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*Published in:*
Marine Structures

*DOI:*
10.1016/j.marstruc.2015.10.004

Published: 01/01/2016

*Document Version*
Publisher's PDF, also known as Version of record

*Please cite the original version:*
Estimating sea bottom shapes for grounding damage calculations

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A R T I C L E   I N F O

Article history:
Received 24 June 2015
Received in revised form 29 September 2015
Accepted 13 October 2015
Available online 30 November 2015

Keywords:
Groundings
Ship damage
Sea bottom shape
Gulf of Finland

A B S T R A C T

Groundings are among the most common and serious maritime accidents. The shape of the sea bottom is one of the important factors that determine the extent of grounding damage on ships, including loss of watertightness. This paper presents a four-step methodology for mathematically analyzing sea bottom shapes, where individual peaks are identified and isolated from larger datasets. To these individual peaks mathematical models are fitted and the goodness of the fit is evaluated. The aim is to develop mathematical rock models that can be used in grounding damage analysis. The method is applied to bottom topography data from the two busiest tanker harbors in Finland. As potential mathematical rock models common assumptions found in literature as well as new suggestions are used. It was observed that rocks vary in shape and size locally as well as between different geographical areas. This highlights the need to have rock models that are flexible enough to model a wide range of sizes and shapes so that the local conditions can be taken into account. This includes modeling asymmetrical shapes, which is not taken into account in the current models. The analysis results suggest using a binormal function to describe the bottom shape: This model gives overall better goodness-of-fit test results than the cone and polynomial models. In the data the binormal model can model well the also the bottom shapes where the polynomial and conical models had a good fit in statistical terms. Although the paper discusses data

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http://dx.doi.org/10.1016/j.marstruc.2015.10.004
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from the Gulf of Finland, the framework for analyzing bottom shapes is expected to be applicable to other sea areas as well.

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1. Introduction

Groundings are among most common and serious maritime accidents. As grounding happens, significant parameters affecting the resulting damage are e.g. ship kinetic energy, loading condition and structural solutions (e.g. double bottom, compartments), the angle of attack to the ground and tides, and the properties of ground itself (e.g. shoal, rock). The relation between kinetic energy, structural solutions, the resulting damage and the residual strength of hull girder has been investigated by several authors using idealized bottom shapes such as cones and parabolas [1–4,8,10–14,18,21–23,33–37,39,40].

However, as stated by several authors (e.g. Refs. [1,11,24,34]) bottom shapes have central role in ship grounding event and therefore need further investigation. As an example of this [1] presented a comparative study on a single ship grounding on rock, reef and shoal. While sharp shape cuts the plating easily, the damage might be limited. As the size of indenter increases, the damage naturally spreads out for larger area in double bottom. Even if there is no hull fracture, the overall damage may harm the global strength of the hull girder [1,5,35]. From a risk analysis perspective this has significant implications; e.g. on oil tankers a widespread damage might result in no oil spill if it is absorbed by the outer hull whereas a narrow but deep cut that penetrates the inner hull might result in a significant oil spill.

Another important issue is the grounding depth. Currently it is assumed to be either triangularly [13,30] or uniformly distributed [13]. In a triangular distribution the theoretical grounding depth range is between sea level \( (z = 0 \text{ m}) \) and vessel draft. The most likely grounding depth is so that the sea bottom barely scrapes the ship bottom and the least likely grounding depth is at 0 m depth. [13] created the distributions based on expert opinions, not on empirical data. Grounding reports usually do not specify the actual grounding depth, thus there is a need for further research. Tikka et al. [30] state that “Distributions for speeds, obstruction depths, and tides were based on the data from charts, NOAA\(^1\), USCG\(^2\) offices, and pilots,” but the exact data or methodology behind arriving at triangular distribution for the grounding depth is not specified.

This paper describes a systematic approach to identify individual peaks from the sea bottom data pool and how the individual peaks can be modeled mathematically for grounding damage calculations. The paper also aims at testing the assumptions regarding grounding depth and the current conical rock shape assumption using bottom topographical data from selected Finnish harbors. Having correct assumptions regarding these factors is important for the damage estimation in the existing grounding models.

1.1. Current rock shape assumptions in literature

Currently the bottom shape during grounding is assumed to be a symmetrical cone-like object with varying diameter and slope. Most commonly the rock is assumed to be a blunted cone, see e.g. Refs. [15,21,26]. There is some variations in how the cone tip looks like, i.e. whether it is simply cut-off or rounded, see Fig. 1. Another rock model found in literature is the polynomial rock used by Ref. [9]; who use a 2nd order polynomial equation to describe the rock, see Fig. 1. The equation is

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\(^1\) National Oceanic and Atmospheric Administration.

\(^2\) United States Coast Guard.
\[ z = -\frac{y^2}{a} \]

where \( a \) is the form parameter of the rock for the 4 different rock sizes illustrated in Fig. 1.

The sea bottom shapes of [1] are relatively similar to the ones in Fig. 1. Other fields have also dealt with geological formations; The geometry of mountainous landscapes (e.g. Refs. [7,20,25]) and rocks (e.g. Refs. [19,27,29,38] has been analyzed in literature. [28] present a method for quantitatively and qualitatively analyzing similarities between surfaces including peak regions. Other relevant publications include [28] alpha peak region analysis, sand grain shape parameter classification of [6] and automatic mound- [32] and glacier valley [25] detection. However, the methods found in literature do not cover extensively this paper’s research question; no comprehensive method to identify, isolate, classify and mathematically model individual peaks from larger topographical data was found.

2. Methodology

The bottom shape analysis based on this data is carried out using the following four-step approach:

Step 1: Define relevant sea area and find bottom data
Step 2: Identify individual peaks from the larger bottom formations
Step 3: Define the cut-off method for the individual peaks
Step 4: Fit mathematical models to the data and evaluate the models

The steps are described in detail as follows:

Step 1: Define relevant sea area and find bottom data

In this case, two fareways from the Gulf of Finland was selected for analysis. On these fareways all rock formations that follow the following criteria are further selected for analysis:
1a) Rock formation is in the immediate vicinity of the fareway so that ships can possibly ground there.

1b) Sufficiently high resolution data is available of the location, e.g., data from multibeam echo soundings.

1c) The formation has at least one bottom shape apex which is at a depth of no less than \( m \) meters and no deeper than the fareway nominal draft clearance.

Restriction c) is due to the draft limitations of the survey vessels, limiting availability of data from shallow waters as they cannot be covered from all sides even with side-sweeping sonar equipment. In this case \( m \) is approximately 5 m. This restriction is illustrated Fig. 2.

In Fig. 2, line 1 is the 5 m depth mark. Bottom formations above this line (f) are theoretically not included in the data as the surveying vessels cannot sail directly above it. However, due to the side-sweeping technology there would most likely be some data from at least one side of f available. If this is the case then f would also be included in the analysis. The formations below line 2 are below the nominal fareway draft and as such are not included in the analysis at all. Using the method described here, peaks a–e (and f) are analyzed separately.

Step 2: Identify individual peaks from the larger bottom formations

From the larger bottom formations, single peaks from the larger bottom formation (a–e in Fig. 2) were cut out as follows:

2a) Selecting the mid-point of the “rock” in the plotted figure (usually the apex) by visual inspection and manual input. Alternatively, an algorithm for peak finding can be used, see Ref. [28].

2b) Defining this point as origo \( x = y = 0 \).

2c) Selecting all the data points within a radius of \( r \) meters from it in the xy-plane for separate analysis.

The apexes had to fulfill the following criteria:

The highest point must be no deeper than fareway nominal depth and the protrusion has to stand out enough from the surrounding features. In practice this means that ~0.5 m “bumps” are included or are included only as a part of a larger bottom formation. Again, an algorithm can be used for large datasets but it has to be tuned properly so that it does not cut the data into too small or large subsets.
Step 3: Define the cut-off method for the individual peaks

In this step the individual peaks are isolated into their own datasets using the following approach:

3a) Selecting all data points less than $r$ distance in the xy-plane from each rock midpoint for separate analysis, see Fig. 3. The radius $r$ to which select the data around the midpoint or apex (2a) was defined iteratively as follows:
3b) Plotting the selected data separately from the bigger data.
3c) Changing the radius until a visual check would show that the whole of the “rock” would be included and all other apexes would be excluded (except protrusions that are not big enough to be analyzed on their own, see 3a).

Alternatively algorithms can be used that e.g. detect changes in gradient. The cut-off formation does not necessarily have to be done as a circle around the apex, working with height/depth curves prominence can be used as a cut-off criteria, see also [28].

Step 4: Fit mathematical models to the data and evaluate the models

4a) Define models to be fitted to the data.
4b) Calculate fit.
4c) Calculate goodness of fit, e.g. using coefficient of determination $R^2$.
4d) Evaluate goodness of fit visually.
4e) Tweak cut-off radius $r$ if it improves 4c) and/or 4d) if it does not compromise on 3b).

Evaluating the goodness of the fit (4d) visually is recommended as it is possible to get mathematically well-fitting models that make no practical sense such rock models with the peak kilometers over the sea level. In the following chapter, this four-step methodology is applied to a case-study in the Gulf of Finland.

3. Case study

The bottom topography data set used in this paper are from hydrographic survey database of Finnish Transport Agency (Liikennevirasto). The data has been collected in controlled hydrographic surveys which fulfill at least Order 1a of IHO’s survey standard S-44, see Ref. [12]. Data collection was done from 1999 to 2012 by multibeam (MBES) and multi transducer echo sounder (MTES) systems as fairway surveys. The data consists of discrete xyz-data points of sonar readings, each point corresponding to one sonar signal echo from a bottom shape or object in the water column.
Horizontal resolution of the data sets varies based on survey technique and water depth. Typical sonar footprint ranges between ca. 1 m for MTES to centimeter level for MBES systems in shallow water. The depth value of a sounding is typically detected from the first significant return echo within the footprint. Vertical uncertainty of sounding is typically less than 0.3 m (given a 95% confidence interval), usually much less. Uncertainties regarding relative position and depth are an order of magnitude smaller, and therefore the bottom shape is very well preserved within limitations of horizontal resolution. The data has been post-processed in order to remove obvious outliers above the bottom. Possible outliers close to the bottom have been removed only if independent second source data validates the removal. Since the data is primarily used for nautical charting the depth values do not always represent the most probable bottom depth but shallowest probable bottom depth. Depth difference between these levels is typically few centimeters and does not affect the bottom shape. Parts of the survey data is classified by legislation therefore the true location of individual rocks has been removed but the local shape has been preserved.

4. Application of the four-step methodology

Steps 1–4 are applied to this data, which is described as follows.

Step 1: Define relevant sea area and find bottom data

The data sets are chosen along two deep draught channels on northern shore of Gulf of Finland that have the highest volume of tanker traffic in Finland:

Sköldvik channel in Porvoo
Design vessel: tanker, $L = 300$ m, $B = 40$ m, $T = 15.3$ m. Maximum authorized draught 15.3 m, safe clearance depth in the outer channel –17.45 m, in the inner channel –16.95 m, minimum width 270 m. Vessel speed 12 knots in the inner channel.

Hamina channel
Design vessel: $L = 210$ m, $B = 32$ m, $T = 12.0$ m. Maximum authorized draught 12.0 m, safe clearance depth in the approach –13.8 m, in the entrance –13.5 m and at the port entrance –13.2 m. Minimum channel width 180 m.

A design vessel indicates the type and the maximum dimensions of vessels for which the fairway is designed.
In Fig. 4 red lines represent the Sköldvik (Porvoo) channel (westernmost) and Hamina (easternmost) channel.

Step 2: Identify individual peaks from the larger bottom formations

For the Hamina fareway, the authors were able to isolate 16 different bottom formations with a total of 144 apexes. For Porvoo, the numbers were 13 different bottom formations with a total of 55 apexes. This brings the total number to 199 apexes spread over 29 different formations. The larger formations were labeled with numbers for Hamina and with capital letters for Porvoo. The lower-case letter in parenthesis denotes the individual apexes of the formation in question. A larger formation is typically of the size order of magnitude of 150 m \times 150 m \times 10 m in the xyz-dimensions. In the data the x-axis values are the easting coordinates and y-axis the northing coordinates for the data points. The z-axis describes the water depth where z = 0 is the sea level. All units are in meters. Looking at the different formations some pattern repeated themselves and the formations in the data were classified into the following three categories:

Type 1: Formations with multiple peaks and one clear apex

![Fig. 5. Type 1: Multiple peaks and one clear apex (formation 8).](image)

Note distorted dimensions: x- and y-axes each cover an area of \(~200\) m while the z-axis only 6 m. In Fig. 5 the difference between the highest and the second highest peak is >2 m.

Type 2: Formations with multiple roughly even peaks

Type 2 formations may or may not include smaller peaks deeper down such as the following example, which has 11 distinguishable individual apexes, illustrated in Fig. 6.

![Fig. 6. Type 2: Multiple roughly even peaks (formation 4).](image)
Please note that the dimensions of the figure are heavily distorted: The depth-axis (z) runs only from \(-6\) to \(-14\) m (\(\Delta z = 8\) m) while the x-axis runs from 0 to 140 m and the y-axis from 0 to 250 m.

Type 3: Solo rounded rock

Besides the multiple peak formations, also some lone rocks were found such as the following example in Fig. 7.

![Fig. 7. Type 3: Solo round rock (formation V) with data points and a fitted surface.](image)

In Fig. 7 the blue dots are the actual data while the surface is a fitted function which is meant to aid in visualization of the form of the rock.

Step 3: Define the cut-off method for the individual peaks

After identifying the peaks in the data, the next step is to define the cut-off radius in the xy-plane for each bottom formation around this apex- or midpoint. The aim is to have each individual bottom shape cut-out into its own data file for separate analysis.

In Figs. 8 and 9 the cut-off radii are presented for the two channels along with other descriptive data. The figures include histograms for the 199 bottom shapes for the following variables:

- \(z_{\text{max}}\) is the highest point of each peak in meters below the sea level, see Fig. 3.
- \(r\) is the cut-off radius in meters in the xy-plane around each peak apex or midpoint, see Fig. 3.
- \(\Delta z\) is the difference in the highest and deepest point in the z-axis for each bottom formation in meters, see Fig. 3.
- Resolution is the number of data points per m\(^2\) in the xy-plane (ie. as seen from directly above).
- \(\Delta z/r\) is the depth change divided by the radius of the bottom formation, which tells us how steep the individual formation is: the lower this values is the blunter the formation and vice versa.
- \(N\) is the number of the xyz-data points for each individual bottom formation, see the blue dots in Fig. 7 for an example.
The $y$-axis is the number of observations in each histogram bin.

4.1. Qualitative description of individual peaks

When looking closer at the individual bottom shapes, there are 199 individual peaks which can be classified as follows and are illustrated in Fig. 10:
1. Sharp small peak

These smaller rocks are usually a few meters wide and deep, which means that their size is of the same order of magnitude as the model rocks most commonly used in grounding damage calculations.

2. Wide and blunt peak

These rocks resemble the “shoal” type by Ref. [1]; where the width is around ten times greater than the depth. These rocks may be so wide that they will unlikely cause a spill due to a grounding on a double-hulled vessel, see Ref. [1].

3. Ridge

Ridges are steep to some sides while remaining quite flat in other directions. The models found in the literature review assume symmetry and as such will have problems modeling this type of bottom shapes.

4. Compilation of multiple smaller formations

The smaller tips of the formation could not be modeled as stand-alone bottom shapes: Practical limit is around 1 m protrusion from surroundings or else there is no reasonable fit with the models used here. These formations were analyzed as a cluster of the surrounding shapes, see Fig. 10.

![Individual bottom shape types.](image)

Fig. 10. Individual bottom shape types.

The blue dots are the actual data while the surface is an approximate fit to the data which is meant as a visualization aid. In Fig. 10 the peak types are as follows: Sharp small peak (top left, Formation 10
a), wide and blunt peak (top right, Formation S (a)), ridge (bottom left, Formation D (b)) and compilation of multiple smaller formations (bottom right, Formation 10 (m)). For all of the 199 peaks found in the data, steps 3 and 4 were applied.

Step 4: Fit mathematical models to the data and evaluate the models

To these individual peaks, various functions were fitted according to the approach explained in 4). This data is used to statistically test the current grounding depth and bottom shape assumptions in terms of how well the models describe the data. The data fitting, filtering and testing was done using MATLAB.

The reasoning of the choice of models to fit to the data is as follows: The models should avoid unnecessary complexity so that coefficient interpretation, practical implementation, dissemination of information to other researchers as well as calculations are practically feasible with reasonable effort. As such splines and other more complex models are left as suggestions for future research, see also [31]. The models should also evaluate the current assumptions in literature and preferably at least one model should have a good fit. Models 1 [9] & 3 (cone) are presented in the literature.

Besides these, the authors add two models to the testing: Model 2 (2nd order polynomial) is a more detailed and flexible version of model 1. The scaled and shifted binormal model (4) is selected as it resembles a cone (model 3) with a blunted peak if standard deviations of x and y are small and a 2nd order polynomial equation if the standard deviations are large. Besides this, many things in nature are approximately normally distributed which makes the choice a “natural” one.

The fitted models are as follows:

1. Heinvee et al. [9] model extended in the x-axis direction

\[ z = b_0 + b_1 (x - x_0)^2 + b_2 (y - y_0)^2 \]

The origo of the rock is already defined approximately visually in 2a) but as this model is quite sensitive to the exact location of the origo, constants \( x_0 \) and \( y_0 \) were added so that potentially better fits could be achieved by moving the origo. Besides this another polynomial equation with more terms was tested out to see if adding terms would improve the model fit:

2. 2nd order polynomial equation

\[ z = b_0 + b_1 x + b_2 y + b_3 x^2 + b_4 x y + b_5 y^2 \]

3. Cone

\[ z = \frac{\frac{(x - x_0)}{h} \cdot \frac{h}{r \cos(\arctan((y - y_0)/(x - x_0)))}}{+ h - b_1} \]

Where \( h \) is the height of the fitted cone and \( r \) the radius of the cone at \( h = 0 \). The constant \( b_1 \) is added to this equation (and equation (4) as well) to counter out that all the observed values of \( z \) are negative.

4. Scaled and shifted binormal function

\[ z = b_0 \frac{1}{2 \pi \sigma_x \sigma_y \sqrt{(1 - \rho)^2}} \exp \left( -\frac{1}{2(1 - \rho^2)} \left( \frac{(x - \mu_x)^2}{\sigma_x^2} + \frac{(y - \mu_y)^2}{\sigma_y^2} - 2 \rho \frac{(x - \mu_x)(y - \mu_y)}{\sigma_x \sigma_y} \right) \right) - b_1 \]
Where \( b_0 \) is the scaling multiplier, \( b_1 \) a constant that counters out that all observed \( z \) are negative, \( \sigma \) the standard deviation, \( \mu \) the mean and \( \rho \) the correlation between \( x \) and \( y \).

Note that none of the aforementioned constants in models 1—4 are taken straight from the bottom data. They are the parameters of the best-fitting binormal distribution, which is determined using nonlinear least squares method in MATLAB with the ‘fittype’ curve- and surface fitting function. This method was applied to all equations (1)—(4). The goodness of the fit was evaluated using the coefficient of determination \( R^2 \), which has a best possible value of 1, see Fig. 11 for an example of a fit with high \( R^2 \).

![Binormal, \( R^2 = 0.919, MSE = 0.0225, b_0 = 8.8, \sigma_x = 0.861, \sigma_y = 0.968, \rho = 0.142 \)](image1)

**Fig. 11.** Example of a rock with binormal fit with \( R^2 = 0.92 \).

The fitting process yielded the following histograms presented in Fig. 12 for \( R^2 \) for the four models that were tested.

![Heinvee et al.](image2)

**Fig. 12.** \( R^2 \) for different models using the Hamina data.
Counting the share of all rocks where $R^2$ exceeded a certain threshold (0.7, 0.8 or 0.9) the following results are obtained, which are summarized in Table 1.

### Table 1
Rock model performance for Hamina.

<table>
<thead>
<tr>
<th></th>
<th>Heinvee et al.</th>
<th>2nd order polynomial</th>
<th>Cone</th>
<th>Binormal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2 \geq 0.7$</td>
<td>0.25</td>
<td>0.36</td>
<td>0.29</td>
<td>0.76</td>
</tr>
<tr>
<td>$R^2 \geq 0.8$</td>
<td>0.10</td>
<td>0.17</td>
<td>0.10</td>
<td>0.53</td>
</tr>
<tr>
<td>$R^2 \geq 0.9$</td>
<td>0.02</td>
<td>0.06</td>
<td>0.01</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Generally the binormal function is superior compared to the other three models in terms of $R^2$: 76% of all modeled rock formations had $R^2 \geq 0.7$, 53% $R^2 \geq 0.8$ and in 19% of cases $R^2 \geq 0.9$. The other three models give smaller shares for the thresholds. In 95.14% of the cases the binormal function had the best $R^2$ of all 4 models when comparing each case separately side-by-side.

Fig. 13. Descriptive statistics of Hamina individual peaks.
In Fig. 13 the upper leftmost corner shows the radius plotted against $R^2$ of the binormal function. Rocks with radii greater than 10 m consistently have an $R^2$ of at least 0.8 while smaller rocks have $R^2$ of everything from less than 0.4 up to 1. There seems to be no clear connection between $R^2$ and the resolution (upper right corner in Fig. 13), individual rock depth change (right middle in Fig. 13) nor the apex minimum depth (lower right corner in Fig. 13). See Appendix 1 for histograms of the parameters of the binormal functions for all analyzed rocks. For Porvoo the results are illustrated in Fig. 14 and Table 2.

For Porvoo the binormal model gives superior results in terms of the share of all cases where $R^2$ exceed a certain threshold: 84% of cases have $R^2 \geq 0.7$, in 65% of cases $R^2 \geq 0.8$ and in 20% $R^2 \geq 0.9$. All other models have a smaller share of cases where these thresholds were exceeded.

The binormal model was the best in terms of $R^2$ in 71% of all cases when comparing the four models to one another case-by-case. The threshold for getting consistently $R^2$ of at least 0.8 for is $r > 17$ m, see Fig. 15.

**Table 2**

Rock model performance for Porvoo.

<table>
<thead>
<tr>
<th></th>
<th>Heinvee et al.</th>
<th>2nd order polynomial</th>
<th>Cone</th>
<th>Binormal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2 \geq 0.7$</td>
<td>0.53</td>
<td>0.76</td>
<td>0.36</td>
<td>0.84</td>
</tr>
<tr>
<td>$R^2 \geq 0.8$</td>
<td>0.27</td>
<td>0.56</td>
<td>0.13</td>
<td>0.65</td>
</tr>
<tr>
<td>$R^2 \geq 0.9$</td>
<td>0.05</td>
<td>0.16</td>
<td>0.02</td>
<td>0.20</td>
</tr>
</tbody>
</table>

![Fig. 14. $R^2$ for different models using Porvoo data.](image-url)
In Fig. 15 the resolution vs $R^2$ shows a slightly increasing trend: Peak datasets with resolution over 8 observations per m$^2$ have consistently a $R^2 \geq 0.8$ while the lower resolution ones have $R^2$ values ranging from $<0.6$ to $\sim 1$.

4.2. Grounding depth based on $z_{\text{max}}$

In order to test whether the grounding depth follows a triangular or a uniform distribution [13,30], the following null- and alternative hypothesis were formulated and tested using the Kolmogorov–Smirnov goodness-of-fit test:

$H_0 = z_{\text{max}}$ follows a specified distribution (triangular or uniform)

$H_1 = z_{\text{max}}$ does not follow the specified distribution

The assumptions for the triangular distribution are as follows: Lower bound and most likely value is the smallest value of $z_{\text{max}}$, the upper bound and least likely value is the largest observed value of $z_{\text{max}}$. This means that it more likely to ground on deeper rocks than on rocks with peaks in shallow waters, see Figs. 8 and 9 for histograms of $z_{\text{max}}$. The following results were obtained when constructing cumulative frequency distributions for the theoretical triangular distribution and the data:
Looking at Fig. 16, the biggest discrepancies between the observed Hamina data (+) and the theoretical triangular distribution (solid line) is at around $-10.5$ m while the uniform distribution shows the largest difference in the $-11$ to $-8$ m range. Porvoo shows the largest difference between the data and the distributions at around $-15$ m to $-14$ m. For Hamina, the p-value of the test is $0.0048$ for the triangular and $3.5 \cdot 10^{-10}$ for the uniform distribution, meaning that the null hypothesis that grounding depth follows a triangular distribution is rejected at a significance level of 0.05.

For Porvoo, the p-value of the test is $0.293$ for the triangular and $0.420$ for the uniform distribution, meaning that the null hypothesis that grounding depth follows any of these two distributions is not rejected at a significance level of 0.05. Combining the data from Porvoo and Hamina the following results are obtained:

Looking at Fig. 17 the largest discrepancy between the theoretical triangular distribution and the observed values is at around $-12$ m mark while the data overlaps quite well with the triangular distribution from $-10$ to $-2$ m. The uniform distribution deviates most from the data in the $-10$ m to $-8$ m depth range. The p-value of the test is $3.19 \cdot 10^{-8}$ for the triangular and $1.02 \cdot 10^{-6}$ for the uniform distribution meaning that the null hypothesis that the combined dataset grounding depth follows any of these two distributions is rejected at a significance level of 0.05.
5. Discussion

The four-step approach to mathematically model bottom shapes was presented and applied to two case studies from Finnish waters. Sea bottom varies in shape and size, range from sharp rocks a few meters wide to shallow round shapes with a radius of several tens of meters; thus in the data examples that size-wise roughly correspond to the conical rock models, the different rock sizes A-D of [9] as well as the three bottom shape cases presented in Ref. [1] were found. In addition, asymmetrical shapes such as ridges and groups of smaller formations right next to one another were observed. The data shows a difference in the size of bottom shapes between Porvoo and Hamina: the bottom shapes in Porvoo tended to be much wider (bigger $r$) and not as sharp (smaller $\Delta z/r$) as the ones in Hamina. As such there is a need to take local conditions into account when performing grounding damage analysis. This shows the need for more rock models to supplement the commonly assumed single cone shapes and symmetrical polynomial equations. An example of a model that fulfills the aforementioned criteria is the binormal function. Note that some of the cones presented in literature (such as [21] are qualitatively assessed very similar to a binormal function; However these cones are symmetrical in the $xy$-plane whereas the binormal function is more flexible in terms of modeling the actual bottom topology. The data shows that this flexibility is needed. A pure, unblunted cone does not fit to the data well in terms of $R^2$ in most cases: it achieved a fit of 0.9 or better in only 1–2% of cases and fit of at least 0.8 in only 10–13% of cases. The results polynomial model fits were mixed: Relatively good fit for Porvoo (over half $R^2 \geq 0.8$) while for Hamina $R^2 \geq 0.8$ only in 17% of cases. The [9] model was on average slightly worse than the 2nd order polynomial model with more terms. The binormal function gives better results and in comparison has the best fit in the vast majority of cases. For larger rocks (10 m for Hamina, 17 m for Porvoo) fits of $R^2 \geq 0.8$ are consistently obtained, whereas the small rocks show $R^2$ ranging from 0.3 to 1. The binormal function a flexible and a good alternative to the conical model and/or polynomial models. The binormal function can generally be used to model the cases where other models have a good fit, see Figs. 18 and 19.

![Fig. 18. Formation 16 with fitted cone (left) and binormal (right) models.](image)

![Fig. 19. Formation D (a) with fitted 2nd order polynomial (left) and binormal (right) models.](image)
As Fig. 18 shows, the cone model gives $R^2 = 0.864$ and the binormal model $R^2 = 0.913$ respectively. Similarly Fig. 19 shows that the 2nd order polynomial model gives $R^2 = 0.933$ and the binormal model $R^2 = 0.938$. Thus, binormal model outperforms slightly the cone and polynomial models. The tested cone type here is a pure, unblunted cone. The $R^2$ could most likely be improved by blunting or otherwise modifying the tip part of the cone. However the fit results are not encouraging; if cones in general does correspond with the data, one would expect higher $R^2$ values than the current results. Any asymmetry in the bottom shape will still result in a bad fit for the cone model and would require the model to be changed to elliptical instead. Blunting the cone heavily such as in Ref. [21] brings the cone very close - if not identical - to a binormal function. The authors recommend using the shifted and scaled binormal function in modeling bottom shapes that can be assumed to be similar to the ones presented in this paper. See Appendix 2 for examples of a small, sharp rock and a wide, blunt rock and their parameters.

Note that the coast of Finland has been strongly shaped by the latest ice age and as such the analysis presented as steps 1–4 should be repeated in other sea areas for more general recommendations.

The incapability of the binormal model to model all of the smaller rocks leads us to suggest investigating the fit of other more complex or non-parametrical models such as the approach presented in Ref. [31]. The selected data here does not cover stranding, which is also left as a suggestion for the future research.

The $R^2$ does not tell the whole truth: A good fit in statistical terms does not warrant a good representation of reality. For example Hamina Formation 2 m has $R^2 = 1$ but the height of the fitted binormal function is several hundred meters. The opposite is also true: what looks qualitatively to be a good fit might be ruined by “outliers” at the base of the rock, yielding a low $R^2$ score. As $R^2$ does not straightforward tell the exact goodness of the fit in applied risk analysis. The real question is whether using these fitted models would result in the same damage as a raw data rock when analyzing ship grounding damage with models such as the one by Ref. [9]. In other words, the authors highly recommend an investigation on the resulting damage between realistic and idealized ground shapes. The numerical methods to estimate the grounding damage are getting mature as the understanding of the failure process of material has increased and computational power increases as well. Thus, the data from this paper should be applied to numerical simulations to show the effects of mathematical idealizations on ground shape on the simulated consequences. This would increase the reliability of the simulations. However, this is left for future work.

Regarding the grounding depth, the Kolmogorov–Smirnoff test results support the triangular assumption for Porvoo data: This means that according to the hypothesis tests any of these two distributions can be used to describe grounding depth in Porvoo. For Hamina and the combined dataset the conclusion is opposite as both distributions were rejected in the hypothesis testing. Overall analyzing more data is suggested as future research.

6. Conclusions

The paper presented a methodology to measure the actual bottom topology of specific ship routes. The methodology was used at two locations in Baltic Sea on the coastal line of Finland. The measured shape was compared with simplified mathematical shapes from literature, i.e. cone and parabolic. The curve fitting show that these simple forms do not fit the measured data very accurately in most cases. This should cause some uncertainty to the input to the Finite Element simulations of the actual grounding damage. Alternative model is presented based on a scaled and shifted binormal function and it is shown that it outperforms the other available models in terms of statistical goodness-of-fit. Regarding the grounding depth, the Kolmogorov–Smirnoff test results support the triangular assumption for Porvoo data but not for Hamina data, leaving the question ambiguous. Analyzing more data is suggested as future research.

This data should be used in conjunction with the Finite Element simulations to clarify how much the absorbed energy is under- or overestimated using these simplified models. This is left for future work. Furthermore, the authors propose the following for future research including: Investigating for different ships how probable it is to hit each the different bottom shapes. The potential effects of grounding angle should be investigated as the data shows asymmetrical bottom shapes. This
asymmetry might be bound to compass heading due to local geological factors; e.g. in Finland glaciers retreated from South-West towards North-East during the last ice age, causing directed erosion. Groundings and strandings on slopes leading up to fast land and non-solid bottom shapes could also be studied. Furthermore the authors propose investigating what spatial correlation and patterns exists between individual peaks in a larger formation, see e.g. Ref. [28]. This is also left for future work.

Acknowledgments

This paper has received funding from Kotka Maritime Research Center Merikotka. The authors would like to thank Finnish Transport Agency for the data and the friendly spirit of co-operation which made this study possible. For programming and analyzing tips the authors would like to thank Johannes Haataja, Jaripekka Juhala and Daniel Laajala. For feedback on writing and how to frame the study the authors would like to thank Otto Puolakka, Heikki Remes, Teemu Manderbacka and Markus Ahola.

Appendix 1. Binormal function parameters

The parameter values for the binormal function

\[ z = b_0 \frac{1}{2\pi \sigma_x \sigma_y \sqrt{1-\rho^2}} \exp \left( -\frac{1}{2(1-\rho^2)} \left( \frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{(y-\mu_y)^2}{\sigma_y^2} - \frac{2\rho (x-\mu_x)(y-\mu_y)}{\sigma_x \sigma_y} \right) \right) - b_1 \]

for all analyzed Hamina cases are illustrated in Figs. 20–22.

The parameter values for the binormal function for all analyzed Porvoo cases are illustrated in Figs. 23–25.

![Fig. 20. Standard deviations (σ) for x and y (excluding values over 20, 8 cases) for Hamina.](image)
Fig. 21. Histogram of correlation ($\rho$) for Hamina.

Fig. 22. $b_0$ vs $b_1$ for Hamina, excluding 4 cases where $b_0 > 5 \times 10^4$. 
Fig. 23. Standard deviations ($\sigma$) for $x$ and $y$ for Porvoo.

Fig. 24. Histogram of correlation ($\rho$) for Porvoo.
Appendix 2. Example rocks and their parameters

The following rock models can be used an examples in grounding damage calculations. They have a decent $R^2$ combined with a good visual agreement between the data and the fitted model as well as being tall enough to be used to calculate inner hull damage on even the thickest 1.8 m double bottoms. Example 1: Large blunt rock.

Fig. 25. $b_0$ vs $b_1$, for Porvoo.

Fig. 26. Formation $S$ (a).
Example of a large, rounded rock, where data within 13 m of the xy-origo was selected for analysis. \( R^2 \) is 0.877 and the coefficients for the binormal function

\[
z = b_0 \frac{1}{2 \pi \sigma_x \sigma_y \sqrt{1 - \rho^2}} \exp \left( -\frac{1}{2(1 - \rho^2)} \left[ \frac{(X - \mu_x)^2}{\sigma_x^2} + \frac{(Y - \mu_y)^2}{\sigma_y^2} - 2\rho \frac{(X - \mu_x)(Y - \mu_y)}{\sigma_x \sigma_y} \right] \right) - b_1
\]

are

\[
b_0 = 1524, \quad b_1 = 12.96, \quad \mu_x = -2.269, \quad \mu_y = -0.5717, \quad \rho = -0.3006, \quad \sigma_x = 7.604, \quad \sigma_y = 8.396.
\]

Example 2: Small sharp rock.

\[
\text{Binormal, } R^2 = 0.946, \text{ MSE} = 0.0354, b_0 = 28.2, \sigma_x = 1.36, \sigma_y = 1.53, \rho = 0.18
\]

This is an example of a small, quite sharp rock with a good \( R^2 \) of 0.946. The coefficients for the binormal function are

\[
b_0 = 28.21, \quad b_1 = 13.97, \quad \mu_x = 0.7694, \quad \mu_y = 0.315, \quad \rho = 0.1799, \quad \sigma_x = 1.361, \quad \sigma_y = 1.525.
\]

References


