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Towards probabilistic models for the prediction of a ship performance in dynamic ice

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A B S T R A C T

For safe and efficient exploitation of ice-covered waters, knowledge about ship performance in ice is crucial. The literature describes numerical and semi-empirical models that characterize ship speed in ice. These however often fail to account for the joint effect of the ice conditions on ship’s speed. Moreover, they omit the effect of ice compression. The latter, when combined with the presence of ridges, can significantly limit the capabilities of an ice-strengthened ship, and potentially bring her to a halt, even if the actual ice conditions are within the design range for the given ship.

This paper introduces two probabilistic, data-driven models that predict a ship’s speed and the situations where a ship is likely to get stuck in ice based on the joint effect of ice features such as the thickness and concentration of level ice, ice ridges, rafted ice, moreover ice compression is considered.

To develop the models, two full-scale datasets were utilized. First, the dataset about the performance of a selected ship in ice is acquired from the automatic identification system. Second, the dataset containing numerical description of the ice field is obtained from a numerical ice model HELMI, developed in the Finnish Meteorological Institute.

The collected datasets describe a single and unassisted trip of an ice-strengthened bulk carrier between two Finnish ports in the presence of challenging ice conditions, which varied in time and space. The relations between ship performance and the ice conditions were established using Bayesian networks and selected learning algorithms.

The obtained results show good prediction power of the models. This means, on average 80% for predicting the ship’s speed within specified bins, and above 90% for predicting cases where a ship may get stuck in ice.

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1. Introduction

Ship performance in ice has been a lot of attention in the recent years, especially among northern maritime countries i.e. Canada, Finland, Norway, Russia and Sweden. However due to global warming resulting in the opening of the northern sea route in the Arctic, the issue becomes of global interest.

This increased attention has led to the development of semi-empirical methods that estimate ship resistance and ship speed in ice, see for example (Kotovirta et al., 2009; LaPrairie et al., 1995; Lapp et al., 1997; Lindqvist, 1989; Lubbad and Leset, 2011; Mulherin et al., 1996; Naegle, 1980; Riska et al., 1997; Su et al., 2010). However the ice conditions, which are considered in these models, are often limited to level ice and ice channel. In some cases, the effect of ice ridges is taken into account (Riska et al., 1997), (Juva and Riska, 2002), (Keinonen, 1996).

Further studies are required for example concerning the quantification of the joint effect of the relevant ice features on ship speed including the effect of ice compression (Kubat, 2012; Kubat et al., 2013; Kubat et al., 2015; Kulaots et al., 2013; Tomac et al., 2013). Moreover, suggestions have been made to move from the deterministic, quantity-oriented models towards probabilistic and event-oriented models (Kotovirta et al., 2009). An event-oriented model reflects the ice features under which an event of interest occurs e.g. ship proceeding with certain speed or a ship getting stuck in ice, see for example E. T. S. Inc et al. (1996). This type of modeling, unlike the commonly adopted quantity-oriented approach, quantifies the joint effect of various ice features on ship’s speed. However, it does not provide an insight into the physics of the process of ice breaking. Moreover, if appropriate probabilistic modeling techniques are applied, they lead to a model which is...
computationally fast, easy to validate and can be updated if new knowledge about the conditions/inputs is gained. Such models can be used for prediction of the vessel performance in an operational setting, or in e-navigation services, including route optimization.

This paper introduces two probabilistic, event-oriented models for predicting performance of a ship navigating in ice. In this paper we measure ship performance using two indicators. The first is the probability for a ship to attain certain speed (model A). The second is the probability for a ship being best in ice (model B). The following ice features are considered: thickness and concentration of various types of ice (level ice, ridged ice, and rafted ice), ice compression and its relative direction with respect to a ship.

To develop the models linking the ice field features with certain events which reflect ship performance in ice, machine-learning algorithms were applied to a predefined and carefully selected learning dataset. The latter contained full-scale data about performance of a certain ship in specific ice field along a specific route. This means a single and unassisted trip of an ice-strengthened bulk carrier between two Finnish ports in the presence of challenging ice conditions, which varied in time and space.

The learning dataset was obtained, by combining in a tempo-spatial fashion a set containing data about ice field, obtained from a state-of-the-art numerical ice model, called HELMI (Haapala et al., 2005), with a set describing ship performance in this ice field—obtained from the automatic identification system (AIS).

As a result, two probabilistic models were developed, a.k.a. Bayesian belief networks (BBNs). A major advantage of BBNs over many other types of predictive models, such as neural networks, is that the BBN's structure represents the inter-relationships among the dataset attributes in a probabilistic fashion. Moreover, if experts are involved in the process of model development they can easily understand the model structures and if necessary modify them to increase the predictive power of the models.

The remainder of the paper is organized as follows: Section 2 presents full-scale datasets used for the development of the models. Section 3 introduces the adopted modeling techniques. The developed models and the obtained results are shown in Section 4, and discussed in the following section. Section 6 concludes the paper and summarizes the main findings.

2. Data

The approach taken towards development of the probabilistic models presented in this paper, capable of forecasting ship performance in ice, utilizes techniques of Bayesian learning from data. These models first determine the relations between all the analyzed explanatory and response variables, and second they quantify the joint effects of ice features on ship's speed, allowing probabilistic analysis of ship performance in ice.

To develop the models, two data sources were used. First the reanalyzed ice forecast was taken, called hindcast, for the sea area under consideration. This dataset provides information about ice features for the interval of 1 hour and spatial resolution of 1 NM by 1 NM. Second, the database containing the state vectors of an analyzed ship was constructed. This included ship course and speed obtained from AIS, recorded with an interval varying between 2 sec and 3 min, depending on the speed of the vessel and operational status, see US Department of Homeland Security (2013). Then, these two data sources were matched in tempo-spatial fashion spacing equally the entries time-wise, and a database was created that reflected ship performance in dynamic ice over the analyzed time span. However such database required further analysis before it is used for modeling purpose, as the entries that may deteriorate predictive power of the models shall be removed. Such entries are: the time instances where a ship remains beset in ice for longer period or situations where changes in ship's speed are due to operational settings (ship navigating an ice channel, assisted by ice breaker or slowing down to board a pilot) not environmental conditions. In the first case, all the entries between the two time instances where a ship became beset in ice and she was released were removed, to keep only entries where the effect of ice features on ship's speed is observable. In the second case, videos were made, and experts were consulted to understand properly the analyzed voyage, identify and remove the undesired entries.

This resulted in the development of database, which contained 4040 rows, which are not equally distributed in time, but they reflect as far as possible the causality that exists between the environmental conditions and the performance of a ship. Therefore each row in the database comprises of 12 columns representing the following parameters—including two response variables:

1. ship speed [kn]—response variable,
2. ship beset in ice [yes/no]—response variable,
3. level ice concentration [%],
4. level ice thickness [cm],
5. ridged ice concentration [%],
6. ridged ice thickness [cm],
7. rafted ice concentration [%],
8. rafted ice thickness [cm],
9. relative direction of ice compression [deg],
10. ice compression magnitude [0–4],
11. relative direction of wind [deg],
12. wind speed [kn].

Finally, two learning algorithms were applied to the database to determine the models' structure and estimate the parameters. The obtained probabilistic models and their results are valid for a particular ship type, which is an ice going bulk carrier with ice class of IA Super—see Table 1—which is navigating under a certain set of hydro-meteorological conditions.

2.1. Date and area of interest

The date and area of interest have been selected specifically to capture challenging ice conditions, meaning high concentration of level ice, the presence of ridged ice and ice compression that changes in time. For this reason, the day of 6th of March 2011 was chosen, and the sea area between two Finnish harbors, in the Bay of Bothnia (the Baltic Sea), namely Vaasa and Kokkola were selected. The case study presented here is based on records of a single trip of the bulk carrier between two positions of boarding a pilot, meaning that the stage of the high-seas navigation is considered, where the ship is supposed to proceed with full engine power. In Fig. 1 the trajectory of the ship is overlaid on the ice chart, therein the ship track is marked with the blue circles, whereas the locations where the ship was brought to a halt are marked with yellow crosses.

The ship covered a distance of 94 NM in 14 hours, and the ice conditions hampered her significantly, making her ram the ice several times, and forced her to idle in ice for three hours. The recorded data contain significant variability and strong effect of environmental conditions on ship's speed, as depicted in Fig. 2, thus the created dataset can be considered appropriate for the model development.

| Table 1 |
| Ship particulars. |
| Type | Bulk carrier |
| Ice class | IAS |
| DWT | 21353 t |
| Length | 149.3 m |
| Breadth | 24.6 m |
| Draught | 9.4 m |
| Power | 9720 kW |
| Year of construction | 2006 |
2.2. Ship data

The chosen ship is a bulk carrier having the DNV ice class of +1A1 Ice 1A super, which is equivalent to IA super according to Finnish–Swedish Ice Class rules, (TRAFI, 2010). This means that she has such structure, engine output and other properties, which make her capable of navigating in difficult ice conditions without the assistance of icebreakers. The design requirement for this ice class is a minimum speed of 5 knots in 1 m thick brash ice channels with a 0.1 m thick consolidated layer of ice on top, see TRAFI (2010). The ship is equipped with an “ice knife” at the bow, which additionally eases the process of ice breaking. The ship particulars are presented in Table 1.

However, one important parameter, which describes ship’s inertia and her ability to break the ice, meaning mass of the ship (displacement) cannot be determined accurately. Based on our knowledge about the route of the ship and harbors visited we made an assumption about half-load conditions during the analyzed journey.

In order to quantify the joint effect of ice conditions on ship’s speed, the following parameters of ship motion were retrieved from the AIS records: time, ship position, speed over ground, course over ground, and true heading. Then for each time step and position, the relevant ice characteristics were obtained from the ice model. Once the ship parameters had been aligned with the ice model, one additional parameter...
was calculated, called “relative direction of compression”. This is a dynamic parameter, which is an angle between ship’s centerline and the resultant direction of the ice compression. This parameter is expressed on a scale from 0 deg (ice pressing from the bow) to 180 deg (ice pressing from the stern), the value of 90 deg means that the ice compression acts perpendicularly to the ship.

2.3. Ice data

The ice data were obtained from the hindcasts performed with the HELMI multicategory sea-ice model, see Haapala et al. (2005). The model resolves ice velocity, internal ice stress, ice concentration and ice thickness. Thickness is resolved for seven categories: five level ice categories, rafted ice and ridged ice. The ice model is discretized in a curvilinear coordinate c-grid, a common solution when there are both fields of velocities and velocity-dependent properties to be solved. The grid has 415 nodes from west to east and 556 nodes from south to north. The SW lower corner coordinates are 56.74° N 16.72° E, NE corner coordinates 65.99° N 30.48° E, and the increment is 1/30 degrees eastwards and 1/60 degrees northwards. This is approximately 1 NM in both directions at 60°N.

The ice forecasts take thermodynamic and dynamic forcing from weather prediction model HIRLAM. The forecast is made every 6 hours or after each HIRLAM run. The length of the forecast is 54 hours and interval of 3 hours. Sea surface temperature (SST), including ice edge information, is prescribed and updated once a day. This is obtained from digital ice and SST charts that are based on daily SAR images, satellite SST data and observations from ships. Ice forecasts have been validated against the observed ice situations, and good agreement was found, see for example Lehtiranta et al. (2012). On the other hand, hindcasts use HIRLAM reanalyzes and are stored at 1-hour intervals. Their ice edge is not reinitialized by observations but rely solely on the model physics throughout the ice season. The present set-up of the ice prediction system does not include any dynamical ocean component, thus ocean currents are neglected. Although ocean currents in the Baltic are negligible for ice drift magnitude, they may have effect on the compression magnitude, especially on compression relief when water level gradient induces off-coast currents after a stormy period. This may be one reason for the discrepancies, observed in the validation exercises, between modeled compression and observations close to the fast ice edge.

Ice motion is determined by the momentum balance equation, which takes into account the Coriolis force, wind and water stresses, sea surface tilt term and internal friction of ice, which is the divergence of internal stress tensor. The magnitude of internal friction is used as the principal model variable to describe compression. It is to be noted that the viscous-plastic rheology does not describe elastic stresses and the internal stress arises from the interactions of moving ice. Forces arising in a static ice field are included by assuming a negligibly slow viscous creep. Roughly, the internal friction term can be interpreted to describe the forces arising when ice floes are pushed and sheared against each other, or broken and heaped into ridges. Thus it is a good descriptor for the interaction between dynamical ice cover and an ice-going ship. This is manifested as ice forces against the ship hull and as the closing of channels, or other phenomena that navigators associate to compressive ice conditions. The internal friction magnitude has typical values ranging from 0 to 10 N/m², as depicted in Fig. 2. The magnitude acts as a proxy for ice compression, scaled to semi-empirical compression numeral 0–4, where 0 means no compression and 4 stands for extreme severe compression, see Table 2. However, to estimate the actual local

![Time series of the analyzed parameters.](image)

### Table 2

<table>
<thead>
<tr>
<th>Internal friction magnitude obtained from HELMI model [N m⁻²]</th>
<th>Meaning</th>
<th>Practical scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–1.5</td>
<td>No significant compression</td>
<td>0</td>
</tr>
<tr>
<td>1.5–2.5</td>
<td>Mild pressure</td>
<td>1</td>
</tr>
<tr>
<td>2.5–5.5</td>
<td>Moderate pressure</td>
<td>2</td>
</tr>
<tr>
<td>5.5–9</td>
<td>Severe pressure</td>
<td>3</td>
</tr>
<tr>
<td>&gt;9</td>
<td>Extreme severe pressure</td>
<td>4</td>
</tr>
</tbody>
</table>
forces additional scaling arguments must be taken into account such as floe size and other ice cover geometry.

2.4. Data resolution

The resolution of data describing ship motion is much finer—10 sec for most of the time—than the data obtained from the ice forecast, which is 1 hour time interval and 1 NM × 1 NM in space. This difference in resolutions causes the variability in the ship’s speed even if the modeled ice conditions remain unchanged, see Fig. 2. For example, when a ship proceeds with a speed of 10 kn, it takes her 6 minutes to make 1 NM. Knowing that her movements are recorded every 10 seconds, it means that there are 36 records of her speed and course during 6 minutes. The ice model provides a discrete description of the ice field, by gridding the sea area into 1 NM by 1 NM cells, and updates each cell every hour. This means, that during ship transit over a cell, there are dozens of various records of ship movements, whereas there is a constant set of parameters describing the ice features existing in this particular cell. When a ship enters a new cell or new time interval, the constant set of new ice parameters is retrieved.

The effect of different resolutions is to a large degree removed at the stage of model development, as the variables are discretized and divided into classes, as required by Bayesian learning algorithms. Correspondingly, the speed of the ship has been binned into three classes as follows: below 5 kn, between 5 and 10 kn and above 10 kn. The variability of speed given constant ice conditions occurs mostly within the classes, see Fig. 3.

2.5. Description of the trip of the analyzed ship

The speed profile of the voyage of the analyzed ship is shown in Fig. 3, along with annotations regarding the navigational status and noteworthy events. The data range used for the model development is shown as well. The detailed information of the vessel’s movement is obtained through the Automatic Information System (AIS). This AIS data was linked to available vessel characteristics regarding vessel type, ice class and tonnage as obtained through the PortNet system and to the local ice conditions as obtained from the HELMI ice model. While the speed profile of Fig. 3 and the ship trajectory of Fig. 1 are insightful, the specific navigational conditions and traffic conditions are better understood with animations of the spatio-temporal data. Thus, videos were made to enable a detailed understanding and analysis of the voyage. The nautical officers of one of the Finnish icebreakers were consulted to assure the proper identification of all maneuvers performed by the analyzed ship and the assisting icebreakers as evident from the animation.

Videos 1 and 2 show the analyzed vessel. The left pane shows the analyzed vessel in the center, along with local vessels within a 2 NM radius around her. Around the left pane, situational parameters are shown, regarding time, location, speed, heading and course over ground as obtained from AIS, as well as the air and sea temperatures, wind speed and

Fig. 3. Annotated time history of the analyzed ship journey.
direction, ice drift speed and direction and ice compression magnitude and direction from the HELMI model. The upper right pane shows the complete trajectory of the analyzed vessel and the traffic situation in the entire area and the ship types are marked with different colors. The lower right pane contains the following: the mean ice thickness and a weighted average of the level, ridge and rafted ice thicknesses, with indication of the current vessel’s position. On the right, local ice parameters at the location of the analyzed vessel and parameters of the closest vessels to the analyzed ship are shown.

As illustrated in Fig. 1, the analyzed vessel left the waiting area in front of the harbor of Vaasa at 0545 UTC. At that time 2 icebreakers assisted her, as depicted in Fig. 3 and shown in Video 1. Fig. 4 shows the vessel under tow in the presence of mild to moderate compression, in line with the taxonomy of Table 2. After decoupling the tow, one of the icebreakers leaves the vessel, while the other icebreaker escorts the vessel to open water conditions under decreasing pressure and ice concentration. The towing operation occurs at a variable speed between 8 and 12 kn, while the escort takes place under variable speed between 12 and 14 kn. From 0730 until 1040, the vessel proceeds alone through open water conditions at a speed of 15.5 kn.

At 1040 the ship enters the ice field and her speed drops and falls in the range between 10 and 12 kn. At 1105 the ship encounters another ship that is stopped in ice, approaches her and cuts her loose; the speed of the analyzed vessel is still high at about 9 kn. At 1130 she passes two other immobile ships at a distance of 1.5 NM, her speed begins to decrease and at 1210 she is brought to a halt for the first time.

![Fig. 4. Snapshot of the situation at which the analyzed ship is towed.](image)

![Fig. 5. Snapshot of the situation at which the analyzed ship besets in ice.](image)
She made an attempt of backing and ramming in order to release but did not succeed. At 1236 an outbound ice breaker towing a ship approaches her on reciprocal course and cuts her loose, so she can resume her voyage and she follows the ice channel made by these two ships. Her speed falls in the range of 8–10 kn for about half an hour, until 1300, when it decreases gradually and at 1422 she is stopped for the second time, some 15 NM from her destination, Kokkola harbour, see Fig. 5 and Video 2. From the analyzed videos, we learn that the other ships in the area are also barely able to proceed due to the severe ice pressure, and that the prevailing ice conditions are rather demanding. After a few attempts of backing and ramming and another nearby vessel are cut loose by an outbound ice breaker at 1440, and they continue approach, passing several stopped ships along their way. At 1540 both ships are brought to a halt again, the analyzed vessel makes one or two attempts to release by herself, but she gives up, and both ships need to wait. They are cut loose at 1800 by an inbound icebreaker, which overtake them and proceeds to the harbour. Since that point the analyzed ship follows the ice channel made by the icebreaker, with a speed of 10 kn. At 1845 she slows down to 2 kn at the pilot boarding position to pick up a pilot, then she speeds up and continues to the harbour, which is finally reached at 1930 UTC.

3. Modeling techniques

The models presented in this paper have been developed with the use of Bayesian belief networks (BBNs). For this purpose we combined Bayesian algorithms for learning from the data with expert knowledge. A major advantage of BBNs over many other types of predictive models, such as neural networks, is that the Bayesian network structure represents the inter-relationships among the dataset attributes in a probabilistic fashion (Goerlandt and Montewka, 2014; Hänninen, 2014; Hänninen et al., 2014; Lehtikoinen et al., 2013; Montewka et al., 2014; Straub and Grêt-Regamey, 2006). Moreover, if experts are involved in the process of network development they can easily understand the model structures and if necessary modify them to obtain better predictive models.

Bayesian belief networks are representational devices that are meant to organize one’s knowledge about a particular situation (Darwiche, 2009). They are probabilistic graphical models that represent a set of random variables and their conditional dependencies via a directed acyclic graph (DAG). For example, a Bayesian network could represent the probabilistic relationships between speed of a ship and surrounding ice features. Given ice features, the network can be used to compute the probabilities for a ship to attain certain speed or the probability for a ship being beset in ice.

3.1. Bayesian belief networks

From a mathematical viewpoint, classical BBNs are a pair \( N = (G, P) \), where \( G = (V,E) \) is a DAG with its set of variables \( V = \{X_1,...,X_j\} \), and set of links between them called edges \( E \). \( P \) is a set of probability distributions of \( V \). Therefore, BBNs representing a set of variables and their dependencies consist of two parts: a quantitative \( \langle P \rangle \) and a qualitative \( \langle G \rangle \). Thus, a network \( N = (G, P) \) is an efficient representation of a joint probability distribution \( P(V) \) over \( V \), given the structure of \( G \), following the formulae; see also (Darwiche, 2009).

\[
P(V) = \prod_{X_i \in V} P(X_i|\text{parents}(X_i))
\]  

(1)

In the above equation, \( \text{parents}(X) \) means all the variables upon which \( X \) is directly conditioned. The direction of links between variables signifies the parent–child relation, with an arrowhead pointing towards a child. BBNs encode the joint probability distribution governing a set of variables by determining a set of conditional probability tables (CPTs). Each variable is annotated with a CPT, which represents the probability of the variable given the values of its parents in the graph. A CPT describes all the conditional probabilities for all the possible combinations of the states of the parent variables. If a variable does not have parents, its CPT reduces to an unconditional probability, also referred to as a prior probability of that variable.

The following Bayesian rules can be defined, which govern the flow of information through a network’s structure, see for example (Nielsen and Jensen, 2007):

- conditional dependence rule:

\[
P(X_i = x_i|X_j = x_j) = \frac{P(X_i = x_i)P(X_j = x_j)}{P(X_j = x_j)}
\]  

(2)

- chain rule:

\[
P(X_1, X_2, ..., X_n) = \prod_{i=1}^nP(X_i|\text{parents}(X_i))
\]  

(3)

- joint probability rule:

\[
P(X_1 = x_1, X_j = x_j) = P(X_1 = x_1)P(X_j = y_j|X_i = x_i)
\]  

(4)

3.2. Reasoning with BBNs

After the model of a domain in question is developed, we can reason about the domain. The process of reasoning, also called probability propagation or belief updating, is performed via a “flow of information” through the network, and the flow is not limited to the directions of the arcs. In our probabilistic system, this becomes the task of computing the posterior probability distribution for a set of query variables, given values for some evidence variables.

BBNs provide full representations of probability distributions over their variables. That implies that BBNs can be conditioned upon any subset of their variables, supporting any direction of reasoning. Therefore, one can perform forward (predictive) reasoning, from new information about causes (explanatory variables) to updated beliefs about the effects (response variables), following the directions of the network edges. Alternatively backward (diagnostic) reasoning is possible, from the effects of interest to the most probable causes, where the information in the model is propagated against the direction of the edges (Korb et al., 2010).

3.3. Constructing BBNs

There are four main methods of constructing BBNs to model a particular situation:

1. by eliciting the experts’ knowledge and organizing it into BBNs;
2. by synthesizing knowledge from other sources into BBNs;
3. by adopting data learning algorithm;
4. mixture of the above.

In this paper we have adopted the fourth technique to develop the models, utilizing available data (model and full scale) enhanced by experts’ knowledge.

Development of BBNs with the use of data learning algorithms, usually, involves two stages. First is to discover the graphical structure \( \langle G \rangle \) and second is to estimate parameters for the structure \( \langle P \rangle \), expressed in a form of CPTs.

In this paper two models are presented. The structure of model A was discovered from the data adopting learning algorithm called PC and with the active involvement of experts. The latter provided their knowledge about the analyzed phenomena and informed about anticipated relations between variables (i.e. the model structure). By incorporating experts’ knowledge into a model structure, we make sure that all
the relevant relations are present in the model. This knowledge might be about the existence or nonexistence of certain edges in the graph, or about the orientation of some of the edges, or about the time order of the variables.

In case of model B, we adopted Naïve Bayes algorithm, which predefines the structure and focuses on parameters estimation. The parameters of both models presented in this paper were estimated based on the dataset described in Section 2.

Detailed description of the adopted algorithms is provided in the Appendix I.

4. Models and results

4.1. Models development process

The process of models development adopted in the presented study is depicted in Fig. 6. It starts with defining the scope of the analysis, which is followed by relevant data acquisition and organization, as described in Section 2. There are twelve variables considered in the models, see Table 3, all variables are discretized into states with the use of hierarchical method imposing background knowledge about the process of ship navigation through the ice field.

Once the variables are discretized and the selected learning algorithms are applied, the obtained models are cross-validated. This provides an estimate of the predictive power of a model with respect to a selected hypothesis. The criteria for passing cross-validation (CV) can be arbitrary set up by a model developer. In our case, several parallel models were developed, and the one that obtained the highest posterior probabilities for a class variable has been selected.

Subsequently, the model, which passes CV, undergoes the behavior analysis tests, (Pitchforth and Mengersen, 2013). The behavior analysis tests address the following question: does the model predict the behavior of the system being modeled? An analyst performs the scenario walk-throughs and checks if the model prediction is consistent with the background knowledge and the understanding about the phenomena, which is being analyzed.

The final stages of model development are devoted to performing the value-of-information analysis. It informs how the model uncertainty is distributed among model parameters. It also identifies the variables, which are the most informative with respect to the model output.

4.2. Probabilistic models for ship performance in ice

The analyses performed in this study have failed to deliver a single unified model, which simultaneously predicts the speed of the ship and conditions under which a ship gets stuck in ice. Therefore, two separate models have been developed, one predicting each event. They are presented in this section.

The models are obtained by applying two different types of Bayesian learning algorithms, thus, the first model predicting the ship’s speed (model A), considers the causality discovered in the data. The model has been developed by applying the PC algorithm to the dataset, moreover the background knowledge about the analyzed phenomena has been imposed to the model structure.

The model is depicted in Fig. 7, in which the edges represent the dependencies among variables, as found by the PC algorithm and imposed by an analyst. The probabilities behind the variables reflect tempo-spatial variability of the analyzed parameters, as found in the dataset. The probabilistic relations between the variables have been determined in the course of learning process, with the use of the EM algorithm.

Model A is rather complex; it encompasses 12 variables, which are connected with 29 arcs, thus producing a large number of conditional probabilities (4415). This makes it impossible to illustrate all parameters and their states in the paper. Consequently, the description and presentation of the model is limited to its qualitative part—model structure—only.

The second model predicts the probability of a ship being beset in ice, given ice field features (model B). It disregards the causality, as it utilizes Naïve Bayes algorithm. The algorithm assumes the structure and just searches for the parameters of a model, such that the posterior probability is the highest for an output (class) variable. The structure of

---

**Table 3** Variables included in the models and their states.

<table>
<thead>
<tr>
<th>No</th>
<th>Variable’s name</th>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ship’s speed [kn]</td>
<td>&lt;5</td>
<td>5–10</td>
<td>&gt;10</td>
</tr>
<tr>
<td>2</td>
<td>Ship stuck in ice</td>
<td>Yes</td>
<td>No</td>
<td>–</td>
</tr>
<tr>
<td>3</td>
<td>Level ice thickness [m]</td>
<td>0.3–0.4</td>
<td>0.4–0.5</td>
<td>–</td>
</tr>
<tr>
<td>4</td>
<td>Level ice concentration [%]</td>
<td>&lt;25</td>
<td>25–75</td>
<td>&gt;75</td>
</tr>
<tr>
<td>5</td>
<td>Ridged ice thickness [m]</td>
<td>2.5–3.0</td>
<td>3.0–3.5</td>
<td>–</td>
</tr>
<tr>
<td>6</td>
<td>Ridged ice concentration [%]</td>
<td>&lt;5</td>
<td>5–15</td>
<td>–</td>
</tr>
<tr>
<td>7</td>
<td>Rafted ice thickness [m]</td>
<td>0.3–0.4</td>
<td>0.4–0.5</td>
<td>–</td>
</tr>
<tr>
<td>8</td>
<td>Rafted ice concentration [%]</td>
<td>&lt;5</td>
<td>5–10</td>
<td>–</td>
</tr>
<tr>
<td>9</td>
<td>Wind speed [m/s]</td>
<td>3.5–5.0</td>
<td>5.0–6.5</td>
<td>–</td>
</tr>
<tr>
<td>10</td>
<td>Compression level</td>
<td>0–1</td>
<td>1–2</td>
<td>2–3</td>
</tr>
<tr>
<td>11</td>
<td>Relative direction of compression [deg]</td>
<td>0–45</td>
<td>45–135</td>
<td>135–180</td>
</tr>
<tr>
<td>12</td>
<td>Relative direction of wind [deg]</td>
<td>0–45</td>
<td>45–135</td>
<td>135–180</td>
</tr>
</tbody>
</table>
model B is simpler than the structure of model A; it contains only 54 conditional probabilities, thus both parts of the model, namely qualitative and quantitative, can be presented here, see Fig. 8 and Table A1 in the appendix.

4.3 Validation of the models

The models developed here are validated adopting K-fold cross-validation (CV), where part of the data is used to develop the model,
The K-fold algorithm works as follows:

1. randomly divide the data set into K subsets;
2. for each subset S:
   a. train on the data but not on the subset S;
   b. test model on the subset S;
3. return the average error over the K subsets.

The results of cross-validity analyses are presented in Tables 4 and 5, where relatively good prediction power for the model A is noted. For the predicted variable ship speed the probability of delivering the right answer by the model varies between 0.75 and 0.93, depending on the state of the variable, see Table 4.

In the case of the model B, its predictive power is even higher, 0.9 and 1.0, depending on the state of the output variable (ship stuck in ice), see Table 5.

### 4.4. Models behavior analysis

The results of behavior analysis for model B are depicted in Fig. 9. Therein the model is evaluated for three scenarios, and the results are presented, as follows:

1. The probabilities of a two-state response variable are provided (ship gets stuck in ice = yes, ship gets stuck in ice = no), given the set of explanatory variables.
2. The probabilities of explanatory variables are calculated, given the response variable is set to one of its states—ship gets stuck in ice = yes.
3. The probabilities of explanatory variables are calculated, given the response variable is set to another of its states—ship gets stuck in ice = no.

### 4.5. The value of information analysis

The value-of-information analysis identifies the most informative variables, with respect to the output variable. It determines the variables among which the probability mass of the output is scattered. This analysis can be seen as a tool for analyzing the potential usefulness of additional information, before the information source is consulted. For this purpose the concept of Shannon entropy—\( H(X) \)—is utilized. The entropy is defined therein as follows (Kjæraulf and Madsen, 2012):

\[
H(X) = - \sum_{i=1}^{n} p(x_i) \ln p(x_i)
\]

where \( X \) is a random variable with \( n \) states \( \{x_1, ..., x_n\} \), and \( p(x_i) \) is the probability of outcome \( x_i \). In the case of model A, the response variable has three states, and the associated probabilities for each state are depicted in Fig. 7. The probabilities for the two-state response variable in model B are shown in Fig. 8. This allows calculating the entropy of output variable, as shown in Tables 6 and 7.

Entropy describes an amount of information, which is delivered by a model when it is run. The maximum entropy situation is when a realization of a certain process can take \( n \) states, and the prior probabilities for the occurrence of any of these states are all equal \( (1/n) \). This means that an outcome of a model is completely unpredictable prior to this execution, and it needs to be run to gain new information. Zero entropy means that the results of the model are fully predictable prior to its execution; therefore each run of the model does not deliver any new information.

However, in a model, where the response variable \( (X) \) is conditionally dependent on a number of explanatory (parental) variables \( (Y) \), a measure of the uncertainty of \( X \) given an observation of \( Y \) needs to be estimated. This is done by applying the conditional entropy \( H(X|Y) \), following the formulæ:

\[
H(X|Y) = \sum_{i,j} p(x_i, y_j) \ln \frac{p(y_j)}{p(x_i, y_j)}
\]

where \( p(x_i, y_j) \) is the probability that \( X = x_i \) and \( Y = y_j \). Conditional entropy is calculated for each pair of variables \((X|Y)\) that exists in a model.

The results of the value-of-information analysis with respect to the output of the models developed in this paper are gathered in Tables 6 and 7. Therein the actual value of entropy—\( H(X) \)—and the values of conditional entropies—\( H(X|Y) \)—are shown for all the variables included in the models. Additionally the maximum entropy—\( \max \ H(X) \)—that a given outcome variable can take is presented. This together with the actual entropy informs where on the entropy scale the analyzed model sits.

### 5. Discussion

The results, which are obtained in the course of validation analyses, allow the statement that the models, obtained in the course of presented study, show rather good agreement with the recorded data and general understanding of the analyzed processes of ship progress through the ice field.

Both models feature good prediction power, which however, depends on the discretization level of the variables used in the models to some degree. In the study presented in this paper, variables discretization is an iterative, heuristic process, where the variables are divided into classes so the discretization level reflects the essential features of the analyzed system. The discretization of the variable ship’s speed which is present in model A is performed based on analysis of the time series of the ship’s speed, and the division lines between classes are made to avoid “jumps” of the variable between classes if there are no significant changes in ice conditions. However any other way of data discretization may lead to different prediction powers with respect to a given class of an outcome variable, even if the model structure remains unchanged.

Another relevant assumption, which is made here, is that the ship is proceeding with her full power and the speed changes, which are recorded, are the results of encountered ice conditions and ship’s crew does not intentionally evoke them. Therefore, the high-seas navigation is considered in the models, and detailed investigation of ship navigation has been performed eliminating from the dataset the phases of navigation, which required speed reductions—for instance at the pilot boarding positions or situations where a ship is escorted by icebreakers.

An essential part of the models, which are presented here, is reliable information about ice forecast. In this paper we have used the state-of-the-art HELMI model.
Fig. 9. Results of behavior analysis for model B, for the description of the states of variables the reader is referred to Table A1.
5.1. Discussion on cross-validation

Model A tends to overestimate the modeled parameter—ship’s speed—for the lowest speed category (below 5 kn), where in 22% of the cases the model classifies the speed wrongly, assigning it to the higher speed category (between 5 and 10 kn). This means that the model may deliver results, which are too optimistic for a ship. In other speed categories the model tends to slightly underestimate the ship’s speed. The accuracy of the model is 78%, 75% and 93% for the low, medium and high-speed categories respectively, see Table 4.

The discrepancy for the wrong classification of the speed category between 5 and 10 kn can be explained by some speed drops just below 5 kn. If we change the threshold values for the speed categories, and take the lower as below 4 kn, and the second as between 4 and 10 kn, then the prediction power of the model with respect to this middle state of the variable increases from 75% up to 85%. However this manipulation does not reduce the prediction error for the ship’s speed belonging to the lower category. On the contrary, the prediction power of the model with respect to this category drops by 16% if the variable is re-discretized. The model has problems with proper estimation of the speed category in the locations where ship speed fluctuates significantly, and better accuracy than 78% cannot be attained with the presented set of variables.

Considering model B, if we allow the following hypothesis: ship stuck in ice = yes, then the probability for the model to deliver the right prediction is 1, and the probability of false prediction is 0. However, if the alternative hypothesis is adopted, meaning: ship stuck in ice = no, then the probability for the model to deliver the correct response is 0.9. This means that there exists the probability 0.1 for the incorrect prediction, see Table 5. This means, that the model classifies certain combinations of ice features as being probable to stop a ship, whereas in the reality the ship has not been beset in ice under this set of conditions. We found following three reasons for the model B delivering the answer different from the observed data:

1. The model does take into account navigation in ice channel, which took place at least two times during approaching harbor of destination. It is visible in the data, where the recorded ship’s speed is above 10 kn, while the model predicts the speed belonging to the lowest category (0–5 kn)—see Fig. 10. The effect of ice channel navigation is also recognized in Fig. 11, where results of model B are presented. Therein model B predicts the ship being stopped in ice, given existing ice feature, however the ship was still underway. However, in the dataset analyzed ice-channel navigation was rather minor, as a significant level of compression was observed, and the ice channels could not remain open for a long time.
2. The model tends to classify the cases where the ice conditions are challenging and the ship’s speed drops below 4 kn as “ship stuck in ice”, which in the reality is not always the case. According to the recorded data, in some instant of time the ship’s speed dropped dramatically from 8 kn to 3 kn—in Fig. 11 marked as “speed drop”—however the ship managed to continue for some time before she eventually stopped.
3. The HELMI ice model data are insightful for the ice conditions in geographical scale, but the exact conditions at the ship location may fluctuate beyond the HELMI output. The HELMI data are uncertain estimates of the local conditions, given the spatio-temporal grid of 1 NM × 1 NM × 1 hour. Despite these drawbacks, HELMI model is one of the best choices for delivering the ice information for the analyzed sea area.

5.2. Discussion on behavior analysis

One variable in model B, called wind speed, is found not to behave as expected; otherwise the variables react as expected when the outcome variable is set to either of its states. It means that the model delivers higher probability for a ship to get stuck in the presence of lower wind speed. This is not coherent with the available knowledge, as the higher
wind speed may create higher resistance as a ship is pressed against the ice, which increases friction. But it also shows that the effect of wind is not dominant in this case, as when the variable wind speed is removed from the model, its accuracy does not change, as there are other variables having a stronger influence, see Table 7.

5.3. Discussion on the value-of-information analysis

In the case of model A, its actual entropy is as high as 67% of its theoretical, maximum entropy, see Table 6. This means that by running the model, a large amount of information is gained. This comes from the fact that the model’s outcome has three states, with the following prior probabilities: 0.25; 0.32; 0.44 for each, meaning that the states are not so far from being distributed evenly.

The conditional entropy informs how much information about the model outcome can be gained by knowing a variable in the model. Analyzing Table 6, we can conclude that there is not much to be gained by observing only one or even two variables, which should not be surprising knowing the complexity of the analyzed process. Instead, the outcome variable to be explained needs most of the explanatory variables to be present.

In the case of model B, its entropy is relatively small (11% of its theoretical, maximum entropy), which means that the outcome of the model is to large degree predictable, see Table 7. This should not be surprising either, as the outcome of the model has two states with the following probabilities: 0.03 for a ship getting stuck in ice and 0.97 otherwise. Such an imbalance already suggests the probable result of the model for a given dataset, before running the model. Thus the amount of new information gained by running the model is relatively small. However, by learning about certain variables a significant amount of new information about the model outcome can be gained. This means that the model is explained to a large degree by five variables: ship’s speed, ice compression level, level ice concentration, ridged ice concentration and rafted ice thickness, as depicted in Table 7.

6. Conclusions

In this paper we have introduced two probabilistic, event-oriented models predicting ship performance in ice, meaning ship’s speed and the situations where a ship is likely to get stuck in ice. We assume that the knowledge about ice conditions in the moment of making the prediction comes from the ice model only, and information about location of ice channels, which make the ice navigation easier, is not available at this time instant. This holds in the presence of ice compression or ice drift, where the ice channels tend to close rapidly.

The models presented in this paper feature several novelties, first they have been developed with the use of full-scale data; second they predict the ship performance in a probabilistic fashion; third the models consider the joint effect of various ice features on ship performance, finally the ice compression which is known to have significant negative correlation with ship’s speed has been taken into account.

The models have been developed with the use of the techniques of Bayesian learning from the data, where information about ship track was retrieved from the AIS and the ice conditions were obtained from the state-of-the-art numerical model HELMI. Finally, the models have been cross-validated with the use of recorded dataset and by performing set of analyses. Good predictions of the outcome variables for both models have been found.

Notwithstanding all the assumptions, the results obtained are promising as they can help to understand the joint effect of ice features including ice compression on ship’s speed in addition to the conditions associated with ships getting stuck in ice.

The models can be used to determine the probability that a ship will attain a certain speed class or to specify the conditions under which she may get stuck in ice. The latter is especially important for ships navigating the ice-covered sea, where assistance of an icebreaker is not available immediately. Thereby, we expect this new approach to facilitate the optimal route selection problem for ice-covered waters where the ship performance is reflected by an objective function (Guinness et al., 2014).

It should be noted, that the models, which are presented here, are valid only for a specific ship (ice going bulk carrier), and the specific ice model (HELMI). Moreover, the findings can be interpreted only within the limits of the ice features adopted for the analysis. These features reflect harsh ice conditions existing in the Northern Baltic Sea, where the first-year ice is present along with ice ridging and ice compression.

Therefore, further work could focus on analyzing the performance of ships of various types and ice classes. As the results of the presented analyses are promising, and show the potential for the BBNs to assist in the modeling of ship performance in ice, it would be of high interest to perform such analyses for other sea areas, where other types of sea ice exist.

It would also be valuable to compare the prediction of the probabilistic models introduced here with the results obtained with the use of quantity-oriented models for ship performance in ice applying a particular set of data.

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Appendix I. Classifiers adopted for BBNs development

BBNs can be used to compute the conditional probability of one variable, given values assigned to the other variables. Therefore, it can also serve as a classifier, performing classification. The latter is the task to identify the class labels for instances of response variable based on a set of attributes belonging to explanatory variables (Cheng and Greiner, 2001). A classifier gives the posterior probability distribution of the response variable (a.k.a. class node) given the values of explanatory variables. In our case the class labels correspond to:

- the speed intervals of the ship’s speed in model A (0–5 kn; 5–10 kn; above 10 kn),
- the instances of ship being beset in ice in model B (beset = yes; beset = no).

Various classifiers can be applied for developing BBNs; in this study we adopted two Bayesian classifiers, namely Naïve Bayes (NB) and PC.

1.1. Naïve Bayes classifier

The NB is specifically used for classification problems, and the graph structure is predefined, so the algorithm does not determine a structure, it just estimates parameters. The class variable in our case is a Boolean variable ship getting stuck in ice, see Fig. 8. An NB classifier considers each of explanatory variables to contribute independently to the class variable.

An advantage of the NB classifier is that it only requires a small amount of training data to estimate the classifier. Despite its naive design and apparently oversimplified assumptions, this type of classifiers...
has worked quite well in many complex real-world situations, see for example Cheng and Greiner (2001).

In classification, the goal of a learning algorithm is to construct a classifier given a set of training examples (ice features) with class labels (ship stuck or not).

A data record \( D \), used for training the network is represented by a tuple of attribute values \((x_1, ..., x_n)\), where \( x_i \) is the value of attribute \( X_i \). We consider 12 attributes \((n = 12)\), each containing 4040 entries, as specified in Section 2. \( S \) represents the classification variable (ship stuck in ice), and \( s \) is the value of \( S \). In this paper, we assume that the classification variable can take two classes: \( s = 1 \) (ship is beset) or \( s = 0 \) (ship is not beset).

A classifier is a function that assigns a class label \( s \) to a data record. From the probability perspective, according to Bayes rule, the probability of a data record \( D = (x_1, ..., x_n) \) being class \( s \) is:

\[
P(s|D) = \frac{p(D|s)p(s)}{p(D)} \tag{7}
\]

\( D \) is classified as the class \( S = 1 \) if and only if:

\[
f_B(D) = \frac{p(S = 1|D)}{p(S = 0|D)} \geq 1 \tag{8}
\]

where \( f_B(D) \) is called a Bayesian classifier.

Assuming the independency between all the attributes, the Naïve Bayes classifier takes the following form, see (Zhang, 2004), (Spirtes et al., 2001):

\[
f_{NB}(D) = \frac{p(S = 1|D) \prod_{i=1}^n p(x_i|s = 1)}{p(S = 0|D) \prod_{i=1}^n p(x_i|s = 0)} \tag{9}
\]

The function \( f_{NB}(D) \) is called the Naïve Bayesian classifier. Finally, the function combines this probabilistic model with a decision rule to select the hypothesis (model parameters) that is most probable. One common rule is known as the maximum a posteriori (MAP) decision rule, as follows:

\[
\text{classify}(x_1, x_2, ..., x_i) = \max_{s} p(S = s) \prod_{i=1}^n p(X_i = x_i|S = s) \tag{10}
\]

1.2. PC classifier

Unlike the NB classifier, PC is an algorithm based on independence tests, and it is a general algorithm to learn BBNs. The basic procedure of the PC algorithm is as follows (Spirtes et al., 2001), (Acid et al., 2004):

1. it starts with organizing variables in a completely connected, undirected graph.
2. then it performs conditional independence tests, to remove edges from the graph, as follows:
   a. the pairwise independence and after a full iteration, when all the edges are checked, the size of the conditioning set is increased;
   b. the independence test is rerun conditioning on one variable, then increasing the number of variables by one;
   c. if the algorithm finds that two variables are independent it records the separator, which is the set of conditioning variables that was used for the test.
3. After the algorithm has determined a skeleton structure then it finds the orientation of the edges. First the algorithm looks for the following v-structure: \( a \rightarrow b \leftarrow c \), by looking for triplets \((a,b,c)\) where:
   a. there is an edge between \( a \) and \( b \), \( b \) and \( c \);
   b. no edge between \( a \) and \( c \);
   c. \( b \) is not in a separator set of \( a \) and \( c \).
4. After the directed and undirected arcs have been determined, the algorithm tries to orient the remaining undirected arcs by applying two rules to the graph until both of them can no longer match:
   a. orient \( x \rightarrow y \rightarrow z \) as \( x \rightarrow y \rightarrow z \);
   b. orient \( x \rightarrow z \) as \( x \rightarrow z \) if there is a path \( x \rightarrow ... \rightarrow z \).

5. At this point, the developed BBNs may or may not have a directed acyclic graph. The remaining undirected edges are either oriented by hand or randomly.

A user of PC algorithm is allowed to incorporate the background knowledge to the model that could constrain the search. This knowledge might be about the existence or nonexistence of certain edges in the graph, or about the orientation of some of the edges, or about the time order of the variables.

This background knowledge incorporation is realized at the stage of definition of the model’s variables, where the relations between variables can be specified, i.e. forced or forbidden. For instance, a relation between level ice and ship’s speed can be forced, as this is supported by formulae based on physical laws. Also, the relation between wind speed and relative direction of compression is forbidden, as such dependency does not exist.

If prior belief forbids an adjacency, for example, the algorithms need not bother to test for that adjacency; if prior belief requires a direct influence of one variable on another, the corresponding directed edge is imposed and assumed in the orientation procedures for other edges. These procedures assume that prior belief should override the results of unconstrained search. Therefore, PC algorithm represents one way of learning Bayesian networks, called constraint search-based learning (CSBL), where an algorithm searches the data for independence relations to determine the causal relations.

The parameters corresponding to a graph determined by the PC algorithm are estimated with the use of the expected maximization (EM) method for finding maximum likelihood or maximum a posteriori (MAP) estimates of parameters in statistical models, see also Madsen et al. (2003).

The EM iteration alternates between performing an expectation (E) step and a maximization (M) step. The former creates a function for the expectation of the log-likelihood evaluated using the current estimate for the parameters. The latter computes parameters maximizing the expected log-likelihood found on the E step. These parameter-estimates are then used to determine the distribution of the variables in the next E step.

Appendix II

<table>
<thead>
<tr>
<th>Variable’s name</th>
<th>Variable’s states</th>
<th>Output variable ship stuck in ice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ship’s speed [kn]</td>
<td>5</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>5–10</td>
<td>0.25</td>
</tr>
<tr>
<td>Level ice thickness [m]</td>
<td>10</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>0.3–0.4</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>0.4–0.5</td>
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</tr>
<tr>
<td>Level ice concentration [%]</td>
<td>25</td>
<td>0.32</td>
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<tr>
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<td>0.25</td>
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<tr>
<td>Ridged ice thickness [m]</td>
<td>&gt;75</td>
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</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td>3.0–3.5</td>
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<tr>
<td>Ridged ice concentration [%]</td>
<td>&gt;5</td>
<td>0.43</td>
</tr>
<tr>
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<td>0.58</td>
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<tr>
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<td>&lt;5</td>
<td>0.29</td>
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### References


