numerical result in Eq. (21). The solid line in Fig. 8 shows \( \lambda/\lambda_c \sim (L_f T_f)^{-3} \). To lend support to the validity of Eq. (32), in the Appendix we present another argument that gives the same result based on purely geometric considerations.

**IV. SUMMARY AND CONCLUSIONS**

In this paper we have studied the kinetic roughening of surfaces of random fiber deposits in one dimension through a simple model. In the model, fibers of length \( L_f \) are randomly deposited on a lattice and allowed to bend down upon touching the deposit by an amount determined by the flexibility \( T_f \). For any finite value of \( T_f \), following an initial random deposition regime, a nontrivial pore structure develops in the bulk and the consequent kinetic roughening of the growing surface is characterized by the KPZ universality class. We have also studied in detail the dependence of the nonlinear coefficient \( \lambda \) on the parameters of the model and developed analytic arguments to explain the tilt dependence of the growth velocity \( v \). Results from numerical simulations are in good agreement with these arguments.

The results of this paper point out the possibility of studying kinetic roughening in the deposition of fiberlike or plate-like objects, similarly to Ref. [16]. Although we have not discussed the 2D version of the deposition model here, many of the conclusions presented for the 1D case can be extended to higher dimensions as well. In particular, due to the tilt dependence of growth inherent in the model, the kinetic roughening should be described by the KPZ equation.

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**APPENDIX: GEOMETRIC ARGUMENT FOR \( \lambda \) AT SMALL TILTS**

We have developed another argument to support the validity of the inverse power law of Eq. (32) by using geometric reasoning similar to what was applied successfully in the large tilt regime. Consider a fiber lying on a horizontal substrate in such a way that its left end is supported by a column of height \( a \) [see Fig. 10(a)]. The fiber is sufficiently long so that the right end touches the substrate. The empty space between the substrate and the fiber has an area of

\[ A = \sum_{i=1}^{a/T_f} (a - i T_f) \frac{a^2 - a T_f}{2 T_f}. \]  

Next the substrate is tilted by \( m \) so that both ends of the fiber still have something to touch beneath them [Fig. 10(b)]. The condition for this is
Thus the tilting introduces a change of

\[ A_2 = \sum_{i=1}^{a(T_f-m)} \frac{a+am-aT_f}{2(T_f-m)}. \]

(A3)

The area of the empty space between the fiber and the substrate is now

\[ \frac{a}{T_f-m} < L_f - 1. \]  

(A2)

The change in area for a fiber supported at its right end is therefore

\[ \Delta A_f = A_2 - A_1 = \frac{a^2 + am - aT_f}{2(T_f-m)} - \frac{a^2 - aT_f}{2T_f} \]

in the area. By using the Taylor expansion for \( m \ll 1 \) we obtain

\[ \Delta A_f = \frac{a^2}{2T_f} m + \frac{a^2}{2T_f^3} m^2. \]  

(A4)

A reversal of the fiber is clearly equivalent to reversing the sign of the tilt. The change in area for a fiber supported at its right end is therefore

\[ \Delta A_r = \frac{a^2}{2T_f} m + \frac{a^2}{2T_f^3} m^2. \]  

(A5)

By symmetry, the number of fibers of the two types is the same for an untitled substrate. The contributions to Eqs. (A4) and (A5) that are linear in \( m \) therefore cancel and the net change in area is

\[ \Delta A \sim \Delta A_f + \Delta A_r - \frac{1}{L_f} m^2. \]  

(A6)

The corresponding change in growth rate is then

\[ \nu(m) = \frac{\Delta A}{L_f} \sim \frac{1}{L_f T_f^3} m^2. \]  

(A7)

which by Eq. (16) leads to Eq. (32).

[20] For stiff fibers with \( T_f = 0 \), it is clear from dimensional considerations that \( k \sim L_f \). We therefore expect a crossover from \( k \sim T_f^2 \) to \( k \sim 1/L_f \) at \( T_f \sim 1/\sqrt{L_f} \).