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A Two Phase Approach for the Bi-Objective Non-convex Combined Heat and Power Production Planning Problem

Aiying Rong* José Rui Figueira[†] Risto Lahdelma[‡]

April 3, 2015

In this paper, we deal with the bi-objective non-convex combined heat and power (CHP) planning problem. A medium and long term planning problem decomposes into thousands of single period (hourly) subproblems and dynamic constraints can usually be ignored in this context. The hourly subproblem can be formulated as a mixed integer linear programming (MILP) model. First, an efficient two phase approach for constructing the Pareto Frontier (PF) of the hourly subproblem is presented. Then a merging algorithm is developed to approximate the PF for the multi-period planning problem. Numerical results with real CHP plants demonstrate the effectiveness and efficiency of the solution approach using the CPLEX based ε -constraint method as benchmark.

Keywords: multi-objective optimization, combined heat and power production, mixed integer linear programming, two phase method.

1 Introduction

Energy and environmental policies are inextricably linked. The growing awareness about the environmental impacts of energy has significantly broadened the policy goals set in the energy sector. Combined heat and power (CHP) production is universally accepted as one of the energy efficient technologies with lower fuel consumption and fewer emissions. CHP can provide viable solutions to mitigate environmental impacts of energy production. The European Union (EU) is strongly promoting clean production technologies with fewer emissions [1] to respond to the environmental policy.

When environmental impacts must be incorporated into the planning problem, multiple criteria decision making (MCDM) is a good way to deal with the problem. This is a paradigm under which decisions are made not based on the results of optimizing a single criterion (usually the economic cost) but rather on the tradeoff of simultaneously optimizing different and conflicting criteria [2]. MCDM approaches have long been used in energy planning for both traditional power-only and heat-only systems [3–5] as well as poly-generation including CHP systems [6].

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In this paper, we consider explicitly the environmental impact of a medium or long term CHP economic dispatch (CHP-ED) problem under the framework of multi-objective optimization. The ED problem can be viewed as a subproblem of the long term planning problem such as the generation expansion planning problem [7]. The medium or long term planning problem needs to be considered in the context of risk analysis for tactical and strategic decision making [8].

In CHP technology, heat and power generation follow a joint characteristic. Traditional CHP production is usually applied in backpressure plants, where the joint characteristic can typically be represented by a convex (linear programming, LP) model [9–14]. But the convexity assumption may not be valid for more advanced production technologies such as backpressure plants with condensing and cooling options, gas turbines and combined gas and steam cycles. A non-convex CHP plant can be formulated as a mixed integer linear programming (MILP) model based on the convex partitioning technique [8, 15–17].

Usually a long term planning problem can be decomposed into a sequence of single period subproblems using different decomposition techniques [18]. The natural period length is typically one-hour because power on the market is traded on the hourly basis. In this paper, we refer to the single period subproblem as the hourly subproblem. In a broader context of risk analysis where numerous scenarios need to be considered, usually the simplification of the planning problem (*e.g.*, ignoring dynamic constraints) is allowed [19–23] for at least two reasons. First, dynamic constraints are associated with a short term horizon and do not have much effect on the operation over a long term horizon. Second, dynamic constraints couple the operations from period to period and cause convergence problems for decomposition-based algorithms for a longer planning horizon. In multi-objective optimization context, the situation becomes more serious. A general solver cannot solve the planning problem even with a very short horizon (*e.g.*, a day), in some cases even when dynamic constraints are ignored.

In the following, we briefly review the algorithm development for handling multi-objective CHP-ED and multi-objective MILP problems. For dealing with multi-objective CHP-ED, meta-heuristics were mainly used [24–30]. The interested readers can also refer to comprehensive surveys [31, 32]. In terms of multi-objective MILP problems, the literature in the field includes exact and approximate algorithms. The former are targeted to find the exact Pareto Frontier (PF) while the latter aim at approximating the PF. Exact approaches include different variants of Branch and Bound (BB) algorithms [33–37] and the augmented ε -constraint method [38]. The approximate approaches include meta-heuristics [39, 40], relaxation based heuristics [41], the master slave procedure [42], interactive procedures [43–46], an algorithm for finding well-dispersed subsets of non-dominated outcomes [47] and an algorithm that uses a composite linear function to find all supported non-dominated outcomes [48].

Note that the PF of the multi-objective MILP consists of Non-Dominated Line Segments (NDLSs) and possible isolated points (refer to Figure 2(a) later in Section 3). An NDLS refers to a line segment on which any two elements (points) do not dominate each other. The contributions of the current research are summarized as follows. First, an NDLS instead of a single point is treated as an entity to represent the non-continuous and non-convex PF for the MILP problem. A method for constructing NDLSs is presented by analyzing the model structure. Second, a two phase approach centered around manipulating NDLSs is developed for the hourly subproblem. Up to now, the two phase method has only been applied to pure multi-objective integer programming (combinatorial) problems [49–52] though it is a general approach for dealing with multi-objective mathematical programs with integer variables. Third, the relation between the proposed two phase method and the ε -constraint method is identified and the optimality of the NDLS computed by the two phase method can be verified by checking two points on it using the ε -constraint method. Finally, a merging algorithm (MA) is developed to approximate the PF for the bi-objective non-convex problem by extending the MA for the bi-objective LP problem. [53].

For the single objective optimization case, the utilization of the simplification for ignoring

dynamic constraints is straightforward because the optimal solution of the long term planning problem can be obtained trivially by adding directly the solutions of the hourly subproblems. For bi-objective LP, the *exact* PF for the multi-period problems can also be obtained by merging the PF for the hourly subproblems based on the convexity of the PF [53]. However, for the multi-objective (including bi-objective) non-convex problem, it is an open question how to aggregate the PF of the hourly subproblems.

The paper is organized as follows. Section 2 describes the model of the individual CHP plant as well as the model of the bi-objective CHP planning problem. In Section 3, a two phase method for the hourly subproblem is presented centered around manipulating NDLSs. Then, an MA is presented for approximating the PF of the bi-objective multi-period problem. Section 4 reports the computational results with real CHP plants. A comparison is made between the proposed solution approach and ε -constraint method. At the end of the paper, four appendices are attached. Appendix I gives the formulas for calculating the parameters of an NDLS. Appendix II collects the procedures related to the two phase method and MA. Appendix III gives the data for power plants related to the illustrative example. Appendix IV illustrates the two phase algorithm.

2 Problem Description

Here the problem under study can be viewed as an extension of the single objective non-convex CHP planning problem [17] to accommodate emissions objective or a non-convex version of the bi-objective CHP planning problem studied in [53]. The modeling framework of plants is briefly described as follows. The operating region of a convex plant is modelled as a convex combination of extreme points for the region. A non-convex plant is modelled according to the convex partitioning technique [8, 15], *i. e.*, the characteristic area of a non-convex plant is divided into multiple convex subareas and each subarea is encoded as a convex submodel. Each time only one subarea is active. For detailed discussion about the relation between convex and non-convex plant models, refer to [17]. If emissions of the plant are explicitly considered, it is convenient to use fuel characteristics to represent the extreme points of the operating region (characteristic area). The fuel characteristic of a CHP plant defines the relation between fuel consumption and generated heat and power. The operating cost (production cost) of a fossil fuel based plant is mainly determined by the fuel cost, *i. e.* fuel consumption multiplied by fuel price. Refer to [53] for detailed discussion about the relation between fuel characteristics and caused emissions.

Similar to [53], a mixture of different fuels is used to affect the tradeoff between economic objective and emission objective. For the system under study, the environmental impact of energy production is associated with CO₂ emissions for producing energy. Different types of fuels with different specific CO₂ emissions are combusted at plants. However, to facilitate emission calculation, it is assumed that a plant should only combust one fuel. Usually, the fuel with larger emissions is cheaper than that with smaller emissions. For example, coal is cheaper than natural gas. This implies a tradeoff between fuel cost and emissions.

When both power market and CO₂ emission allowance market coexist, the objective of the CHP planning problem is to simultaneously minimize the overall *net acquisition costs* for power and heat as well as the *emissions costs* associated with providing power and heat. In addition to production units (CHP plant, power-only plant, heat-only plant), a CHP system may include non-production components such as bilateral sales and purchase contracts. All components (plants and contracts) can be modeled based on a unified technique as discussed in [17]. The emissions for the plant are caused by the fuel combusted at the plant. The emissions for the non-production component are based on a reference system (*e. g.*, coal-fired condensing power plants for power component or coal-fired boiler for heat component). The net acquisition costs consist of actual production costs (fuel costs), costs for purchasing components subtracted by

the revenue from selling the produced energy.

To facilitate understanding system model for the planning problem presented later, the non-convex plant model is briefly described first.

2.1 Non-convex CHP Plant Model

Figure 1 illustrates the non-convex characteristic of a backpressure plant with condensing and auxiliary cooling options. The characteristic is projected onto the p - q (power-heat) plane and the π -coordinate (fuel consumption) at each extreme point is shown numerically in Table 1. The characteristic area is divided into three convex subareas: A1, A2 and A3. A1 is formed by extreme points 1, 8, 9, 2 and 3. This subarea includes the normal backpressure operation mode (line between points 1 and 2), gradual shift into the condensing mode (subarea within points 1, 2 and 3) and the reduction mode (subarea within points 1, 8, 9 and 2). The auxiliary cooling operating mode must be split into two convex subareas A2 and A3. A2 is formed by extreme points 1, 3, 6, 5, and 4 and A3 by points 2, 7, 6, and 3. The plant can only operate in one convex subarea each time, but some extreme points may belong simultaneously to several subareas. To enable (activate) a convex subarea means enabling the extreme points that define the subarea and disabling the remaining extreme points.

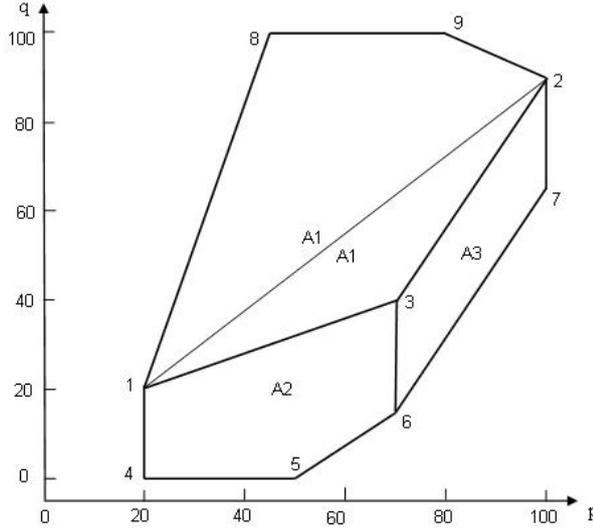


Figure 1: Non-convex characteristic of a backpressure plant with condensing and auxiliary options. p = power, q = heat.

Due to convexity, when subarea a of plant u is active, the hourly power generation $P_{u,t}$, heat generation $Q_{u,t}$, and operating costs $C_{u,t} = C_{u,t}(P_{u,t}, Q_{u,t})$ of plant u can be represented as a convex combination of extreme characteristic points $(c_{j,t}, p_{j,t}, q_{j,t})$ (cost, power, heat) for the subarea below.

$$\begin{aligned}
 C_{u,t} &= \sum_{j \in J_a} c_{j,t} x_{j,t}, \\
 P_{u,t} &= \sum_{j \in J_a} p_{j,t} x_{j,t}, \\
 Q_{u,t} &= \sum_{j \in J_a} q_{j,t} x_{j,t}, \\
 \sum_{j \in J_a} x_{j,t} &= 1, \\
 x_{j,t} &\geq 0, j \in J_a
 \end{aligned} \tag{1}$$

Here variables $x_{j,t}$ are used for forming the convex combination, J_a is the index set of extreme points for subarea a of plant u and $c_{j,t} = \pi_{j,t} c_{f,\varphi(j),t}$, where $\pi_{j,t}$ and $c_{f,\varphi(j),t}$ is fuel consumption and price of fuel $\varphi(j)$ combusted at plant u and the same with $j \in J_u(J_a)$. Note that index set

Table 1: Fuel characteristics and related fuel costs (c_0), net costs (c_1) and emission (c_2) costs as well as allocation of extreme points to convex subareas

Point No.	Fuel			Cost			Allocation		
	π	p	q	c_0	c_1	c_2	A1	A2	A3
1	50	20	20	1000	-500	2000	1	1	
2	238	100	90	4750	-2375	9500	1		1
3	196	70	40	3929	-1964	7857	1	1	1
4	67	20	0	1333	-667	2667		1	
5	152	50	0	3030	-1515	6061		1	
6	218	70	15	4359	-2179	8718		1	1
7	275	100	65	5500	-2750	11000			1
8	156	40	100	3111	-1556	6222	1		
9	212	80	100	4235	-2118	8471	1		

J_a in (1) can be replaced by J_u for problem formulation (refer to later formula (2)-(6)) because $x_{j,t} = 0$ for non-active subareas.

A compact way to represent the convex partition of a non-convex operating area is to tabulate the allocation of the extreme points into different subareas. Table 1 shows the fuel characteristics and related fuel, net and emission costs as well as allocation of extreme points into convex subareas. Refer to later Section 2.3 for computing net and emission costs.

2.2 Problem Formulation

The following notation is introduced to formulate the problem.

- t index of a period or a point in time. The period t is between points $t-1$ and t ,
- T number of periods over the planning horizon,
- p, q super/subscripts or prefixes for power and heat.

Index Sets

- A^N characteristic areas of all non-convex plants ($A^N = \bigcup_{u \in U^N} A_u$),
- A_j characteristic areas that contain extreme point j ,
- A_u characteristic areas of non-convex plant u ,
- J set of extreme points of the operating regions of all components including non-generating components (*e. g.*, contracts). ($J = \bigcup_{u \in U} J_u$),
- J^N set of extreme points of the operating regions of all non-convex plants ($J^N = \bigcup_{u \in U^N} J_u$),
- J_a set of extreme points in subarea a ,
- J_u set of extreme points of the operating region of component $u \in U$,
- U set of all components including non-generating components,
- U^N set of non-convex plants.

Parameters

- $\varphi(j)$ fuel associated with component $u \in U$ and the same for all points $j \in J_u$,
- $(\pi_{j,t}, p_{j,t}, q_{j,t})$ extreme point $j \in J_u$ of operating region of component $u \in U$ (fuel consumption, power, heat) in MWh in period t ,
- $c_{e,t}$ emission allowance price in €/ton for period t ,

$c_{f,\varphi(j),t}$	price for fuel $\varphi(j)$ in €/MWh in period t ,
$c_{p,t}$	power price in €/MWh on the power market in period t ,
$c_{q,t}^+$	heat surplus penalty cost in €/MWh in period t ,
$\eta_{\varphi(j)}$	specific CO ₂ emission in ton/MWh for fuel $\varphi(j)$,
Q_t	heat demand in MWh in period t .

Decision variables

$x_{j,t}$	variables encoding the operating level of each component in terms of extreme points $j \in J$ in period t ,
$x_{p,t}$	net level of power output in MWh in period t ,
$x_{q,t}^+$	heat surplus in MWh in period t ,
$y_{a,t}$	0-1 variables determining if area a is in operation in period t , $a \in A^N$.

When dynamic constraints are ignored, the multi-period CHP planning problem is simply the sum of independent hourly subproblems. The bi-objective planning problem under study is represented as a vmin optimization problem. The operator vmin means vector minimization. The vmin problems arise when two or more objectives are to be minimized over a given feasible region.

$$\text{vmin} \quad \left(\sum_{t=1}^T \left(\sum_{j \in J} \pi_{j,t} c_{f,\varphi(j),t} x_{j,t} - c_{p,t} x_{p,t} + c_{q,t}^+ x_{q,t}^+ \right), \sum_{t=1}^T \sum_{j \in J} \pi_{j,t} \eta_{\varphi(j)} c_{e,t} x_{j,t} \right) \quad (2)$$

subject to

$$\sum_{j \in J_u} x_{j,t} = 1, \quad u \in U, \quad t = 1, \dots, T, \quad (3)$$

$$\sum_{j \in J} p_{j,t} x_{j,t} - x_{p,t} = 0, \quad t = 1, \dots, T, \quad (4)$$

$$\sum_{j \in J} q_{j,t} x_{j,t} - x_{q,t}^+ = Q_t, \quad t = 1, \dots, T, \quad (5)$$

$$x_{j,t} \geq 0, \quad j \in J, \quad t = 1, \dots, T, \quad (6)$$

$$x_{q,t}^+ \geq 0, \quad t = 1, \dots, T, \quad (7)$$

$$x_{j,t} \leq \sum_{a \in A_j} y_{a,t}, \quad j \in J^N, \quad t = 1, \dots, T, \quad (8)$$

$$\sum_{a \in A_u} y_{a,t} = 1, \quad u \in U^N, \quad t = 1, \dots, T, \quad (9)$$

$$y_{a,t} \in \{0, 1\}, \quad a \in A^N, \quad t = 1, \dots, T. \quad (10)$$

The above model (2)-(10) is a bi-objective MILP model where the bi-objective LP model (2)-(7) is embedded in the formulation. The objective (2) is to simultaneously minimize the net cost (the first objective) and the emission cost (the second objective). In the first objective, the first, second and third terms represent cumulative costs from all components, revenue from selling produced power on the market and possible penalty costs from heat surplus, respectively. In the second objective, the emission cost is calculated according to caused emissions from all components and emission allowance price. Constraints (3) and (6) represent the convex combination of the extreme points of the operating region of each plant. Power balances (4) determine the net amount of power that can be traded on the market. The first term on the left-hand side is the cumulative power volumes from all components, and $x_{p,t}$ can be either non-negative or negative. Heat balances (5) state that demand Q_t in each period t must be satisfied and heat surplus $x_{q,t}^+$ imposes penalty cost $c_{q,t}^+$ in the first objective. The first term on the left-hand side is the cumulative heat volumes from all components. Constraints (8)-(10) encode non-convex characteristics. Constraints (8) disallow operations in non-active subareas or between subareas by forcing the corresponding $x_{j,t}$ to zero. Constraints (9) state that the

exactly one subarea is active for a non-convex plant. It means that non-convex model (2)-(10) reduces to convex model (2)-(7) once active subareas in all plants are determined. For the above formulation, power transaction cost is ignored in the power market. This simplification is acceptable in the broad context of risk analysis.

2.3 Net Cost and Emission Cost of Extreme Points

The net cost for the extreme point can be derived from the first objective of (2). The free variables $x_{p,t}$ are solved from constraints (4) and substituted into the first objective, giving $\sum_{t=1}^T \left(\sum_{j \in J} (\pi_{j,t} c_{f,\varphi(j),t} - p_{j,t} c_{p,t}) x_{j,t} + c_{q,t}^+ x_{q,t}^+ \right)$. Then the net cost for point j is defined as

$$c_{1,j,t} := c_{1,j,t}(c_{p,t}) := \pi_{j,t} c_{f,\varphi(j),t} - p_{j,t} c_{p,t} \quad (11)$$

The emission cost for point is computed according to the second objective of (2)

$$c_{2,j,t} := c_{2,j,t}(c_{e,t}) := \pi_{j,t} \eta_{\varphi(j)} c_{e,t} \quad (12)$$

It can be seen that net and emission costs for extreme points are a function of power price and emission allowance price, respectively.

3 Solution Approach

In Section 2.2, the bi-objective non-convex CHP planning problem was formulated as a bi-objective MILP problem. Figure 2(a) shows the non-continuous and the non-convex profile of the Pareto frontier (PF) of the problem characterized by a set of NDLSs. An NDLS can be treated as an entity of the PF. Therefore, our two phase solution approach is developed centered around manipulating NDLSs. In this section, the optimality concept for multi-objective optimization is reviewed first. The concept of point dominance will be extended to the context of NDLSs. Next, the method of constructing NDLSs (feasible outcomes) of the hourly subproblem is given. Then, a two phase solution approach for the hourly subproblem is presented. Next, the merging algorithm (MA) for the multi-period problem is given for approximating the PF.

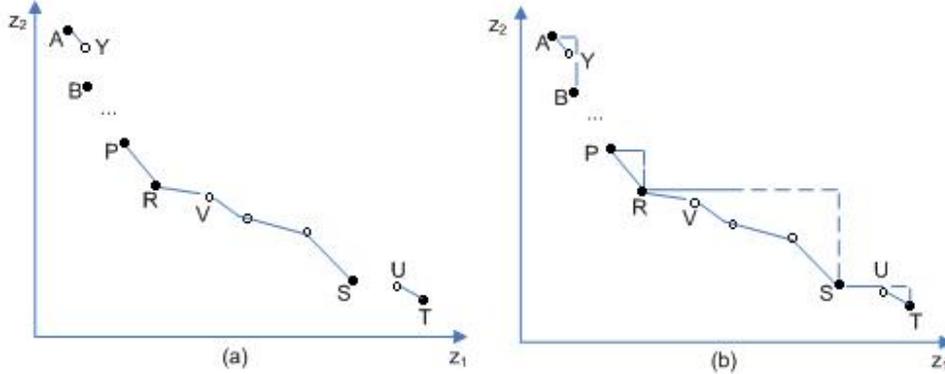


Figure 2: The PF profile for the vmin bi-objective MILP problem
(a): profile; (b): used for the two phase algorithm later.

3.1 Optimality Concept for Multi-Objective Optimization

Let \mathcal{X} denote the set of feasible solutions in the decision space and \mathcal{Z} their images in the objective space. The image of solution $x \in \mathcal{X}$ is $f(x) = (f_1(x), \dots, f_r(x))$, where $r \geq 2$. Solving

multi-objective optimization problem here is interpreted as generating its efficient set \mathcal{X}_E in the decision space and corresponding image $\mathcal{Z}_N = f(\mathcal{X}_E)$ in the objective space \mathbb{R}^r , called *Pareto frontier* (PF) or *non-dominated set*.

The dominance relations are defined based on the componentwise ordering of \mathbb{R}^r . Considering two points $\mathbf{z}^1, \mathbf{z}^2 \in \mathbb{R}^r$,

$$\begin{aligned}\mathbf{z}^1 \leq \mathbf{z}^2 &\Leftrightarrow z_k^1 \leq z_k^2, \quad k = 1, \dots, r \quad \text{and} \quad \mathbf{z}^1 \neq \mathbf{z}^2 \\ \mathbf{z}^1 < \mathbf{z}^2 &\Leftrightarrow z_k^1 < z_k^2, \quad k = 1, \dots, r.\end{aligned}$$

The relations \geq and $>$ are defined accordingly.

Definition 1. (Point dominance) For vmin problem, $f(\bar{\mathbf{x}}) \in \mathbb{R}^r$ is *dominated* by $f(\mathbf{x}) \in \mathbb{R}^r$ if $f(\mathbf{x}) \leq f(\bar{\mathbf{x}})$.

$$\mathcal{X}_E := \{\mathbf{x} \in \mathcal{X} : \text{there exists no } \bar{\mathbf{x}} \in \mathcal{X} \text{ with } f(\bar{\mathbf{x}}) \leq f(\mathbf{x})\}$$

For multi-objective optimization with integer variables, often supported and unsupported non-dominated outcomes are distinguished [54]. Let $\mathbb{R}_{\geq}^r := \{\mathbf{z} \in \mathbb{R}^r, z_i \geq 0, i = 1, \dots, r\}$ denote the non-negative orthant of \mathbb{R}^r and $\mathcal{Z}^{\geq} := \text{conv}\{\mathcal{Z}_N \oplus \mathbb{R}_{\geq}^r\}$. Supported and unsupported non-dominated outcomes are located on the boundary and interior of \mathcal{Z}^{\geq} , respectively. In above Figure 2 (a) points ‘●’ and ‘○’ represent supported and unsupported non-dominated outcomes, respectively.

Now, some definitions related to an NDLS are given. A line segment including NDLS can be presented as $[\mathbf{z}^r, \mathbf{z}^s]$ according to its endpoints \mathbf{z}^r and \mathbf{z}^s , where $z_1^r < z_1^s$, similar to that in [33]. The line segment can be treated as a special set. An NDLS is called *convex segment* if its two endpoints are supported non-dominated outcomes; otherwise, it is called *non-convex segment*. Segment PR is the only convex segment illustrated in Figure 2(a) and the remaining are non-convex. In the following, the dominance relations associated with NDLSs are introduced by extending point dominance in Definition 1.

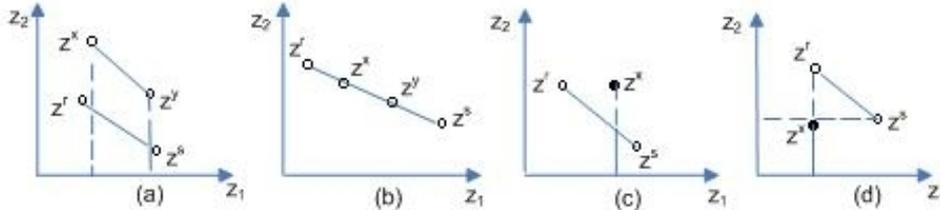


Figure 3: Dominance relations between NDLSs and between an isolated point and an NDLS

Definition 2. (Segment-to-segment dominance) For two NDLSs, $[\mathbf{z}^r, \mathbf{z}^s]$ dominates $[\mathbf{z}^x, \mathbf{z}^y]$ if for any element $\mathbf{z}^i \in [\mathbf{z}^x, \mathbf{z}^y]$, there exists at least one element $\mathbf{z}^j \in [\mathbf{z}^r, \mathbf{z}^s]$ such that $z_k^j \leq z_k^i, k = 1, \dots, r$. On this basis, if $\mathbf{z}^j \neq \mathbf{z}^i$ or $[\mathbf{z}^r, \mathbf{z}^s] - [\mathbf{z}^x, \mathbf{z}^y] \neq \emptyset$, then $[\mathbf{z}^r, \mathbf{z}^s]$ strictly dominates $[\mathbf{z}^x, \mathbf{z}^y]$.

Definition 3. (Segment-to-point dominance) An NDLS $[\mathbf{z}^r, \mathbf{z}^s]$ strictly dominates \mathbf{z}^x if there is no element $\mathbf{z}^j \in [\mathbf{z}^r, \mathbf{z}^s]$ such that $z_k^j > z_k^x, k = 1, \dots, r$.

Definition 4. (Point-to-segment dominance) An isolated point \mathbf{z}^x strictly dominates an NDLS $[\mathbf{z}^r, \mathbf{z}^s]$ if for any element $\mathbf{z}^j \in [\mathbf{z}^r, \mathbf{z}^s]$, the conditions $z_k^x \leq z_k^j$ hold for $k = 1, \dots, r$.

Figure 3 illustrates dominance relations between NDLSs as well as between an isolated point and an NDLS for the bi-objective case, where (a) and (b) illustrate Definition 2 and (c) and (d) illustrate Definitions 3 and 4 respectively.

For the bi-objective case, the slope of an NDLS $[z^r, z^s]$ (always negative) is defined as

$$\gamma(r, s) = \frac{z_2^r - z_2^s}{z_1^r - z_1^s} \quad (13)$$

Finally, the notation for the current problem is introduced. Let \mathbf{x}_t and \mathbf{x} denote the decision variable vector in period t and over the entire planning horizon, respectively.

$$\begin{aligned} z_{1,t} &:= f_1(\mathbf{x}_t) = \sum_{j \in J} \pi_{j,t} c_{f,\varphi(j),t} x_{j,t} - c_{p,t} x_{p,t} + c_{q,t}^+ x_{q,t}^+ \\ z_{2,t} &:= f_2(\mathbf{x}_t) = \sum_{j \in J} \pi_{j,t} \eta_{\varphi(j)} c_{e,t} x_{j,t} \\ z_1 &:= f_1(\mathbf{x}) = \sum_{t=1}^T f_1(\mathbf{x}_t) \\ z_2 &:= f_2(\mathbf{x}) = \sum_{t=1}^T f_2(\mathbf{x}_t) \end{aligned} \quad (14)$$

The weighted-sum function with a weight vector (λ_1, λ_2) for period t subproblem, where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ is defined as

$$f_\lambda(\mathbf{x}_t) = \lambda_1 f_1(\mathbf{x}_t) + \lambda_2 f_2(\mathbf{x}_t) \quad \text{or} \quad z_\lambda(\mathbf{x}_t) = \lambda_1 z_{1,t} + \lambda_2 z_{2,t} \quad (15)$$

3.2 A Method for Constructing NDLSs for the Hourly Subproblem

Constructing NDLSs occurs in the second phase of the two phase procedure presented later, where a series of weighted-sum scalarization problems (single objective) need to be solved with the aid of bounding techniques. The result of solving the individual weighted-sum scalarization problem is a discrete point, called *reference outcome*. The idea of constructing NDLSs is to construct the neighbourhood of the reference outcome as shown in Figure 4(a), where point ‘A’ represents the reference outcome. The final NDLSs of a given reference outcome are obtained by applying dominance relations among multiple NDLSs according to Definition 2 as shown in Figure 4(b). The weighted-sum scalarization problem can be solved by specialized efficient algorithms [15, 17] or general solvers such as CPLEX.

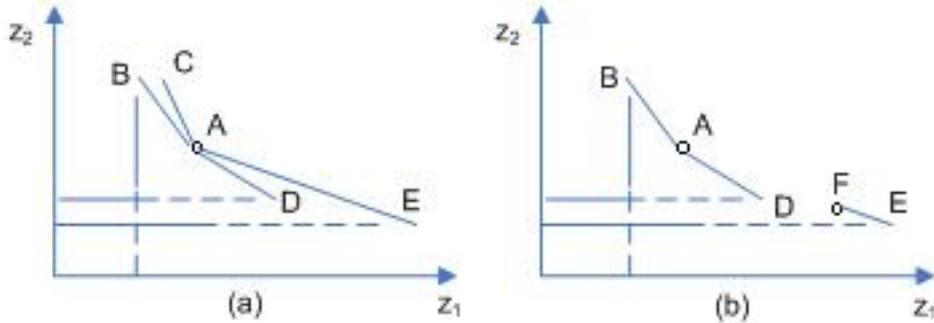


Figure 4: The neighborhood of a reference outcome

As mentioned in Section 2.2, solving the original non-convex problem can reduce to solving a convex (LP) subproblem once an active subarea is known. The basis of the LP problem comes from variables of model (2)-(7), *i. e.*, heat surplus variables and extreme points of plants. Then there are $|U| + 2$ elements in any feasible basis according to (2)-(7). However, for the single objective problem, the feasible basis has only $|U| + 1$ elements because constraints (4) can be dropped. If $f_1(\mathbf{x}_t)$ is taken as an objective, then the free variable $x_{p,t}$ can be solved according to constraints (4) and substituted into the objective as shown in (11). If $f_2(\mathbf{x}_t)$ is taken as an objective, then constraints (4) are redundant. It also means that feasible basis of any weighted-sum scalarization problem (15) has $|U| + 1$ elements. Consequently, the neighbourhood can be constructed by introducing an additional element (called *adjustment element*) into the basis of the reference outcome (called *basis for short*) to form the feasible basis of the bi-objective problem, called *neighbourhood basis*.

3.2.1 Neighbourhood basis

Among the solvers for solving the single objective version of the hourly subproblem (2)-(10), the envelope based Branch and Bound (EBB) algorithm [17] is the most efficient. In addition, it is easy to identify adjustment elements to form the neighbourhood basis. The envelope is the least cost curve of providing heat. In the convex case, it is a piecewise linear convex function (refer to [14] for the algorithm of constructing the envelope.) In the non-convex case, the envelope is a generic piecewise linear function constructed according to individual convex subareas. If the heat surplus variable is in the basis, each plant contributes exactly one element to the basis. Otherwise, only one plant (called *regulating plant*) contributes two elements. The elements of the plants in the basis come from the envelope of the plants. Here it is worth mentioning that it is possible that linear relaxation of the original problem can produce feasible (optimal) solution to the original problem. In this case, the whole plant is treated as a convex subarea, numbered as the $(|A_u| + 1)^{\text{th}}$ subarea.

Note that the essence of the optimality concept for multi-objective optimization is the tradeoff between objectives. The candidates of adjustment elements should have potential to cause the tradeoff between $f_1(\mathbf{x}_t)$ and $f_2(\mathbf{x}_t)$. They can be on envelopes of $f_1(\mathbf{x}_t)$, $f_2(\mathbf{x}_t)$ and $f_\lambda(\mathbf{x}_t)$ or in the subareas or in the plant next to the elements in the basis or *heat surplus variable*. If the elements are selected according to the envelope of $f_\lambda(\mathbf{x}_t)$, the corresponding elements have to be on the envelopes of $f_1(\mathbf{x}_t)$ or $f_2(\mathbf{x}_t)$.

If one or more plants have active subareas $(|A_u| + 1)^{\text{th}}$, when the adjustment elements are in the same (natural) subareas as those of the basis elements, the selection has no difference from that of the natural subareas as just mentioned. Otherwise, the selection needs additional steps. First, the corresponding adjustment elements are also selected if they are on the envelopes of $f_1(\mathbf{x}_t)$ or $f_2(\mathbf{x}_t)$. These elements can be used to change the reference outcome (see discussion in the next subsection). In addition, all natural subareas associated with the basis element need to be checked and the elements will be selected according to the relative scale of the envelope slopes.

3.2.2 Constructing NDLSs

According to the above discussion, it is possible that partial adjustment elements are used to change the reference outcome while the remaining adjustment elements are used to construct the neighbourhood. In the following, we first describe how to construct the NDLSs for a given reference outcome. Then, we describe how to change the reference outcome. Note that line segments with non-negative slopes will be discarded automatically during the construction process.

To simply the notation, period index t is dropped. For the sake of convenience, the extreme point of the plant is redefined by introducing a plant index explicitly. Let $(c_{1,k,u}, c_{2,k,u}, q_{k,u})$ denote extreme point k in plant u , where $c_{1,k,u}$, $c_{2,k,u}$ and $q_{k,u}$ are net cost, emission cost and heat value of point k . Let $(z_{1,0}, z_{2,0})$ denote a reference outcome obtained by solving the weighted-sum scalarization problem (15). At least one point $(c_{1,k(0,u),u}, c_{2,k(0,u),u}, q_{k(0,u),u})$ of each plant u are in basis of $(z_{1,0}, z_{2,0})$.

The salient nature of the envelope-based algorithm is that all of calculations are based on primary operations. However, formulas are different depending on whether heat surplus variable x_q^+ is in the basis or not. For calculating discrete points, the formula are directly adapted from [14]. In the following, we give the results without detailed explanation.

If heat surplus variable x_q^+ is in the basis of $(z_{1,0}, z_{2,0})$,

$$\begin{cases} x_q^+ = \sum_{u \in U} q_{k(0,u),u} - Q \\ z_{1,0} = \sum_{u \in U} c_{1,k(0,u),u} + x_q^+ c_q^+ \\ z_{2,0} = \sum_{u \in U} c_{2,k(0,u),u} \end{cases} \quad (16)$$

If regulating plant s contributes another point $(c_{1,k(1,s),s}, c_{2,k(1,s),s}, q_{k(1,s),s})$ with $q_{k(1,s),s} > q_{k(0,s),s}$. Let q_s denote heat provided by plant s , then

$$\begin{cases} q_s = Q - \sum_{u \in U, u \neq s} q_{k(0,u),u} \text{ with } q_{k(0,s),s} \leq q_s \leq q_{k(1,s),s} \\ z_{i,0} = \sum_{u \in U, u \neq s} c_{i,k(0,u),u} + \frac{c_{i,k(0,s),s}(q_{k(1,s),s} - q_s) + c_{i,k(1,s),s}(q_s - q_{k(0,s),s})}{q_{k(1,s),s} - q_{k(0,s),s}}, \quad i = 1, 2 \end{cases} \quad (17)$$

An NDLS associated with $(z_{1,0}, z_{2,0})$ can be represented as $(z_{1,0}, z_{2,0}, d_1, d_2, v)$, where $v \geq 0$, $d_1 d_2 < 0$, $\gamma = d_2/d_1$ is the slope of the line segment. Parameter v is the length of the segment and an isolated point is treated as a special case of the NDLS with $v = 0$. The formula for the NDLS is given below.

$$\begin{cases} z_1 = z_{1,0} + v d_1 \\ z_2 = z_{2,0} + v d_2 \end{cases} \quad (18)$$

Similarly, formulas for calculating the parameters (d_1 , d_2 and v) associated with the NDLS are also different depending on whether heat surplus variable x_q^+ is in the basis of $(z_{1,0}, z_{2,0})$ or not. The results are given in Appendix I.

Next, we consider the situation when the reference outcome $(z_{1,0}, z_{2,0})$ changes into a new reference outcome $(\hat{z}_{1,0}, \hat{z}_{2,0})$, where the point in some plant j changes. Again, results are different depending on whether heat surplus variable x_q^+ is in the basis of $(z_{1,0}, z_{2,0})$ or not. Let $\hat{q}_u (u \in U)$ denote the heat provided by regulating plant u for $(\hat{z}_{1,0}, \hat{z}_{2,0})$ if heat surplus variable is not in its basis.

If heat surplus variable x_q^+ is in the basis of $(z_{1,0}, z_{2,0})$, then it is possible to generate a new reference outcome with the basis not including x_q^+ if there exists a plant j with point $(c_{1,w,j}, c_{2,w,j}, q_{w,j})$, where $q_{w,j} < q_{k(0,j),j}$. Let \hat{q}_j denote the heat provided by plant j , if $q_{w,j} \leq \hat{q}_j$, then plant j is eligible to be a regulating plant. \hat{q}_j and $(\hat{z}_{1,0}, \hat{z}_{2,0})$ can be obtained according to a similar formula of (17). The neighborhood of $(\hat{z}_{1,0}, \hat{z}_{2,0})$ can be generated according to (A.4) and (A.6) in Appendix I if relevant adjustment elements exist.

If heat surplus variable x_q^+ is not in the basis of $(z_{1,0}, z_{2,0})$, there are three cases for changing the reference outcome.

- (a) Both the regulating plant s and two basis points of plant s remain unchanged while plant $j \neq s$ changes its basis point into point w , *i. e.*, $w \neq k(0, j)$. Then \hat{q}_s and $(\hat{z}_{1,0}, \hat{z}_{2,0})$ can be obtained according to a similar formula of (17).
- (b) The regulating plant s remains unchanged while one or both basis points of plant s are changed while the basis points for the remaining plants remain unchanged, then the heat provided by all plants remains unchanged. If q_s falls between the q values of two new basis points for plant s , then $(\hat{z}_{1,0}, \hat{z}_{2,0})$ can be obtained according to the second formula of (17).
- (c) The regulating plant changes to a new plant $j \neq s$ and a new point of plant j is introduced into the basis. The basis point of plant s will be either point $k(0,s)$ or $k(1,s)$ and basis points of the remaining plants remain unchanged, then \hat{q}_j and $(\hat{z}_{1,0}, \hat{z}_{2,0})$ can be obtained by a similar formula of (17).

Then, the neighborhood of $(\hat{z}_{1,0}, \hat{z}_{2,0})$ can be generated according to (A.4), (A.6) and (A.8) in Appendix I if relevant adjustment elements exist.

3.2.3 Relations between NDLSs and results of the ε -constraint method

If the hourly subproblem is solved using the ε -constraint method where the second objective of (2) is transformed into a constraint while the first objective is minimized. Then, for a given $y_{a,t}$, the number of constraints is $|U| + 2$ because constraints (4) can be dropped as described

in Section 2.3. It means that the basis structure for constructing NDLSs should be similar to that of the ε -constraint method.

Lemma 5. *For a given NDLS generated by the method described in Section 3.2.2, if two endpoints on the NDLS are elements of the PF, then the NDLS are elements of the PF.*

Proof. The NDLSs generated by the method described in Section 3.2.2 are feasible outcomes of the problem. Each adjustment can at most generate one NDLS. The elements on the PF will be generated by certain adjustments. The slope of the line segment is the image of the search gradient in the decision space and two points in the objective space can solely determine slope of the line segment. If two endpoints are the elements of the PF, then the slope of the line segment is the image for the gradient of optimality. \square

According to Lemma 5, the optimality of an NDLS constructed by the method in Section 3.2.2 can be verified by checking two points on the segment.

3.3 A Two Phase Solution Approach for the Hourly Subproblem

The two phase method divides the procedures for finding the non-dominated outcomes into two phases. In the first phase, supported non-dominated outcomes are computed. In the second phase, unsupported non-dominated outcomes are computed. Up to now, the two phase methods are mainly used to solve the multi-objective integer (combinatorial) problem [49–52]. Different from the combinatorial problem, the procedure for dealing with our problem (an MILP) needs to find NDLSs instead of discrete points in the second phase. For the bi-objective case, usually, in the first phase, the dichotomic search algorithm (DSA) can be used to find supported non-dominated outcomes [55, 56] by solving a series of weighted-sum scalarization problems. Here we applied the similar DSA procedure [53] in conjunction with a variant of EBB algorithm in the first phase. The profile of supported outcomes (points ‘•’) has been shown in Figure 2. The envelopes of the first and the second objective are constructed in the first phase according to algorithm in [14] and remain unchanged during the computation process of the second phase. In the following, we mainly focus on the second phase procedure.

Assume that supported dominated outcomes are arranged in an increasing order of the first objective, the second phase procedure explores sequentially the triangle (shown in Figure 2(b)) derived by two consecutive supported non-dominated outcomes for possible convex and non-convex NDLSs as mentioned in Section 3.1. The EBB algorithm in conjunction with the method for constructing NDLSs in Section 3.2.2 is used to solve the problem.

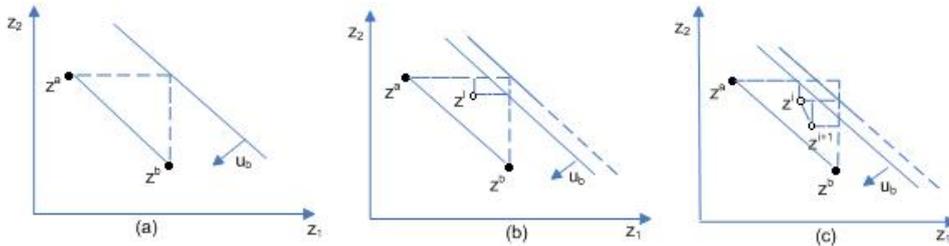


Figure 5: The upper bound for z_λ

3.3.1 The upper bound for the weighted-sum problem

For a given triangle $\Delta z^a z^b$ with $z_1^a < z_1^b$, for constructing NDLSs, reference outcomes need to be found first by solving the weighted-sum problem (15) with $\lambda_1 = z_2^a - z_2^b$ and $\lambda_2 = z_1^b - z_1^a$ using bounding techniques. Then NDLSs will be constructed using the method described in Section

3.2. The upper bound (u_b) for the problem is shown in Figure 5. Initially, $u_b = \lambda_1 z_1^b + \lambda_2 z_2^a$ (Figure 5(a)). For ease of manipulation, only the endpoints of an NDLS are used to compute u_b . If more points on the segment are used, the u_b may be tighter. However, it depends on the profile of the segment. In Figure 5(c), the u_b cannot be reduced if more points on segment $[z^i, z^{i+1}]$ are used. The principle of computing u_b for the vmin problem is similar to that for computing the lower bound for the vmax problem [52]. Suppose that there are m feasible non-dominated outcomes $z^i (i = 1, \dots, m)$ within the triangle with $z_1^1 < \dots < z_1^m$. Let $z^0 := z^a$ and $z^{m+1} := z^b$. The upper bound is given below.

$$u_b = \max_{i=0, \dots, m} (\lambda_1 z_1^{i+1} + \lambda_2 z_2^i) \quad (19)$$

Algorithm 1 in Appendix II gives the procedure for generating the upper bound for the weighted-sum problem (15) according to the endpoints of NDLSs. For an NDLS represented as $(z_{1,0}, z_{2,0}, d_1, d_2, v)$, only one endpoint $(z_{1,0}, z_{2,0})$ is explicitly given. The other endpoint is $(z_{1,0} + d_1 v, z_{2,0} + d_2 v)$. This representation makes the NDLS bear similarity to an isolated point and NDLSs can be arranged according to an increasing order of $z_{1,0}$. However, the original NDLS constructed by the method in Section 3.2.2 may need to be shortened (cut) for different reasons, *e. g.*, dominance checking, feasible bound checking and pseudo segment discussed a little later. For cutting operations, $(z_{1,0}, z_{2,0})$ and v may be subject to change while d_1 and d_2 remain unchanged. The first objective of the other endpoint can be larger or smaller than $z_{1,0}$ depending on whether $d_1 > 0 (< 0)$. It means that the points used to compute the u_b need to be identified dynamically. In addition, the number of points involved in computation is related to the pattern of the segment. As shown in Figure 2(b), if NDLSs assume a continuous pattern (Δ RS), the number of points involved in computation can be reduced because endpoints for two consecutive segments coincide. As shown in Figure 4(b), the same $(z_{1,0}, z_{2,0})$ can be associated with at most two NDLSs after dominance checking and the segment with $d_1 < 0$ is always placed before the segment with $d_1 > 0$ if there are two. Algorithm 1 in Appendix II gives the procedure for computing the u_b . In the algorithm NDLSs are arranged in a non-decreasing of $z_{1,0}$. If $z_{1,0}^i = z_{1,0}^{i+1}$, then $d_1^i < 0$ and $d_1^{i+1} > 0$; $z_{1,0}^i = z_{1,0}^{i+1} \Rightarrow z_{2,0}^i = z_{2,0}^{i+1}$. The tolerance for equality is $\varepsilon_1 = 0.012$ in the algorithm, *i. e.*, $0 \leq \varepsilon_1 \leq 0.012$ implies 0. This relation holds for all the endpoints for NDLSs.

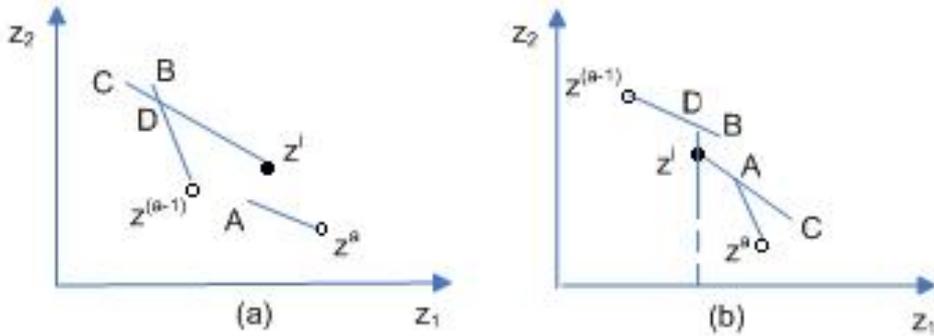


Figure 6: Dominance checking according to slopes of segments

3.3.2 Dominance checking for NDLSs

The dominance checking is conducted according to Definitions 2 to 4 in Section 3.1. For the bi-objective case, it is convenient to proceed the checking according to the slope of the segment as well as the the slope between two consecutive reference outcomes. There are different cases according to the difference between these two slopes and the sign of d_1 . Figure 6 illustrates two cases. Assume that a new segment associated with reference outcome z^i will insert before point

z^a according to $(z_{1,0}^i, z_{2,0}^i)$. In the figure, ‘●’ represents point z^i while ‘o’ represent points z^{a-1} and z^a . The other endpoints of the segments associated with z^a , z^{a-1} and z^i are z^A , z^B and z^C respectively.

In Figure 6(a), $\gamma(z^{a-1}, z^i) > 0$ while $\gamma(z^a, z^i) < 0$. It means that $(z_{1,0}^i, z_{2,0}^i)$ is dominated by $(z_{1,0}^{a-1}, z_{2,0}^{a-1})$. It is better that segment $[z^i, z^C]$ is checked against $[z^{a-1}, z^B]$ first because cutting operations will be significant by $[z^{a-1}, z^B]$. Two segments will cross if v^{a-1} and v^i are sufficiently large because $\gamma(z^{a-1}, z^B) < \gamma(z^i, z^C)$. The cross point is ‘D’ as shown in the figure. Segment $[z^i, z^D]$ is dominated by $[z^{a-1}, z^D]$ while segment $[z^D, z^B]$ is dominated by $[z^D, z^C]$. Afterwards, segment $[z^D, z^C]$ continues to check against the segments prior to $[z^{a-1}, z^D]$. However, it is not necessary to check against segment $[z^a, z^A]$. In Figure 6(b), $\gamma(z^{a-1}, z^i) < 0$ and $\gamma(z^a, z^i) < 0$. It does not matter which segments are checked against first. Suppose that segment $[z^i, z^C]$ is checked against $[z^a, z^A]$ first. Two segments will cross at some point if v^a and v^i are sufficiently large because $\gamma(z^a, z^A) < \gamma(z^i, z^C)$. As shown in the figure, point ‘A’ happens to be the cross point and segment $[z^C, z^A]$ is dominated by $[z^a, z^A]$. Then segment $[z^i, z^A]$ needs to be checked against $[z^{a-1}, z^B]$. It can be seen that segment $[z^D, z^B]$ is dominated by $[z^i, z^A]$. As mentioned in Section 3.3.1, $z_{1,0}^i = z_{1,0}^{i+1} \Rightarrow z_{2,0}^i = z_{2,0}^{i+1}$, the z_1^D should be a little smaller than $z_{1,0}^i$, *i. e.*, $z_1^D := z_{1,0}^i - \varepsilon_2$, $\varepsilon_2 > 0$. In our algorithm, $0.012 < \varepsilon_2 < 0.05$.

3.3.3 Feasible bound checking for NDLSs and a convex segment

As illustrated in Figure 5, it is required that NDLSs are within $\Delta z^a z^b$. Usually, an NDLS within $\Delta z^a z^b$ are generated according to the segments whose one or two endpoints are initially outside the triangle. During the process of checking the feasible bounds for the segment $(z_1 \leq z_1^b$ and $z_2 \leq z_2^a)$, the slope of the segment also needs to be checked. It is pretty easy to check whether supported non-dominated outcomes z^a and z^b are on the current segment by checking the slope of the segment against the slope of the convex segment $(-\lambda_1/\lambda_2)$. If yes, then the current segment is a convex segment (refer to Section 3.1). The construction process within the triangle ends once the convex segment is identified because all the NDLSs within the triangle are dominated by it. This implies that there are no unsupported non-dominated outcomes within the triangle.

3.3.4 A pseudo segment and isolated supported non-dominated outcomes

As illustrated in Figure 2(b), non-convex NDLSs (at least one endpoint is an unsupported outcome) may assume a continuous pattern (*e. g.*, all segments within ΔRS) or a non-continuous pattern (*e. g.*, segment $[z^A, z^Y]$ within ΔAB or segment $[z^T, z^U]$ within ΔST) or a mixed pattern where some segments are continuous while others not. Segments $[z^A, z^Y]$ and $[z^T, z^U]$ are called left and right continuous patterns, respectively for distinguishing purpose. In terms of segment $[z^A, z^Y]$, non-dominated outcomes will be points on $[z^A, z^Y]$ when $z_2^Y \leq z_2 \leq z_2^A$ and reduce to a single point (z_1^B, z_2^B) when $z_2^B \leq z_2 \leq z_2^Y - \varepsilon_3$ ($\varepsilon_3 > 0$). Usually $z_2^Y - z_2^B \gg \varepsilon_3$. For our problem, $\varepsilon_3 = 0.7$ when $z_2^Y - z_2^B \gg 0.7$ and $\varepsilon_3 = 0.01$ when $0.05 < z_2^Y - z_2^B < 0.7$. In terms of segment $[z^T, z^U]$, it means that a small change in z_2 can cause a big jump in z_1 .

Pseudo segment refers to the situation where non-dominated outcomes on the partial segment reduce to a single point. This situation may happen to left continuous non-convex segments and convex segments. Take non-convex segment $[z^A, z^Y]$ an example, let $[z^A, z^Y]$ be the original segment constructed by the method described in Section 3.2.2 and z^W is a point on $[z^A, z^Y]$ where $z_2^W > z_2^Y$. In some situations, non-dominated outcomes will reduce to (z_1^B, z_2^B) when $z_2 \leq z_2^W$. It means that segment $[z^W, z^Y]$ is a pseudo segment. An *isolated point* can be either a supported or an unsupported non-dominated outcome depending on the location of the segment. An isolated supported non-dominated outcome may occur when a pseudo convex segment occurs.

The fundamental causes for the pseudo segment should be attributed to the non-convex and non-continuous characteristics of power plants. However, we did not find an effective way to avoid it during the segment construction process. The pseudo segment can be identified and removed with the aid of the ε -constraint method. According to numerical experiments, the pseudo convex segment occurs occasionally when $\gamma(z^a, z^b) < -M$ and $z_1^b - z_1^a < \varepsilon_4$, where $M > 0$ (a big number) and $\varepsilon_4 > 0$. For our problem, only one occurrence is identified, where $M > 650$ and $\varepsilon_4 < 0.4$. The occurrence of the pseudo non-convex segment is more frequent and it depends on many factors such as the slope of the segment (d_2/d_1), the slope between two consecutive reference outcomes, and the relative gap between these two slopes.

3.3.5 Re-representation of NDLSs

The NDLSs for the hourly subproblem will be subject to further processing to obtain the PF for the multi-period problem after they are constructed. More explicit representation is needed for NDLSs. Two endpoints of each NDLS (*e. g.*, points ‘●’ and ‘○’ as illustrated in Figure 2) used to calculate u_b in Algorithm 1 of Appendix II are explicitly given. The NDLS is re-represented as $(z_{1,0}, z_{2,0}, \gamma, \theta)$, where $(z_{1,0}, z_{2,0})$ is the endpoint, γ is the slope and θ an auxiliary parameter.

The slope associated with a supported non-dominated outcome (including isolated one as mentioned in Section 3.3.4) should be the one with the immediately next supported non-dominated outcome while the slope associated with an unsupported non-dominated should be the one with the immediately previous or next non-dominated outcome. Take $\triangle AB$ in Figure 2(b) as an example, $\gamma(A,B)$ is associated with point ‘A’ while $\gamma(A,Y)$ is associated with point ‘Y’. The slope of the left continuous segment is the one with the immediately previous outcome while the slope of the right continuous and continuous segment is the one with the immediately next outcome. For an isolated unsupported non-dominated outcome and the last supported non-dominated outcome or only one outcome resulted from the only one extreme point in the active convex subareas of all plants, the slope is set to a large positive value.

Parameter θ is used to distinguish between a supported and an unsupported outcome, segment patterns (*e. g.*, continuous, left and right continuous) or identify how many unsupported non-dominated outcomes are used to represent segments within the triangle derived by two consecutive supported outcomes. The θ is defined below.

- b a non-negative integer associated with a supported non-dominated outcome where b unsupported non-dominated outcomes used to represent NDLSs within the triangle derived by it and the immediately next supported outcome
- 1 associated with an isolated supported non-dominated outcome
- 2 associated with the only non-dominated outcome in a hourly subproblem
- 3 associated with an unsupported non-dominated outcome with continuous segment pattern
- 4 associated with an unsupported non-dominated outcome with right continuous segment pattern
- 5 associated with an unsupported non-dominated outcome with left continuous segment pattern
- 6 associated with an isolated unsupported non-dominated outcome

3.3.6 Two phase algorithm

In Appendix II, Algorithm 2 gives the procedure for computing the NDLSs for a given pair of supported non-dominated outcomes (within a given triangle). Algorithm 3 gives the two phase procedure for computing NDLSs.

3.4 Merging Algorithm

Here the practice of obtaining the PF of the convex CHP planning problem [53] is extended to the current non-convex problem, *i. e.*, aggregating the PF of hourly subproblems according to a non-decreasing order of slopes for two consecutive supported non-dominated outcomes while unsupported non-dominated outcomes just move along the supported non-dominated outcomes. The notation used is similar to that in Algorithm 3 of Appendix II but the time index t will be attached for period t subproblem, *i. e.*, $(z_{1,0,t}^i, z_{2,0,t}^i, \gamma_t^i, \theta_t^i)$ is used to denote NDLS i . Let $\mathcal{Z}_{N,t}$ denote the set of non-dominated outcomes used to represent NDLSs, *i. e.*, endpoints of NDLSs. $|\mathcal{Z}_{N,t}| = m + 2$ in Algorithm 3 of Appendix II. If $|\mathcal{Z}_{N,t}| = 1$, then it means that there is only one non-dominated outcome in period t . It is a trivial case to merge. It is simply to add each non-dominated outcome with $(z_{1,0,t}, z_{2,0,t})$ and remaining associated parameters remain unchanged. In the following, assume that $|\mathcal{Z}_{N,t}| \geq 2$.

In Appendix II, Algorithm 4 gives the procedure for updating unsupported non-dominated outcomes associated with a supported non-dominated outcome according to the rule that the unsupported outcomes move along the supported outcome. Algorithm 5 gives the procedure for updating results according to slopes of the two consecutive supported outcomes and Algorithm 6 gives the procedure for merging the PF of a two period problem. Finally, Algorithm 7 gives the merging procedure for approximating the PF of a multi-period problem.

Lemma 6. *Algorithm 7 in Appendix II can guarantee the optimality of supported non-dominated outcomes and convex segments.*

The above result is guaranteed by merging operations of the algorithm, *i. e.*, arranging the non-dominated outcomes according to a non-decreasing order of slopes for two consecutive supported non-dominated outcomes. This is the rationality behind the merging algorithm. However, there is no guarantee for optimality of unsupported non-dominated outcomes.

Finally, Appendix IV illustrates the second phase procedure for the two phase algorithm.

4 Computational Experiments

To evaluate the effectiveness of the two phase approach for the hourly subproblem (Algorithm 3 in Appendix II) as well as the merging algorithm (MA) to aggregate the non-dominated segments for the hourly subproblems (Algorithm 7 in Appendix II). CPLEX [57] based augmented ε -constraint method [38] was implemented as a benchmark. For the two phase approach, the EBB algorithm [17] was adapted to solve the weighted-sum functions in the first phase and to find feasible outcomes for constructing NDLSs in the second phase. The EBB is on average 785 times faster than CPLEX for solving a single objective hourly subproblem [17]. The two phase algorithm and the MA were implemented in C++ in Microsoft Visual Studio 2003 environment. All experiments were carried out on a 2.49 GHz Pentium PC with 2.9 GB RAM under the Windows XP operating system.

Table 2: Dimensions of test problems

Problem	Plants	Binary variables	Continuous variables
D1	3a+1r	38 (3×8+14)	77 (3×16+27+2)
D2	3r	42 (3×14)	85 (2×28+27+2)
D3	3r+1a	50 (3×14+8)	101 (83+16+2)
D4	3a+2r	52 (3×8+2×14)	105 (3×16+28+27+2)
D5	3r+2a	58 (3×14+2×8)	117 (83+2×16+2)
D6	3r+3a	66 (3×14+3×8)	133 (83+3×16+2)

4.1 Test Problem

We used the same test problems as in [17]. A total of 6 test problems were generated according to three real plants (A1, B1 and C1) and three artificially derived plants (A2, B2 and C2) based on three real plants. A1 is a backpressure plant and the other two (B1 and C) are combined steam and gas cycles. Each real plant consists of 14 convex subareas and each derived plant 8 convex subareas. A1, B1 and C1 have 28, 27 and 28 extreme points, respectively. Each derived plant has 16 extreme points. Here we give a slightly different representation of test problems from that in [17]. We mainly focus on the number of binary integer and continuous variables. The number of binary variables is the number of convex subareas for all plants and the number of continuous variables is the number of extreme points for all plants plus the variables for the power output net level and for heat surplus. The dimensions of test problems are shown in Table 2. In ‘Plants’ column of the table, ‘a’ and ‘r’ stand for ‘artificial’ and ‘real’, respectively. *i. e.* ‘3a+1r’ means that D1 consists of three artificially derived plants and one real plant. For a planning problem with T hours, the number of binary integer and continuous variables is proportional to T . *e. g.*, for D2, the number of binary and continuous variables is $42T$ and $85T$, respectively.

In the current study, the fuel combusted at each plant needs to be considered explicitly since emission cost is explicitly considered as an objective. It is assumed that plants A1 (A2), B1(B2), C1(C2) are coal, gas and oil fired, respectively. To form a valid test problem, the heat demand is generated based on history data of a Finnish energy company, power price is generated based on the spot price history of the Nordic power market [58] and emission allowance price is generated based on uniform distribution within $[6, 16]$ €/ton according to the price range on the European market [59].

Note that D2 consists of three real plants. We will report detailed results for this problem later. The hourly subproblem is really hard due to the non-convexity of plant characteristics though the problem size is not large. We did numerical experiments with a limited planning horizon. For the first three problems in Table 2, we solved a planning horizon with 3024 hours instead of 8760 hours (one year) while for the last three problems, we solved a monthly planning horizon (672 hours). There are two reasons for this. On the one hand, as mentioned in Section 3.3.4, it is possible to generate pseudo segments when discontinuous patterns of the PF occur. Numerical experiments showed that the frequency of discontinuous patterns for the PF is quite high. *e. g.*, the PF can consist of about 30 NDLSs, where more than 15 endpoints need to be checked for possible pseudo segments. It is time consuming to check and remove pseudo segments. On the other hand, 3024 hours = 18 weeks = 4.5 months (1 month = 4 weeks) and a little more than one-third year. It is sufficient to capture the performance of the algorithm against the variation of demand patterns, power prices and emission prices as shown later in Figure 7 for problem D2. It can also be seen that the degree of non-convexity for the test problems is reduced when the artificially derived plants are introduced (see RCs measure in Table 3.)

The CPLEX based ε -constraint method cannot solve a daily (24 hour) subproblem in some cases (see later Tables 4 and 5). Therefore, five daily subproblems were formed by choosing evenly distributed starting hour 0, 750, 1500, 2250 and 3000 over the planning horizon for problem D2. These problems are identified by D2(0), D2(750), D2(1500), D2(2250) and D2(3000), respectively. To test the effectiveness of the MA, each daily subproblem was further divided into smaller planning horizons such as 1, 4, 8, 12, 16, 20 and 24 hours. For the ε -constraint method, the range of the second objective f_2 was obtained by computing the lexicographic minimal (lexmin) solutions \mathbf{x}_1 and \mathbf{x}_2 with respect to f_1 and f_2 .

Let $\mathbf{x}_1 := \text{arglexmin}(f_1(\mathbf{x}), f_2(\mathbf{x}), \mathbf{x} \in \mathcal{X})$ and $\mathbf{x}_2 := \text{arglexmin}(f_2(\mathbf{x}), f_1(\mathbf{x}), \mathbf{x} \in \mathcal{X})$. Let $z^1 := f(\mathbf{x}_1)$, $z^2 := f(\mathbf{x}_2)$. Then the right-hand-side coefficients related to f_2 were chosen as $z_2^1 - i(z_2^1 - z_2^2)/N, i = 1, \dots, N - 1$. For the augmented ε -constraint method, no weakly non-dominated outcomes are generated. However, for our problem, it is possible that discontinuous

patterns occur where several right-hand-side coefficients coincide with a single non-dominated outcome as mentioned in Section 3.3.4. Thus, the number of actually generated non-dominated outcomes \hat{N} should be not larger than $N + 1$, including z^1 and z^2 . In the following, the performance indicators (PIs) for evaluating the solution quality of optimizers are briefly reviewed before the computational results are presented.

4.2 Accuracy PIs

In the literature, some researchers proposed or reviewed the PIs for evaluating multi-objective optimizers from different perspectives [60–64]. Usually, the quality of a non-dominated set can be assessed from three aspects: the size of the non-dominated set, the accuracy of the solutions in the set, *i. e.*, the closeness of the solutions to the theoretical PF and distribution and spread of the solutions. In case the theoretical PF is not known, only the relative closeness can be obtained.

Our approach generates a set of NDLSs instead of discrete points in the objective space based on a two phase approach. The size of the non-dominated set is infinite and the NDLSs are well-distributed in the objective space as shown in Figure 2 of Section 3. Therefore, the evaluation mainly focuses on the accuracy of the solution set.

In the literature, there are three PIs that can address the accuracy: error rate (ER), generational distance (GD) [65] and hyper-volume indicator [66]. The ER measures the percentage of solutions from the solution set that are not the member of the theoretical PF. The GD measures the distance between the solution set and the theoretical PF. The hyper-volume indicator measures the volume that is dominated by the solution set. The larger the indicator value, the better the approximation set. Generally speaking, it is difficult to judge the performance of the multi-objective optimization algorithm because there is no universally accepted definition of optimum in the multi-objective optimization cases as in the single optimization [60]. Also, the evaluations according to different PIs may disagree with the common sense of when a multi-objective algorithm is performing better than another [63].

For our problem, it is difficult to calculate the hyper-volume indicator because the solution set consists of NDLSs with the nature of non-continuity and non-convexity as shown in Figure 2. The ER and GD are used to evaluate the accuracy of the solution set. It is straightforward to calculate the distance between the points generated by the ε -constraint method to the NDLSs generated by our approach. Let g_i denote the GD between point i on the theoretical PF: $PF := \{(\hat{z}_1^i, \hat{z}_2^i), i = 1, \dots, \hat{N}\} (\mathcal{Z}_N)$ to the approximate PF: $PF := \{(z_1^i, z_2^i), i = 1, \dots, m + 1\}$ generated by our approach. Let j and k be two consecutive non-dominated outcomes on the approximate PF with $z_1^j < \hat{z}_1^i < z_1^k$. Then, $g_i = |\gamma(j, k)(\hat{z}_1^i - z_1^j) + z_2^j - \hat{z}_2^i| / \sqrt{1 + \gamma^2(j, k)}$ if there is a line segment between j and k and $g_i = \min \left(\sqrt{(\hat{z}_1^i - z_1^j)^2 + (\hat{z}_2^i - z_2^j)^2}, \sqrt{(\hat{z}_1^i - z_1^k)^2 + (\hat{z}_2^i - z_2^k)^2} \right)$ otherwise.

A natural PI is average GD (\overline{GD}). Here we also introduce max GD (\widehat{GD}), which is used to evaluate the worst case performance of the algorithm. In addition, in most cases, the relative GD (GD_r) is more meaningful than the absolute value of GD. The distance of the non-dominated outcomes z^1 and z^2 mentioned at the end of Section 4.1 is used as the reference distance.

$$\overline{GD} = \frac{\sqrt{\sum_{i=1}^{\hat{N}} g_i^2}}{\hat{N}} \quad (20)$$

$$\widehat{GD} = \max_{i=1, \dots, \hat{N}} g_i \quad (21)$$

$$GD_r = 100 \frac{\widehat{GD}}{\sqrt{(z_1^1 - z_1^2)^2 + (z_2^1 - z_2^2)^2}} (\%) \quad (22)$$

Finally, the ER is associated with GD, defined as follows.

$$\text{ER} = 100 \frac{|(\hat{z}_1^i, \hat{z}_2^i) \in \mathcal{Z}_N, g_i > 0, i = 1, \dots, \hat{N}|}{\hat{N}} (\%) \quad (23)$$

Table 3: The results for the two phase method in conjunction with the merging algorithm

Problem	672 hour			3024 hour		
	NDs	CPU (s)	RCS	NDs	CPU (s)	RCS
D1	6614.3	0.38	0.42	26913.2	4.16	0.34
D2	9588.0	0.57	0.211	42854.0	5.17	0.173
D3	12029.3	0.62	0.228	53943.4	6.23	0.182
D4	13827.1	0.98	0.254	N/A	N/A	N/A
D5	16553.8	1.22	0.224	N/A	N/A	N/A
D6	20527.5	1.74	0.235	N/A	N/A	N/A

Table 4: Number of non-dominated outcomes generated by the CPLEX based ε -constraint method for problem D2

Instance	T								
	1	2	3	4	8	12	16	20	24
D2(0)	21	41	61	81	161	241	321	401	481
D2(750)	16	35	56	76	161	241	321	401	481
D2(1500)	21	41	61	81	161	241	321	396	473
D2(2250)	21	41	58	78	142	215	297	382	464
D2(3000)	21	41	61	81	158	236	317	396	N/A

Table 5: Solution time (s) for the CPLEX based ε -constraint method for problem D2

Instance	T								
	1	2	3	4	8	12	16	20	24
D2(0)	1.484	4.078	6.406	11.55	34.95	91.36	150.688	266.8	495.86
D2(750)	1.235	2.984	5.344	7.937	26.75	59.94	132.844	301.8	880.14
D2(1500)	1.39	4.063	7.609	12.7	51.72	234	1242.5	5144	20789
D2(2250)	2.188	5.953	11.453	17.8	124.1	341.1	570.344	1489	32485
D2(3000)	1.953	5.359	10.39	23.55	147.4	913.5	2405.58	9140	N/A

4.3 Computational Results

We have obtained the exact PF of the test problems in Table 2 for the chosen planning horizon by using the two phase approach with the aid of the ε -constraint method because the two phase approach cannot avoid pseudo segments as mentioned in Section 3.3.4. The results are given in Table 3. In the table, NDs, CPU and RCS represent the number of NDLSs, solution time in seconds and the ratio of convex line segments, respectively. For counting NDLSs, a jump is treated as an NDLS. For example, there are two NDLSs within triangle $\triangle AB$ in Figure 2(b). One is segment AY and the other is a jump from Y to B. This is consistent with the convention that an isolated point is treated as a special type of NDLS. In addition, the number of NDLSs is directly related to the number of non-dominated outcomes recorded in the algorithm using this

counting method. It can be seen that the overall solution efficiency of the two phase method in conjunction with MA is not bad according to Table 3.

As mentioned in Lemma 6, the convex segments for the hourly subproblems can maintain their optimality in the multi-period planning problem by our MA. The convex segments show the partially convex nature of the problem. RCs can partially reflect the hardness of the problem: the smaller the RCs, the harder the problem. As shown in Table 3, RCs has tendency to increase when the number of artificially derived plants increases.

In the following, the report mainly focuses on problem D2. The results of the CPLEX based ε -constraint method are reported first. Let $N = 20T$, *i. e.*, the sampling right-hand-side coefficients related to f_2 are proportional to T . The maximum generated non-dominated outcomes should be $20T + 1$. Tables 4 and 5 give the number of non-dominated outcomes and solution time (s) for a given sampling interval for different T for problem D2, respectively. For the CPLEX solver, only the relative objective gap is set explicitly to a suitable level according to the range of the objective values to guarantee the optimality of the solutions. Based on numerical results, for problem D2(3000), CPLEX can only solve partial instances of 24 hour planning horizon with larger right-hand-side coefficients related to f_2 . As the right-hand-side coefficients become smaller, the solver exits. A possible reason is that some default limits of the solver are exceeded before the optimal solution is found.

For problem D2, we have solved 18 weekly planning problems with non-overlapping periods over the planning horizon of 3024 hours. Figure 7 gives the ratio of convex segments for weekly planning problems of problem D2 for our approach. It can be seen that D2(0) and D2(750) are in the weeks with higher RCs while D2(1500), D2(2250) and D2(3000) are in the weeks with lower RCs. More specifically, the RCs of D2(0), D2(750), D2(1500), D2(2250) and D2(3000) are 0.69, 0.57, 0.2, 0.18, and 0.11. It means that hardness of the problem is also related to the interaction between load profiles and plant characteristics on top of the plant characteristics.

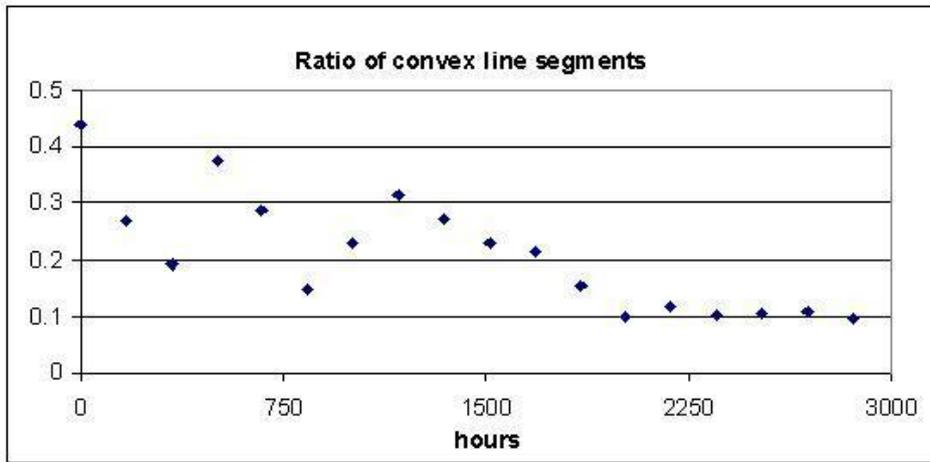


Figure 7: Ratio of convex line segments for weekly planning problems for problem D2

When the results of Table 3 for D2 are compared with those for Tables 4 and 5, our approach is clearly superior to the ε -constraint method in terms of solution efficiency. Next, the accuracy of the non-dominated set for the proposed approach is evaluated against the ε -constraint method. Tables 6 and 7 give the ER and GD_r for different planning horizons, respectively.

According to Table 6, the ER has tendency to increase as the planning horizon increases except D2(0). D2(0) is an instance showing strong convex nature and the ER is kept stable regardless of the planning horizon because the MA can generate the exact PF for the convex problem. The tendency of the \widehat{GD} (not shown explicitly) also shows the similar tendency. However, it seems that GD_r does show this kind of tendency according to Table 7. It means

Table 6: ER (%) for the two phase method in conjunction with the MA in comparison with the CPLEX based ε -constraint method for problem D2

Instance	T								
	1	2	3	4	8	12	16	20	24
D2(0)	0	4.88	4.92	8.64	8.70	6.64	4.36	4.49	3.33
D2(750)	0	5.71	5.36	6.58	7.45	11.20	17.13	22.19	29.11
D2(1500)	0	29.27	31.15	40.74	47.21	51.04	65.42	68.94	65.12
D2(2250)	0	51.22	48.28	47.44	71.13	75.81	65.99	69.90	74.78
D2(3000)	0	26.83	32.79	44.44	72.78	83.05	90.54	92.42	N/A

Table 7: GD_r (%) for the two phase method in conjunction with the MA in comparison with the CPLEX based ε -constraint method for problem D2

Instance	T								
	1	2	3	4	8	12	16	20	24
D2(0)	0	0.15	0.15	0.16	0.09	0.10	0.07	0.04	0.04
D2(750)	0	0.61	0.38	0.38	0.18	0.15	0.20	0.29	0.25
D2(1500)	0	0.34	0.29	0.22	0.28	0.22	0.16	0.18	0.14
D2(2250)	0	0.52	0.29	0.20	0.26	0.27	0.26	0.18	0.15
D2(3000)	0	0.75	1.06	1.26	0.58	0.51	0.39	0.27	N/A

that accuracy of the MA is not bad in a relative sense.

5 Conclusion

This paper presented a two phase method for generating the exact PF for the single period (hourly) bi-objective non-convex CHP planning problem according to the envelope of the plant. Then a merging algorithm (MA) was developed to approximate the PF for the multi-period planning problem. The envelope of a non-convex CHP plant is a generic piecewise linear function. It is structurally identical with the cost function reflecting variation between specified breakpoints or economies of the scale or congestion-related delay in different production, economics and transportation context [67]. Thus, the two phase method in the current study can be extended to above applications.

The numerical results with real plants showed that the accuracy of the two phase method in conjunction with MA is not bad in a relative sense though the evaluation is conducted in a limited scale due to the limitation of the benchmark CPLEX based ε -constraint method. The ε -constraint method sometimes cannot handle a daily planning problem let alone a tactical or a strategic planning problem with thousands of hourly subproblems. The MA is efficient enough and thus it is applicable to the risk analysis for medium term planning problem (several months) for tactical decision making. In addition, the two phase method for the hourly subproblem should be valuable for the short term planning problem in conjunction with coordination techniques for dealing with dynamic constraints. However, for dealing with a longer term planning, it is necessary to ignore non-convex characteristics as commented by [53].

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Appendix I

Formulas for calculating the parameters of NDLSs

- (A) If heat surplus variable is the basis of $(z_{1,0}, z_{2,0})$, then the adjustment element should come from point $(c_{1,w,j}, c_{2,w,j}, q_{w,j})$ of plant j with $q_{w,j} < q_{k(0,j),j}$ to absorb the heat surplus. Objective (2) denoted as (z_1, z_2) , constraints (3) (convex combination) and (5) (heat balance) are simplified as below.

$$\begin{cases} z_1 = \sum_{u \in U, u \neq j} c_{1,k(0,u),u} + x_q^+ c_q^+ + c_{1,k(0,j),j} x_{k(0,j),j} + c_{1,w,j} x_{w,j} - v c_q^+ \\ z_2 = \sum_{u \in U, u \neq j} c_{2,k(0,u),u} + c_{2,k(0,j),j} x_{k(0,j),j} + c_{2,w,j} x_{w,j} \\ x_{k(0,j),j} + x_{w,j} = 1 \\ q_{k(0,j),j} x_{k(0,j),j} + q_{w,j} x_{w,j} = q_{k(0,j),j} - v \end{cases} \quad (\text{A.1})$$

The above subproblem can be solved taking v as a parameter and at the same time checked against equations (16) and (18). The final results are given below.

$$\begin{cases} d_1 = (c_{1,k(0,j),j} - c_{1,w,j}) / (q_{w,j} - q_{k(0,j),j}) - c_q^+ \\ d_2 = (c_{2,k(0,j),j} - c_{2,w,j}) / (q_{w,j} - q_{k(0,j),j}) \\ v = \min(x_q^+, q_{k(0,j),j} - q_{w,j}) \end{cases} \quad (\text{A.2})$$

For this specific situation, it is required that $d_1 < 0$ and $d_2 > 0$.

- (B) If regulating plant s contributes another point in the basis of $(z_{1,0}, z_{2,0})$, discussions address three cases.

- (a) If the adjustment element $(c_{1,w,s}, c_{2,w,s}, q_{w,s})$ also comes from plant s and points w , $k(0,s)$ and $k(1,s)$ are in the same subarea if plant s is a non-convex plant. The change only occurs in plant s , which can be computed according to objective (2), constraints (3) and (5) as below.

$$\begin{cases} z_i = \sum_{u \in U, u \neq s} c_{i,k(0,u),u} + c_{i,k(0,s),s} x_{k(0,s),s} + c_{i,k(1,s),s} x_{k(1,s),s} + v c_{i,w,s}, \quad i = 1, 2 \\ x_{k(0,s),s} + x_{k(1,s),s} + v = 1 \\ q_{k(0,s),s} x_{k(0,s),s} + q_{k(1,s),s} x_{k(1,s),s} + q_{w,s} v = q_s \end{cases} \quad (\text{A.3})$$

Similarly, the above subproblem can be solved by taking v as a parameter. The max v can be obtained by considering $0 \leq x_{k(0,s),s}, x_{k(1,s),s}, v \leq 1$ at the same time. The values of d_i ($i = 1, 2$) are given below.

$$d_i = \frac{c_{i,k(0,s),s}(q_{w,s} - q_{k(1,s),s}) + c_{i,w,s}(q_{k(1,s),s} - q_{k(0,s),s}) + c_{i,k(1,s),s}(q_{k(0,s),s} - q_{w,s})}{q_{k(1,s),s} - q_{k(0,s),s}} \quad (\text{A.4})$$

- (b) If the adjustment element $(c_{1,w,j}, c_{2,w,j}, q_{w,j})$ comes from plant $j \neq s$ and points w and $k(0,j)$ are in the same subarea if plant j is a non-convex one. The change only occurs at plants j and s , which can be computed according to objective (2), constraints (3) and (5) as below.

$$\begin{cases} z_i = \sum_{u \in U, u \neq s, u \neq j} c_{i,k(0,u),u} + \sum_{w \in \{0,1\}} c_{i,k(w,s),s} x_{k(w,s),s} + c_{i,k(0,j),j} x_{k(0,j),j} \\ \quad + c_{i,w,j} v, \quad i = 1, 2 \\ x_{k(0,j),j} + v = 1 \\ x_{k(0,s),s} + x_{k(1,s),s} = 1 \\ q_{k(0,s),s} x_{k(0,s),s} + q_{k(1,s),s} x_{k(1,s),s} = q_s \\ q_{k(0,j),j} x_{k(0,j),j} + q_{w,j} v = q_{k(0,j),j} \end{cases} \quad (\text{A.5})$$

Again, the above subproblem can be solved by taking v as a parameter. The maximum v can be obtained by considering $0 \leq x_{k(0,s),s}, x_{k(1,s),s}, v \leq 1$. The values of d_i ($i = 1, 2$) (the coefficients of v in z_i) are given below.

$$d_i = (c_{i,w,j} - c_{i,k(0,j),j}) + \frac{(c_{i,k(1,s),s} - c_{i,k(0,s),s})(q_{k(0,j),j} - q_{w,j})}{q_{k(1,s),s} - q_{k(0,s),s}}, \quad i = 1, 2 \quad (\text{A.6})$$

- (c) Heat surplus variable is an adjustment element. As mentioned before, it is possible that the envelop slope of the first objective for plant s in the basis of $(z_{1,0}, z_{2,0})$ is negative, *i. e.*, producing additional heat may cause the first objective to decrease. In this situation, The adjustment is completed by plant s in conjunction with heat surplus. It can be treated as the dual case of (A.1) in the sense that it attempts to generate additional heat. The subproblem is give below.

$$\begin{cases} z_1 = \sum_{u \in U, u \neq s} c_{1,k(0,u),u} + c_{1,k(0,s),s} x_{k(0,s),s} + c_{1,k(1,s),s} x_{k(1,s),s} + v c_q^+ \\ z_2 = \sum_{u \in U, u \neq s} c_{2,k(0,u),u} + c_{2,k(0,s),s} x_{k(0,s),s} + c_{2,k(1,s),s} x_{k(1,s),s} \\ x_{k(0,s),s} + x_{k(1,s),s} = 1 \\ q_{k(0,s),s} x_{k(0,s),s} + q_{k(1,s),s} x_{k(1,s),s} = q_s + v \end{cases} \quad (\text{A.7})$$

The final results of the above subproblem are given below.

$$\begin{cases} d_1 = (c_{1,k(1,s),s} - c_{1,k(0,s),s}) / (q_{k(1,s),s} - q_{k(0,s),s}) + c_q^+ \\ d_2 = (c_{2,k(1,s),s} - c_{2,k(0,s),s}) / (q_{k(1,s),s} - q_{k(0,s),s}) \\ v = q_{k(1,s),s} - q_s \end{cases} \quad (\text{A.8})$$

Appendix II

Procedures related to two phase algorithm

At step 13 of Algorithm 2, the u_b computed according to Algorithm 1 may not decrease monotonically as the updating process of NDLSs goes on because only the endpoints of known NDLSs are chosen for computing u_b according to formula (27). Thus, the old u_b still needs to be considered for updating u_b .

Algorithm 1 Procedure for generating the upper bound for the weighted-sum problem (15)

Input: $\lambda_1, \lambda_2, (z_{1,0}^i, z_{2,0}^i, d_1^i, d_2^i, v^i) (i = 0, \dots, m + 1)$

Output: u_b

```

1: Set  $u_b := -M^B, \varepsilon_1 := M^S$ .  $\{M^B (M^S)$  is a big (small) positive value $\}$ 
2: if ( $d_1^0 > 0$ ) then
3:    $u_b := \lambda_1(z_{1,0}^0 + d_1^0 v^0) + \lambda_2 z_{2,0}^0$ .
4: end if
5: for ( $i = 0$  to  $m$ ) do
6:   if ( $v^{i+1} = 0$ ) then
7:     if ( $d_1^i \leq 0$ ) then
8:        $u_x := \lambda_1 z_{1,0}^{i+1} + \lambda_2 z_{2,0}^i$ .
9:     else
10:       $u_x := \lambda_1 z_{1,0}^{i+1} + \lambda_2 (z_{2,0}^i + d_2^i v^i)$ .
11:    end if
12:   else
13:     if ( $d_1^{i+1} > 0$ ) then
14:       if ( $d_1^i = 0$  or ( $d_1^i < 0$  and  $|z_{1,0}^i - z_{1,0}^{i+1}| > \varepsilon_1$ )) then
15:          $u_x := \max(\lambda_1 z_{1,0}^{i+1} + \lambda_2 z_{2,0}^i, \lambda_1 (z_{1,0}^{i+1} + d_1^{i+1} v^{i+1}) + \lambda_2 z_{2,0}^{i+1})$ .
16:       else if ( $(d_1^i < 0$  and  $|z_{1,0}^i - z_{1,0}^{i+1}| \leq \varepsilon_1)$  or ( $d_1^i > 0$  and  $|z_{1,0}^i + d_1^i v^i - z_{1,0}^{i+1}| \leq \varepsilon_1$ )) then
17:          $u_x := \lambda_1 (z_{1,0}^{i+1} + d_1^{i+1} v^{i+1}) + \lambda_2 z_{2,0}^{i+1}$ .
18:       else
19:          $u_x := \max(\lambda_1 z_{1,0}^{i+1} + \lambda_2 (z_{2,0}^i + d_2^i v^i), \lambda_1 (z_{1,0}^{i+1} + d_1^{i+1} v^{i+1}) + \lambda_2 z_{2,0}^{i+1})$ .
20:       end if
21:     else
22:       if ( $d_1^i = 0$  or ( $d_1^i < 0$  and  $|z_{1,0}^i - (z_{1,0}^{i+1} + d_1^{i+1} v^{i+1})| > \varepsilon_1$ )) then
23:          $u_x := \max(\lambda_1 (z_{1,0}^{i+1} + d_1^{i+1} v^{i+1}) + \lambda_2 z_{2,0}^i, \lambda_1 z_{1,0}^{i+1} + \lambda_2 (z_{2,0}^{i+1} + d_2^{i+1} v^{i+1}))$ .
24:       else if ( $(d_1^i < 0$  and  $|z_{1,0}^i - (z_{1,0}^{i+1} + d_1^{i+1} v^{i+1})| \leq \varepsilon_1$ ) or ( $d_1^i > 0$  and  $|z_{1,0}^i + d_1^i v^i - (z_{1,0}^{i+1} + d_1^{i+1} v^{i+1})| \leq \varepsilon_1$ )) then
25:          $u_x := \lambda_1 z_{1,0}^{i+1} + \lambda_2 (z_{2,0}^{i+1} + d_2^{i+1} v^{i+1})$ .
26:       else
27:          $u_x := \max(\lambda_1 (z_{1,0}^{i+1} + d_1^{i+1} v^{i+1}) + \lambda_2 (z_{2,0}^i + d_2^i v^i), \lambda_1 z_{1,0}^{i+1} + \lambda_2 (z_{2,0}^{i+1} + d_2^{i+1} v^{i+1}))$ .
28:       end if
29:     end if
30:   end if
31:   if ( $u_x > u_b$ ) then
32:      $u_b := u_x$ .
33:   end if
34: end for

```

Algorithm 2 Procedure for generating NDLSs for a given pair of supported non-dominated outcomes

Input: z^a, z^b

Output: $(z_{1,0}^i, z_{2,0}^i, \gamma^i, \theta^i)(i = 0, \dots, m + 1)$

- 1: Set $\lambda_1 := z_2^a - z_2^b, \lambda_2 := z_1^b - z_2^a, u_b := \lambda_1 z_1^b + \lambda_2 z_2^a$.
 - 2: Set $m := 0, \text{flag}:=1, \text{cflag}:=0, z_{1,0}^0 := z_1^a, z_{2,0}^0 := z_2^a, z_{1,0}^{m+1} := z_1^b, z_{2,0}^{m+1} := z_2^b$.
 - 3: **while** ($\text{flag}=1$ **and** $\text{cflag}=0$) **do**
 - 4: **if** (feasible solutions satisfying the upper bound u_b exist for the weighted-sum problem (15)) **then**
 - 5: Find adjustment elements according to Section 3.2.3
 - 6: Construct NDLSs for each adjustment element according to the method described in Section 3.2.3.
 - 7: Conduct feasible bound checking according to Section 3.3.3.
 - 8: **if** (A convex segment is identified) **then**
 - 9: $\text{cflag}:=1, m := 0$.
 - 10: **else**
 - 11: Conduct dominance checking according to Section 3.3.2.
 - 12: Update m and $(z_{1,0}^i, z_{2,0}^i, d_1^i, d_2^i, v^i)(i = 0, \dots, m + 1)$.
 - 13: $u_b := \min(u_b, \text{Algorithm 1}(\lambda_1, \lambda_2, (z_{1,0}^i, z_{2,0}^i, d_1^i, d_2^i, v^i)(i = 0, \dots, m + 1)))$.
 - 14: **end if**
 - 15: **else**
 - 16: $\text{flag}:=0$.
 - 17: **end if**
 - 18: **end while**
 - 19: Set (γ^i, θ^i) according to Section 3.3.5 and remove $(d_1^i, d_2^i, v^i), i = 0, \dots, m + 1$.
-

Algorithm 3 Two phase procedure

Input: Instance of a hourly subproblem

Output: $(z_{1,0}^i, z_{2,0}^i, \gamma^i, \theta^i)(i = 0, \dots, m + 1)$

- 1: Generate supported non-dominated outcomes $z^i(i = 1, \dots, n)$ according to DSA.
 - 2: Set $m_1 := 0$.
 - 3: **for** ($i = 1$ to $n - 1$) **do**
 - 4: $(\hat{z}_{1,0}^i, \hat{z}_{2,0}^i, \hat{\gamma}^i, \hat{\theta}^i)(i = 0, \dots, m + 1):=\text{Algorithm 2}(z^i, z^{i+1})$.
 - 5: **for** ($j = 0$ to $m + 1$) **do**
 - 6: $(z_{1,0}^{j+m_1}, z_{2,0}^{j+m_1}, \gamma^{j+m_1}, \theta^{j+m_1}) := (\hat{z}_{1,0}^j, \hat{z}_{2,0}^j, \hat{\gamma}^j, \hat{\theta}^j)$.
 - 7: **end for**
 - 8: $m_1 := m_1 + m + 1$.
 - 9: **end for**
 - 10: $m := m_1 - 1$.
-

Procedures related to the MA

The input outcome j is a supported non-dominated outcome of Algorithm 4. The condition at step 1 of Algorithm 4 means that outcome i is a supported outcome according to the state parameter (refer to Section 3.3.5). At step 2, the supported outcome is updated. From steps 4 to 6, the unsupported outcomes are updated. It can be seen that the operation at step 5 is the same as that at step 2. This means that unsupported outcomes move along the supported outcome. The index update at step 7 points to the next supported outcome.

Algorithm 4 Procedure for updating unsupported non-dominated outcomes associated with a supported non-dominated outcome for a two period problem

Input: $u_1, t_1, t_2, i, j, k, (z_{1,0,t_1}^i, z_{2,0,t_1}^i, \gamma_{t_1}^i, \theta_{t_1}^i), (z_{1,0,t_2}^j, z_{2,0,t_2}^j, \gamma_{t_2}^j, \theta_{t_2}^j)$
Output: $i, j, k, (z_{1,0}^k, z_{2,0}^k, \gamma^k, \theta^k)$

- 1: **if** $(\theta_{t_1}^i \geq -1)$ **then**
- 2: $z_{1,0}^k := z_{1,0,t_1}^i + z_{1,0,t_2}^j, z_{2,0}^k := z_{2,0,t_1}^i + z_{2,0,t_2}^j, \gamma^k := \gamma_{t_1}^i, \theta^k := \theta_{t_1}^i, k := k + 1.$
- 3: **if** $(u_1 > 0)$ **then**
- 4: **for** $(l = 1$ to $u_1)$ **do**
- 5: $z_{1,0}^k := z_{1,0,t_1}^{l+i} + z_{1,0,t_2}^j, z_{2,0}^k := z_{2,0,t_1}^{l+i} + z_{2,0,t_2}^j, \gamma^k := \gamma_{t_1}^{l+i}, \theta^k := \theta_{t_1}^{l+i}, k := k + 1.$
- 6: **end for**
- 7: **end if**
- 8: $i := i + 1 + u_1.$
- 9: **end if**

Algorithm 5 Procedure for updating non-dominated outcomes according to slopes of supported non-dominated outcomes for a two period problem

Input: $u_1, u_2, t_1, t_2, i, j, k, (z_{1,0,t_1}^i, z_{2,0,t_1}^i, \gamma_{t_1}^i, \theta_{t_1}^i), (z_{1,0,t_2}^j, z_{2,0,t_2}^j, \gamma_{t_2}^j, \theta_{t_2}^j)$
Output: $i, j, k, (z_{1,0}^k, z_{2,0}^k, \gamma^k, \theta^k)$

- 1: **if** $(\theta_{t_1}^i \geq -1)$ **then**
- 2: **if** $(\theta_{t_2}^j \geq -1)$ **then**
- 3: **if** $(\gamma_{t_1}^i = \gamma_{t_2}^j)$ **then**
- 4: **if** $(\theta_{t_1}^i = 0$ and $\theta_{t_2}^j = 0)$ **then**
- 5: $z_{1,0}^k := z_{1,0,t_1}^i + z_{1,0,t_2}^j, z_{2,0}^k := z_{2,0,t_1}^i + z_{2,0,t_2}^j, \gamma^k := \gamma_{t_1}^i, \theta^k := \theta_{t_1}^i.$
- 6: $i := i + 1, j := j + 1, k := k + 1.$
- 7: **else**
- 8: $(i, j, k, (z_{1,0}^k, z_{2,0}^k, \gamma^k, \theta^k)) :=$
 Algorithm 4($u_1, t_1, t_2, i, j, k, (z_{1,0,t_1}^i, z_{2,0,t_1}^i, \gamma_{t_1}^i, \theta_{t_1}^i), (z_{1,0,t_2}^j, z_{2,0,t_2}^j, \gamma_{t_2}^j, \theta_{t_2}^j)$).
- 9: **end if**
- 10: **else if** $(\gamma_{t_1}^i < \gamma_{t_2}^j)$ **then**
- 11: $(i, j, k, (z_{1,0}^k, z_{2,0}^k, \gamma^k, \theta^k)) :=$
 Algorithm 4($u_1, t_1, t_2, i, j, k, (z_{1,0,t_1}^i, z_{2,0,t_1}^i, \gamma_{t_1}^i, \theta_{t_1}^i), (z_{1,0,t_2}^j, z_{2,0,t_2}^j, \gamma_{t_2}^j, \theta_{t_2}^j)$).
- 12: **else**
- 13: $(j, i, k, (z_{1,0}^k, z_{2,0}^k, \gamma^k, \theta^k)) :=$
 Algorithm 4($u_2, t_2, t_1, j, i, k, (z_{1,0,t_2}^j, z_{2,0,t_2}^j, \gamma_{t_2}^j, \theta_{t_2}^j), (z_{1,0,t_1}^i, z_{2,0,t_1}^i, \gamma_{t_1}^i, \theta_{t_1}^i)$).
- 14: **end if**
- 15: **end if**
- 16: **end if**

Algorithm 6 Procedure for merging PF for a two period problem

Input: $t_1, t_2, (z_{1,0,t_1}^i, z_{2,0,t_1}^i, \gamma_{t_1}^i, \theta_{t_1}^i), (z_{1,0,t_2}^j, z_{2,0,t_2}^j, \gamma_{t_2}^j, \theta_{t_2}^j)$
($i = 0, \dots, |\mathcal{Z}_{N,t_1}| - 1, j = 0, \dots, |\mathcal{Z}_{N,t_2}| - 1$)

Output: $(z_{1,0}^k, z_{2,0}^k, \gamma^k, \theta^k)(k = 0, \dots, |\mathcal{Z}_N| - 1)$

- 1: Set $i := 0, j := 0, k := 0, u_1 := 0, u_2 := 0$.
- 2: **while** ($i < |\mathcal{Z}_{N,t_1}| - 1$ **or** $j < |\mathcal{Z}_{N,t_2}| - 1$) **do**
- 3: **if** ($\theta_{t_1}^i > 0$) **then**
- 4: $u_1 := \theta_{t_1}^i$.
- 5: **end if**
- 6: **if** ($\theta_{t_2}^j > 0$) **then**
- 7: $u_2 := \theta_{t_2}^j$.
- 8: **end if**
- 9: **while** ($i < |\mathcal{Z}_{N,t_1}| - 1$ **and** $j < |\mathcal{Z}_{N,t_2}| - 1$) **do**
- 10: $(i, j, k, (z_{1,0}^k, z_{2,0}^k, \gamma^k, \theta^k)) :=$
Algorithm 5($u_1, u_2, t_1, t_2, i, j, k, (z_{1,0,t_1}^i, z_{2,0,t_1}^i, \gamma_{t_1}^i, \theta_{t_1}^i), (z_{1,0,t_2}^j, z_{2,0,t_2}^j, \gamma_{t_2}^j, \theta_{t_2}^j)$).
- 11: **end while**
- 12: **while** ($i < |\mathcal{Z}_{N,t_1}| - 1$) **do**
- 13: $(i, j, k, (z_{1,0}^k, z_{2,0}^k, \gamma^k, \theta^k)) :=$
Algorithm 4($u_1, t_1, t_2, i, j, k, (z_{1,0,t_1}^i, z_{2,0,t_1}^i, \gamma_{t_1}^i, \theta_{t_1}^i), (z_{1,0,t_2}^j, z_{2,0,t_2}^j, \gamma_{t_2}^j, \theta_{t_2}^j)$)
- 14: **end while**
- 15: **while** ($j < |\mathcal{Z}_{N,t_1}| - 1$) **do**
- 16: $(j, i, k, (z_{1,0}^k, z_{2,0}^k, \gamma^k, \theta^k)) :=$
Algorithm 4($u_2, t_2, t_1, j, i, k, (z_{1,0,t_2}^j, z_{2,0,t_2}^j, \gamma_{t_2}^j, \theta_{t_2}^j), (z_{1,0,t_1}^i, z_{2,0,t_1}^i, \gamma_{t_1}^i, \theta_{t_1}^i)$).
- 17: **end while**
- 18: **end while**
- 19: $z_{1,0}^k := z_{1,0,t_1}^i + z_{1,0,t_2}^j, z_{2,0}^k := z_{2,0,t_1}^i + z_{2,0,t_2}^j, \gamma^k := \gamma_{t_1}^i, \theta^k := \theta_{t_1}^i, k := k + 1$.

Algorithm 7 Procedure for generating PF for a multi-period problem

Input: T

Output: $(z_{1,0}^k, z_{2,0}^k, \gamma^k, \theta^k)(k = 0, \dots, |\mathcal{Z}_N| - 1)$

- 1: Set $t := 1$, call Algorithm 3 to generate PF: $= \{(z_{1,0,t}^i, z_{2,0,t}^i, \gamma_t^i, \theta_t^i), i = 0, \dots, |\mathcal{Z}_{N,t}| - 1\}$ for period t subproblem, $t_2 := t, t := t + 1$.
- 2: **while** ($t < T + 1$) **do**
- 3: Call Algorithm 3 to generate PF: $= \{(z_{1,0,t}^i, z_{2,0,t}^i, \gamma_t^i, \theta_t^i), i = 0, \dots, |\mathcal{Z}_{N,t}| - 1\}$ for period t subproblem, $t_1 := t$.
- 4: Call Algorithm 6 to generate PF: $= \{(z_{1,0}^i, z_{2,0}^i, \gamma^i, \theta^i), i = 0, \dots, k - 1\}$ by merging PF: $= \{(z_{1,0,t_1}^i, z_{2,0,t_1}^i, \gamma_{t_1}^i, \theta_{t_1}^i), i = 0, \dots, |\mathcal{Z}_{N,t_1}| - 1\}$ and PF: $= \{(z_{1,0,t_2}^i, z_{2,0,t_2}^i, \gamma_{t_2}^i, \theta_{t_2}^i), i = 0, \dots, |\mathcal{Z}_{N,t_2}| - 1\}$.
- 5: **if** ($t < T$) **then**
- 6: $|\mathcal{Z}_{N,t_2}| := k$.
- 7: **for** ($i = 0$ to $k - 1$) **do**
- 8: $z_{1,0,t_2}^i := z_{1,0}^i, z_{2,0,t_2}^i := z_{2,0}^i, \gamma_{t_2}^i := \gamma^i, \theta_{t_2}^i := \theta^i$.
- 9: **end for**
- 10: **end if**
- 11: **end while**

Appendix III

Partial extreme points of plants

- Plant 1 (1680.12, 424.5882, 58), (1861.48, 382.3723, 88), (2320.94, 448.6264, 110.001), (3712.188, 766.2298, 173.001), (4307.06, 898.3487, 198)
- Plant 2 (401.4, 256.9367, 0), (299.22, 229.4326, 17.64), (521.22, 445.1696, 17.641), (486.855, 279.9389, 31), (542.85, 259.5612, 38), (753.09, 507.5828, 45), (670.935, 329.4883, 45.1), (884.025, 471.9268, 57), (892.11, 438.6529, 60), (1140.345, 696.9019, 71), (1272.57, 796.3035, 78), (1195.23, 435.1645, 81), (1423.14, 666.6724, 102)
- Plant 3 (166.45, 623.994, 0), (47.34, 565.5968, 37.6), (−160.92, 1018.792, 37.601), (293.72, 686.392, 63), (442.83, 630.3579, 76), (−216.04, 1559.9827, 81), (−144.1, 1595.2535, 89.002), (560.47, 1940.69, 156), (1290.45, 1045.929, 160), (1033.83, 1837, 193.001), (1221.47, 1619.0664, 205)

Envelopes of plants

- $$E_{1,14}^1 := \{(58, 6.0453), (88, 20.8836), (110.001, 22.0833), (173.001, 23.7958), 198\}$$
- $$E_{1,14}^2 := \{(88, 3.0114), (110.001, 5.0413), (173.001, 5.285), 198\}$$
- $$E_{1,14}^\lambda := \{(58, 500.47), (88, 2210.32), (110.001, 2450.86), (173.001, 2631.91), 198\}$$
- $$E_{2,(7,8,9,11,14)}^1 := \{(17.64, 14.0445), (31, 19.0168), 45\}, \{(0, -5.7925), (17.64, 222000), 17.641\}, \\ \{(31, 15.2758), (57, 18.3086), 71\}, \{(31, 13.0553), (45.1, 14.844), 60\}, \\ \{(31, 7.9993), (38, 13.7545), 102\}$$
- $$E_{2,(7,8,9,11,14)}^2 := \{(17.64, 3.7804), (31, 16.2603), 45\}, \{(17.64, 215737), 17.641\}, \\ \{(31, 7.3841), (57, 16.0696), 71\}, \{(31, 3.5141), (45.1, 7.3265), 60\}, \\ \{(38, 4.0838), (81, 11.0242), 102\}$$
- $$E_{2,(7,8,9,11,14)}^\lambda := \{(17.64, 1593.81), (31, 2839.42), 45\}, \{(0, -657.35), (17.64, 34732018), 17.641\}, \\ \{(31, 1933.65), (57, 2759.05), 71\}, \{(45.1, 1888.23), 60\}, \\ \{(31, 598.08), (38, 1721.72), (81, 1727.14), 102\}$$
- $$E_{3,(14,15)}^1 := \{(63, 5.6931), (193.001, 15.638), 205\}, \\ \{(0, -8.7064), (37.601, -1.27), (81, 8.9903), (89.002, 10.5163), (156, 12.7932), (193.001, 15.638), 205\}$$
- $$E_{3,(14,15)}^2 := \{(76, 4.9473), (160, 12.7364), 205\}, \{(37.6, 1.6865), (76, 4.9473), (160, 12.7364), 205\}$$
- $$E_{3,(14,15)}^\lambda := \{(63, 849.25), (76, 1054.33), 205\}, \{(0, -402.33), (37.6, 812.37), (81, 1141.8), (89.002, 1154.73), 205\}$$

Appendix IV

Illustration

Here an instance with three plants is used to illustrate the second phase procedure of the two phase algorithm for constructing NDLSs for a given pair of supported non-dominated outcomes for a hourly subproblem. Each plant has 14 subareas. Plants 1, 2 and 3 contain 28, 27 and 28 extreme points, respectively. The heat limits for the plants are 198, 102 and 205. The heat demand, power price and CO₂ emission allowance price are $Q = 318.08$ MW, $c_p = 26.34$ €/MW and $c_e = 14.42$ €/ton. For the current instance, the envelope points for a few subareas in each plant are sufficient to obtain NDLSs. The notation of extreme points is the same as that in Section 3.2.2. Let $E_{u,a}^i (E_{u,(a_1,\dots,a_2)}^i)$ ($i = 1, 2$) denote the envelope of the i^{th} objective for subarea a (from subarea a_1 to a_2) of plant u and $E_{u,a}^\lambda (E_{u,(a_1,\dots,a_2)}^\lambda)$ denote the envelope of the weighted-sum function for subarea a (from subarea a_1 to a_2) of plant u . The heat-slope pair (q_i, α_i) is used to denote points on the envelope. The relevant extreme points and envelopes of plants are given in Appendix III.

Numerical experiments show that four decimal points for the second objective of the non-dominated outcome are needed to guarantee that the jump for the discontinuous segment occurs at the correct value. *e. g.*, two consecutive unsupported non-dominated outcomes should be (750.32, 7985.6364) and (999.65, 7985.6224). If 7985.636 is used to check the correctness by the ε -constraint method, it may generate (999.42, 7985.636). For the continuous segment, the tolerance ' $\varepsilon_1 = 0.012$ ' for equality needs to be respected as mentioned in Section 3.3.1. For the current instance, heat surplus variable is not in the basis and the feasible basic basis takes the form $(q_{k(0,1),1}, q_{k(0,2),2}, q_{k(0,3),3}, q_{k(1,s),s})$, where $s = \{1, 2, 3\}$ is a regulating plant. In Table A1, the bold font for $q_{k(0,u),u}$ ($u = 1, 2, 3$) indicates $s = u$. The given supported non-dominated pair is (3486.6613, 2258.9975) and (3547.8153, 2161.9756), where $\lambda_1 = 97.0219$, $\lambda_2 = 61.154$ and initial $u_b = 482362$ according to step 1 of Algorithm 2 in Appendix II.

Table A1: Adjustment elements for computing NDLSs

Reference outcome	Basis	Subarea	u_b	Adjustment Elements
(3486.6613, 2258.9975)	(88, 17.64 , 205, 31)	(14, 7, 15)	482362	$\{58^1, 110.001^1, 45^2, 76^3, 160^3, 193.001^3\}$
(3537.544, 2253.2761)	(88 , 17.64, 205, 110.001)	(14, 8, 15)	482360	$\{58^1, 173.001^1, 17.641^2, 76^3, 160^3, 193.001^3\}$
(3547.8153, 2161.9756)	(88, 38, 76 , 205)	(14, 14, 14)	482360	$\{58^1, 110.001^1, 31^2, 81^2, 63^3, 160^3, 193.001^3\}$
(3534.0721, 2236.0042)	(88, 31, 76 , 205)	(14, 9, 14)	480658	$\{58^1, 110.001^1, 57^2, 63^3, 160^3, 193.001^3\}$
(3534.0721, 2236.0042)	(88, 31, 76 , 205)	(14, 11, 14)	480658	$\{58^1, 110.001^1, 45.1^2, 63^3, 160^3, 193.001^3\}$

Table A1 gives the set of adjustment elements for computing NDLSs. q values of elements are given in column 'Adjustment elements' and the superscript associated with q indicates the plant. These elements can be found according to the neighbourhood described in Section 3.2.1. For the current instance, all adjustment elements are on envelopes shown in Appendix II. Take the first row of the table as an example, the basis (88, **17.64**, 205, 31) indicates that $s = 2$. The active subareas of plant 1, 2 and 3 are 14, 7 and 15 respectively as shown in column 'Subarea'. For plant 1, elements with $q = 58$ and 110.001 are next to the basis element with $q = 88$ on $E_{1,14}^1$ and $E_{1,14}^\lambda$ while on $E_{1,14}^2$ only the element with $q = 110.001$ next to it. Plant 2 is a regulating plant, the elements next to the two basis elements with $q = 17.64$ and 31 need to be found. Only the element with $q = 45$ is next to the element with $q = 31$ on $E_{2,7}^1$, $E_{2,7}^2$ and $E_{2,7}^\lambda$. For plant 3, the active subarea is 15, which treats the whole plant as a subarea. The elements with $q = 193.001, 60$ and 89.002 are next to the basis element on $E_{3,15}^1$, $E_{3,15}^2$ and $E_{3,15}^\lambda$, respectively. However, the element with $q = 89.002$ is neither on $E_{3,15}^1$ nor on $E_{3,15}^2$ and thus will not be selected. In addition, it needs checking the true subareas of the basis element. Only subarea 14 contains the element with $q = 205$. The element with $q = 76$ is selected according to $E_{3,14}^\lambda$.

because it is also on $E_{3,14}^1$.

Next, again taking the first row of Table A1 as an example to illustrate how to determine parameters v , d_1 and d_2 as well as the reference outcome $(z_{1,0}, z_{2,0})$ in formula (18). If adjustment element 160^3 is chosen, *i. e.*, $j = 3, q_{w,3} = 160$, then the subproblem (A.5) needs to be solved. The heat provided by regulating plant $s = 2$ is $q_2 = Q - q_{k(0,1),1} - q_{k(0,3),3} = 318.08 - 88 - 205 = 25.08$. Then, $x_{k(0,3),1} = 1 - v$, $x_{k(0,2),2} = (q_{k(1,2),2} - q_2 + (q_{w,3} - q_{k(0,3),3})v) / (q_{k(1,2),2} - q_{k(0,2),2}) = ((31 - 25.08) + (160 - 205)v) / (31 - 17.64)$, $x_{k(1,2),2} = (q_2 - q_{k(0,2),2} + (q_{k(0,3),3} - q_{w,3})v) / (q_{k(1,2),2} - q_{k(0,2),2}) = ((25.08 - 17.64) + (205 - 160)v) / (31 - 17.64)$. $v = 0.1316$ considering $0 \leq x_{k(0,3),1}, x_{k(0,2),2}, x_{k(1,2),2} \leq 1$. $d_1 = 700.9841$ and $d_2 = -403.0187$ according to (A.6)) and extreme points (1290.45, 1045.929, 160) and (1221.47, 1619.0664, 205) of plant 3 as well as (299.22, 229.4326, 17.64) and (486.855, 279.9389, 31) of plant 2. The reference outcome can be obtained as (3486.6613, 2258.9975) as shown on the first row, the first column of Table A1 according to (17). It can be seen that that it is straightforward to apply an adjustment element to the original reference outcome. A little challenge about constructing NDLS is to change the reference outcome because combinations can be large. Table A2 gives combinations of adjustment elements which can lead to the NDLSs within the triangle. These segments will be subject to dominance checking. In the table, the NDLSs in bold can lead to the PF of the problem after dominance checking. The NDLSs outside the triangle were discarded immediately after construction. For simplification, these segments are not listed in the table.

Table A2: Combinations of adjustment elements for computing NDLSs

Basis	Combinations	NDLSs
(88, 17.64 , 205, 31)	76^3	(3486.6613, 2258.9975, 1033.1051, -501.035, 0.04589)
	160^3	(3486.6613, 2258.9975, 700.9841, -403.0187, 0.1316)
	110.001^1	(3486.6613, 2258.9975, 150.4662, -16.9188, 0.3382)
(88, 17.64, 205, 110.001)	76^3	(3537.544, 2253.2761, 1915.3439, -600.236, 0.1129)
	160^3	(3537.544, 2253.2761, 1008.7418, -437.6237, 0.3236)
(88, 38, 76 , 205)	63^3	(3547.8153, 2161.9756, -70.6424, 155.6714, 0.091)
	193.001^3	(3547.8153, 2161.9756, -115.2144, 309.9045, 0.9921)
	31^2	(3547.8153, 2161.9756, -13.7432, 74.0286, 1)
	$63^3, 193.001^3, 205^3$	(3541.3878, 2176.1395, -109.2451, 296.7502, 0.9929)
(88, 31, 76 , 205)	$63^3, 76^3, 193.001^3$	(3432.9166, 2470.7874, 75.0996, -171.0946, 0.007872)
	63^3	(3534.0721, 2236.0042, -70.6424, 155.6714, 0.0417)
	160^3	(3534.0721, 2236.0042, 340.5986, -228.239, 0.1316)
	193.001^3	(3534.0721, 2236.0042, -115.2144, 309.9045, 0.4934)
	58^1	(3534.0721, 2236.0042, -0.2809, 272.1482, 0.1973)
	57^2	(3534.0721, 2236.0042, 240.2348, -7.2867, 1)
(88, 31, 76 , 205)	45.1^2	(3534.0721, 2236.0042, 98.9728, -58.5187, 1)

According to Table A2 (*i. e.*, the procedure of Algorithm 2 in Appendix II), the PF consists of three continuous NDLSs represented as (3486.6613, 2258.9975, 700.9841, -403.0187, 0.043834), (3541.3878, 2176.1395, -109.2451, 296.7502, 0.219687) and (3547.8153, 2161.9756, -70.6424, 155.6714, 0.090986) in the beginning of the last step, where the first NDLS is found by the neighborhood of the first reference outcome and the last two NDLSs by the neighborhood of the third reference outcome in Table A2. The final representation of NDLS (refer to Section 3.3.5) are (3486.6613, 2258.9975, -1.5865, 2), (3517.3883, 2241.3316, -2.7164, -3), (3541.3878, 2176.1395, -2.2037, -3) and (3547.8153, 2161.9756, -0.6701, 0). The representation for the last non-dominated outcome is temporary and the slope value can be determined according to the next supported non-dominated outcome but the state value will be updated after the NDLS computation of the next triangle is finished.

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