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3-D EDDY CURRENT MODELLING OF STEEL LAMINATIONS
TO ANALYZE EDGE EFFECTS

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Abstract — The correct estimation of iron losses is still a challenging task in the numerical analysis of electrical machines. For estimation of eddy current losses, various formulations based on 1-D and 2-D models are mentioned in literature which neglect effect of current density at the edges of steel laminations. This paper compares such simplified 1-D/2-D eddy current loss model with a 3-D model to analyze the effect of edges on eddy current loss calculation. Thickness of the lamination along with frequency of field excitation were determined where considerable deviation in eddy current losses among loss models is observed due to edge assumption.

I. INTRODUCTION

Prediction of losses in electrical machines with high accuracy is a challenge which engineers face when developing more efficient machines. Among the electrical losses, core losses contribute a significant part. Both analytical and numerical models have been developed for calculating these losses. This paper will focus only on eddy current losses in the laminations.

Two dimensional finite element method (FEM) is a popular tool for the numerical analysis of electrical machines and traditionally eddy current losses are modelled by inclusion of 1-D coupled 2-D loss models. These loss models assume symmetry about the middle plane of lamination and does not account the effect of current density at the edges along the laminations in eddy current loss calculation as shown in Figure 1.

![Eddy currents in a lamination](image)

![Edge assumption in 1D model](image)

Fig. 1. Eddy current modelling along edges

Bertotti provided three components of core losses named as hysteresis loss, classical eddy current loss and excess loss.

\[ p_{fe} = c_{hy} f_{1} B_{p}^{2} + c_{d} f_{1}^{2} B_{p}^{2} + c_{ex} f_{1}^{2.5} B_{p}^{1.5} \]  

Where \( B_{p} \) is the peak magnetic flux density and \( f_{1} \) is the frequency of excitation. \( c_{hy}, c_{d} \) and \( c_{ex} \) are constants which depend on material properties. Further, skin effect can be included in eddy current losses by multiplying the classical eddy current loss by skin effect factor \( F_{sk} \) as presented in [1].

\[ F_{sk} = \frac{3 \sinh \lambda - \sin \lambda}{\lambda} \left( \frac{1}{\sinh \delta - \sin \delta} \right) \]  

For a lamination of thickness \( d \) the skin depth \( \delta \) is dependent on material permeability \( \mu \) and conductivity \( \sigma \).

II. 1-D COUPLED 2-D EDDY CURRENT MODEL

Time dependent fields which are responsible for eddy currents in a conducting medium are represented by Maxwell equations.

\[ \nabla \times E = -\frac{\partial B}{\partial t} \]  
\[ \nabla \times H = J \]  

These equations can be modified with the help of material law (\( J = \sigma E \)) related to current density \( J \) and electric field intensity \( E \) and a relation between the field strength \( H \) and time derivative of magnetic flux density can be derived. Here \( \sigma \) is conductivity of lamination material.

\[ \nabla \times \nabla \times H = -\sigma \frac{\partial B}{\partial t} \]  

A simple 1-D iron loss model was considered in [2]. Electrical conductivity was considered constant throughout the lamination and magnetic flux density distribution along the thickness \( z \in [-d/2, d/2] \) was represented by Fourier cosine series with \( N_{b} \) terms.

\[ B(z, t) = \sum_{n=0}^{N_{b}-1} B_{n}(t) \cos \left( 2n\pi \frac{z}{d} \right) \]  

III. 2-D COUPLED 3-D EDDY CURRENT MODEL

Various formulations for 3-D modelling of eddy currents are summarized in [3]. In this paper, AV-A formulation is used for the 3-D calculation of eddy currents with FEM software ELMER. The formulation can be derived with help of a magnetic vector potential \( A (\mathbf{B} = \nabla \times \mathbf{A}) \) and scalar potential \( V \).

\[ E = -\frac{\partial A}{\partial t} - \nabla V \]
Unique field distribution within the lamination is obtained by setting the normal component of flux density on its boundaries along with the normal component of current density [3]. As explained in [4] tangential component of $A$ will be applied on the boundaries to specify normal component of $B$. The 2-D field solution at a node gives the average magnetic flux density along the lamination thickness. The average magnetic flux density (2-D field solution) is then applied at axial boundaries of 3-D model by specifying tangential component of $A$ which is extrapolated from the 2-D vector potential and assumed to be constant along the thickness. Due to symmetry only half of the lamination needs to be simulated and the current density can be assumed to be zero at middle of the lamination. This condition of zero current density is applied by constant scalar potential and setting a free value to scalar potential will provide zero normal component of current density ($J$) on other lamination surfaces.

IV. RESULTS

A case of cuboid lamination ($l \times w \times d$ mm) which has the sinusoidal variation of magnetic flux density ($B = 1.5 \sin (2\pi f t) \textbf{u}_z$) is considered. This flux density value is applied in the 1-D eddy current model (Section II) and corresponding losses for a range of lamination thicknesses are observed with linear and nonlinear materials. Further a case with similar average flux density was simulated in 3-D. The axial boundaries of the lamination are supplied by vector potential $A_z = -1.5x \sin (2\pi f t)$ to generate sinusoidal $B$ of peak value 1.5 T.

First a case of $f = 50$ Hz supply with linear material is tested and results are compared with analytical model for 1-D model and 3-D model. Further lamination thicknesses of range 0.5 mm to 2.5 mm are considered keeping the radial lamination area constant ($l = w = 20$ mm). The results can be seen in Figures 2 and 3. Similarly nonlinear material was analyzed (Table I). Based on the results of both linear and nonlinear materials at 50 Hz it can be observed that eddy current losses at the edges have negligible effect with studied test cases at 50 Hz.

Further, the effect of frequency is observed with the supply of 1000 Hz with a nonlinear lamination and comparative analysis can be analyzed in Table II.

### Table I

#### Eddy Current Losses at 50 Hz with Nonlinear Material

| Lamination thickness (mm) | $w/d$ | 1-D Numerical (kW/m$^3$) | 3-D Numerical (kW/m$^3$) | Difference with respect to 1-D Numerical
<table>
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</thead>
<tbody>
<tr>
<td>1.5</td>
<td>13.3</td>
<td>196.89</td>
<td>197.73</td>
<td>0.43%</td>
</tr>
<tr>
<td>2</td>
<td>10.0</td>
<td>360.44</td>
<td>357.31</td>
<td>-0.87%</td>
</tr>
<tr>
<td>2.25</td>
<td>8.9</td>
<td>460.78</td>
<td>442.81</td>
<td>-3.90%</td>
</tr>
<tr>
<td>2.5</td>
<td>8.0</td>
<td>573.19</td>
<td>546.87</td>
<td>-4.59%</td>
</tr>
</tbody>
</table>

Further, similar behaviour of eddy current losses is observed at 5000 Hz.

### Table II

#### Eddy Current Losses at 1000 Hz with Nonlinear Material

| Lamination thickness (mm) | $w/d$ | 1-D Numerical (MW/m$^3$) | 3-D Numerical (MW/m$^3$) | Difference with respect to 1-D Numerical
<table>
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<tbody>
<tr>
<td>1.5</td>
<td>13.3</td>
<td>84.21</td>
<td>80.62</td>
<td>-4.26%</td>
</tr>
<tr>
<td>2</td>
<td>10.0</td>
<td>144.23</td>
<td>133.24</td>
<td>-7.62%</td>
</tr>
<tr>
<td>2.25</td>
<td>8.9</td>
<td>178.75</td>
<td>162.18</td>
<td>-9.27%</td>
</tr>
<tr>
<td>2.5</td>
<td>8.0</td>
<td>216.42</td>
<td>195.09</td>
<td>-9.86%</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

Analyzing the results of nonlinear material with 50 Hz and 1000 Hz supply cases, we can predict that the presented 1-D eddy current loss model is sufficiently accurate for thinner laminations at lower excitation frequencies as current density at edges can be neglected. However, it overestimates the eddy current losses when $w/d$ ratio is below 10 and frequencies are above 1 kHz with more than 5% difference. The laminations in rotor poles of salient pole synchronous machines are comparatively thick (2 mm) and area between damper bars are observed to have $w/d$ ratio around 10. Based on the results, we may observe considerable difference in eddy current loss calculations under studied loss models for such machines.

REFERENCES


