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# On the Resiliency of Randomized Routing Against Multiple Edge Failures<sup>\*†</sup>

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## Abstract

We study the Static-Routing-Resiliency problem, motivated by routing on the Internet: Given a graph  $G = (V, E)$ , a unique destination vertex  $d$ , and an integer constant  $c > 0$ , does there exist a static and destination-based routing scheme such that the correct delivery of packets from any source  $s$  to the destination  $d$  is guaranteed so long as (1) no more than  $c$  edges fail and (2) there exists a physical path from  $s$  to  $d$ ? We embark upon a study of this problem by relating the edge-connectivity of a graph, i.e., the minimum number of edges whose deletion partitions  $G$ , to its resiliency. Following the success of randomized routing algorithms in dealing with a variety of problems (e.g., Valiant load balancing in the network design problem), we embark upon a study of randomized routing algorithms for the Static-Routing-Resiliency problem. For any  $k$ -connected graph, we show a surprisingly simple randomized algorithm that has expected number of hops  $O(|V|k)$  if at most  $k-1$  edges fail, which reduces to  $O(|V|)$  if only a fraction  $t$  of the links fail (where  $t < 1$  is a constant). Furthermore, our algorithm is deterministic if the routing does not encounter any failed link.

**1998 ACM Subject Classification** C.2.2 Network Protocols

**Keywords and phrases** Randomized, Routing, Resilience, Connectivity, Arborescences

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## 1 Introduction

Routing on the Internet (both within an organizational network and between such networks) typically involves computing a set of *destination-based* routing tables (i.e., tables that map the destination IP address of a packet to an outgoing link). Whenever a link or node fails, routing tables are recomputed by invoking the routing protocol to run again (or having it

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\* Additional details of the results in this note appear in [9].

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run periodically, independent of failures). This produces well-formed routing tables, but results in relatively long outages after failures as the protocol is recomputing routes.

As critical applications began to rely on the Internet, such outages became unacceptable. As a result, “fast failover” techniques have been employed to facilitate immediate recovery from failures. The most well-known of these is Fast Reroute in MPLS where, upon a link failure, packets are sent along a precomputed alternate path without waiting for the global recomputation of routes [23]. This, and other similar forms of fast failover thus enable rapid response to failures but are limited to the set of precomputed alternate paths. Most of existing approaches protect only from a single failure, however in many scenarios (e.g. overlay networks [11], highly-connected large datacenter networks [15]) multiple failures at the same time may be a common occurrence.

The goal of this paper is to perform a theoretical study of failover routing. The fundamental question is, how resilient can failover routing be? That is, how many link failures can failover routing schemes tolerate before connectivity is interrupted (i.e., packets are trapped in a forwarding loop, or hit a dead end)? The answer to this question depends on both the structural properties of the graph, and the limitations imposed on the routing scheme.

Clearly, if it is possible to store arbitrary amount of information in the packet header, perfect resiliency can be achieved by collecting information about every failed link that is hit by a packet [19, 26]. Such approaches are not feasibly deployable in modern-day networks as the header of a packet may be too large for today’s routing tables. Our focus is thus on failover routing schemes that do not involve any change in the packet headers. Another traditional approach to achieving high resiliency is implementing stateful routing, i.e., storing information at a node every time a packet is seen being received from a different incoming link (see, e.g., link reversal [14] and other approaches [20, 21]). As current routing protocols do not allow network operators to implement such stateful failover routing, our goal is to design protocols that correspond to a stateless, or *static*, failover routing.

Specifically, we consider a particularly simple and practical form of static failover routing: for each incoming link, a router maintains a destination-based routing table that maps the destination address of a packet and the set of non-failed (“active”) links, to an output link. The router can locally detect which outgoing links are down and forwards packets accordingly. One should note that maintaining such per-incoming-link destination-based routing tables is necessary; not only is destination-based routing unable to achieve robustness against even a single link failure [18], but it is even computationally hard to devise failover routing schemes that maximize the number of nodes that are protected [2, 5, 18, 24]. We only consider link failures, not router failures (which are not always detectable by neighboring routers, and so such fast failover techniques may not apply).

A failover routing algorithm is responsible for computing, for each node (vertex) of a network (graph), a *routing function* that *matches* an incoming packet to an outgoing edge. A set of such routing functions for each vertex guarantees *reachability* between a pair of vertices,  $u$  and  $v$ , for which there exists a connecting path in the graph, if any packet directed to node  $v$  originated at node  $u$  is correctly routed from  $u$  to  $v$ .

We are interested in routing functions that rely solely on information that is locally available at a node (e.g., the set of non-failed edges, the incoming link along which the packet arrived, and any information stored in the header of the packet).

While it is known that every  $k$ -connected network cannot be partitioned by deleting at most  $k - 1$  links, it is not known whether any static “deterministic” routing (i.e., the outgoing port of each packet is always uniquely determined at a vertex  $v$  by its incoming link and the failed edges incident at  $v$ ) achieves such resiliency.

On the other hand, routing based on random walks, i.e., choosing the outgoing link at random, achieves the best possible resilience as they will eventually deliver a packet to the destination as long as the network is connected. But, it comes with a huge cost. Namely, a random walk might traverse the whole network even when there is no single failed link. In fact, the expected delivery time of a packet would be as large as  $\Theta(|V|^3)$  in some network topologies [6]. Furthermore, a random walk almost never reaches the destination following the shortest path. So, although extremely robust, when it comes to the time needed to deliver a packet to the destination, the behaviour of random walks is undesirable.

## 1.1 Our Results

In this paper, we show how randomness can be used to achieve  $k - 1$  resilient routing in  $k$ -connected networks while significantly outperforming random walks in terms of number of traversed nodes. Namely, we introduce **Randomized failover routing (RND)** in which outgoing edge is chosen for packets in a probabilistic manner based on the destination label, the incoming edge, and the set of non-failed edges. The randomized protocol that we present provides bound on the expected delivery time that gracefully grows with the number of actual link failures.

Our randomized routing functions provide delivery in case of any  $k - 1$  link failures for any  $k$ -connected graph. We achieve that by leveraging the standard decomposition of  $k$ -connected graphs into  $k$  arc-disjoint spanning arborescences  $\mathcal{T}$  [10]. We also provide a bound on the expected number of hops that our algorithm performs, which is  $O(Hk)$  for any  $k - 1$  failures and  $O(H)$  for  $\alpha k$  failures, where  $H$  is the length of the longest branch of any arborescence of  $\mathcal{T}$  and  $\alpha < 1$  is a constant. Furthermore, our routing functions are deterministic as long as the routing does not encounter any failure. Hence, packets belonging to the same logical connection are routed along the same path, minimizing reordering complexity at the receiver side.

Motivated by the fact that one can protect against  $k - 1$  failures in  $k$ -connected graphs using randomness, we make the following general conjecture, whose proof eludes us despite much effort.

- **Conjecture:** For any  $k$ -connected graph, one can find deterministic failover routing functions that are robust to any  $k - 1$  failures.

## 1.2 Organization

The rest of the paper is organized as follows. Section 2 provides background on existing works. In Section 3, we introduce our routing model and formally state the STATIC-ROUTING-RESILIENCY problem. The summary of our routing techniques that are leveraged throughout the whole paper are presented in Section 4. In Section 5, we focus on studying the relation between arborescences our input graph decomposes into and failed links. Section 6 builds on Section 5 and is devoted to designing an algorithm that, for any  $k$ -connected graph, computes randomized routing functions that are robust to  $k - 1$  edge failures and have bounded expected delivery time.

## 2 Related Work

Past work [1, 29] (1) designed such routing functions with guaranteed robustness against *only* a single link/node failure [12, 13, 22, 28, 30, 32], (2) achieved robustness against  $\lfloor \frac{k}{2} - 1 \rfloor$

edge failures for  $k$ -connected graphs [11], and (3) proved that it is impossible to be robust against any set of edge failures that does not partition the network [13].

Thanks to its flexibility and oblivious behavior, another line of study was motivated by randomization. Namely, some of the previous work developed randomized routing schemes, usually to directly or indirectly achieve low congestion and/or balance the network load. In particular, Busch et al. [7] use randomization to adjust packet priorities, which in turn allows them to control deflection of packets.

Valiant [27] proposed a randomized routing algorithm with the goal to balance the load of the underlying network. Since then, that scheme is called *Valiant Load-Balancing* (VLB), whose one of the main ingredients is randomization. VLB was extensively used in designing networks. Zhang-Shen et al. [31] employed VLB to design fault-tolerant networks with guaranteed no congestion under few router or link failures. Greenberg et al. [16] adopt VLB to reduce volatility of traffic and failure pattern of their data centers. In [25], Shepherd et al. extend VLB in order to build cost-effective networks robust to changes in demand patterns.

Beraldi [3] presents a search protocol for mobile networks that is based on modified random walks, i.e. based on biased random walks with look-ahead. Motivated by the success of ant-colonies in their search for food, Günes et al. [17] studied ant algorithms, which in their heart rely on randomization, as an approach to designing on-demand ad-hoc routing algorithms.

Chiesa et al. [8] studied resilience under link failures in  $k$ -connected networks. They devise static routing schemes that are resilient under  $k - 1$  failures in the following regimes: (1) if the routers are allowed to use three bits in the packet header for read/write operation, or (2) if the network supports broadcasting. A building block of those schemes is the result that every  $k$ -connected graph contains  $k$  arc-disjoint arborescences rooted at the same vertex [10].

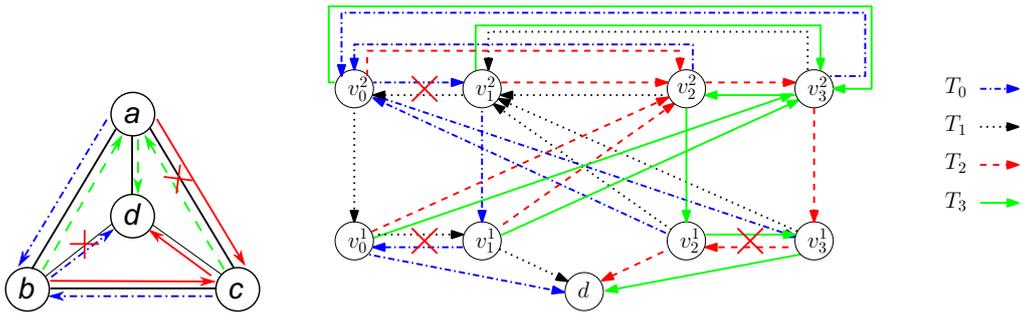
### 3 Model

We represent our network as an undirected multigraph  $G = (V(G), E(G))$ , where each router in the network is modeled by a vertex in  $V(G)$  and each link between two routers is modeled by an undirected edge in the multiset  $E(G)$ . When it is clear from the context, we simply write  $V$  and  $E$  instead of  $V(G)$  and  $E(G)$ . We denote an (undirected) edge between  $x$  and  $y$  by  $\{x, y\}$ . A graph is  *$k$ -edge-connected* if there exist  $k$  edge-disjoint paths between any pair of vertices of  $G$ .

Each vertex  $v$  routes packets according to a *routing function* that matches an incoming packet to a sequence of forwarding actions. Packet *matching* is performed according to the set of active (non-failed) edges incident at  $v$ , the incoming edge, and any information stored in the packet header (e.g., destination label, extra bits), which all are *locally* available at a vertex.

Since our focus is on per-destination routing functions, we assume that there exists a unique destination  $d \in V$  to which every other vertex wishes to send packets and, therefore, that the destination label is not included in the header of a packet. Forwarding *actions* consist of routing packets through an outgoing edge, rewriting some bits in the packet header, and creating duplicates of a packet.

In this paper we consider *randomized* routing functions, in which a vertex forwards a packet through an outgoing edge with a probability based only on the incoming port and the set of active outgoing edges. We present the formal definitions of the randomized routing model in Section 6.



■ **Figure 1** A 3-connected graph with 3 arc-disjoint arborescences colored red, blue, and green.

■ **Figure 2** Graph used in the proof of Theorem 7 for  $k = 2$ .

**The Static-Routing-Resiliency (Srr) problem.** Given a graph  $G$ , a routing function  $f$  is  $k$ -resilient if, for each vertex  $v \in V$ , a packet originated at  $v$  and routed according to  $f$  reaches its destination  $d$  as long as at most  $k$  edges fail and there still exists a path between  $v$  and  $d$ . The input of the SRR problem is a graph  $G$ , a destination  $d \in V(G)$ , and an integer  $k > 0$ , and the goal is to compute a set of resilient routing functions that is  $k$ -resilient.

#### 4 General Routing Techniques and Randomized Algorithm

**Definition and notation.** We denote a directed arc from  $x$  to  $y$  by  $(x, y)$  and by  $\vec{G}$  the directed copy of  $G$ , i.e. a directed graph such that  $V(\vec{G}) = V$  and  $\{x, y\} \in E$  if and only if  $(x, y), (y, x) \in E(\vec{G})$ .

A subgraph  $T$  of  $\vec{G}$  is an  $r$ -rooted arborescence of  $\vec{G}$  if (i)  $r \in V$ , (ii)  $V(T) \subseteq V$ , (iii)  $r$  is the only vertex without outgoing arcs and (iv), for each  $v \in V(T) \setminus \{r\}$ , there exists a single directed path from  $v$  to  $r$  that only traverses vertices in  $V(T)$ . If  $V(T) = V$ , we say that  $T$  is a  $r$ -rooted spanning arborescence of  $\vec{G}$ . When it is clear from the context, we use the word “arborescence” to refer to a  $d$ -rooted spanning arborescence, where  $d$  is the destination vertex. We say that two arborescences  $T_1$  and  $T_2$  are arc-disjoint if  $(x, y) \in E(T_1) \implies (x, y) \notin E(T_2)$ . A set of  $l$  arborescences  $\{T_1, \dots, T_l\}$  is arc-disjoint if the arborescences are pairwise arc-disjoint. We say that two arc-disjoint arborescences  $T_1$  and  $T_2$  do not share an edge  $\{x, y\} \in E$  if  $(x, y) \in E(T_1) \implies (y, x) \notin E(T_2)$ .

For example, consider Fig. 1, in which each pair of nodes is connected by an edge (ignore the red crosses) and three arc-disjoint ( $d$ -rooted spanning) arborescences Red, Green, and Blue are depicted by colored arrows.

**Arborescence-based routing.** Throughout the paper, unless specified otherwise, we let  $\mathcal{T} = \{T_1, \dots, T_k\}$  denote a set of  $k$   $d$ -rooted arc-disjoint spanning arborescences of  $\vec{G}$ . All our routing techniques are based on a decomposition of  $\vec{G}$  into  $\mathcal{T}$ . The existence of  $k$  arc-disjoint arborescences in any  $k$ -connected graph was proven in [10], while fast algorithms to compute such arborescences can be found in [4]. We say that a packet is routed in canonical mode along an arborescence  $T$  if a packet is routed through the unique directed path of  $T$  towards the destination. If the packet hits a failed edge at vertex  $v$  along  $T$ , it is processed by  $v$  (e.g., duplication, header-rewriting) according to the capabilities of a specific routing function and it is rerouted along a different arborescence. We call such routing technique arborescence-based routing. One crucial decision that must be taken is the next arborescence

to be used after a packet hits a failed edge. In this paper, we propose two natural choices that represent the building blocks of all our routing functions. When a packet is routed along  $T_i$  and it hits a failed arc  $(v, u)$ , we consider the following two possible actions:

- **Reroute along some available arborescence**, e.g., reroute along  $T'$ , where  $T'$  is chosen randomly from distribution that we define in the sequel. Observe that, if the outgoing arc belonging to  $T'$  failed, we randomly pick another arborescence  $T''$ , and so on.
- **Bounce on the reversed arborescence**, i.e., we reroute along the arborescence  $T_{next}$  that contains arc  $(u, v)$ .

To grasp how bouncing enters in our picture for obtaining  $k - 1$  resiliency, consider the following case. Assume that in the network there are  $k/2$  failed links, such that every single out of  $k$  arborescences contains one of the links. (As a reminder, arborescences that we construct might share links, but not arcs.) So, this example might suggest that there are scenarios in which already  $k/2$  failed links make all the arborescences not very useful, and that no algorithm can cope with that. But, there is a twist. Let  $k = 2$ , and  $T_i$  and  $T_j$  be the two arborescences and let them share the same failed edge  $a$ . Furthermore, let  $a$  be the only failed edge  $T_i$  and  $T_j$  contain. If a packet hits  $a$  while routed along  $T_i$  or  $T_j$ , then after bouncing on  $a$  the packet will reach  $d$  without any further interruption! So, we have just found a way to resolve a case in which every arborescence contains one failed link, and that is not an isolated scenario, as we discuss in the sequel.

From a different point of view, bouncing is a way of recycling arborescences that contain one failed link. This observation is crucial to obtain an efficient and a simple randomized  $(k - 1)$ -resilient routing scheme, which we are now ready to present. The algorithm is parametrized by  $q$  that we define later.

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**Algorithm 1** Definition of RAND-BOUNCING-ALGO.

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RAND-BOUNCING-ALGO: Given  $\mathcal{T} = \{T_1, \dots, T_k\}$

1.  $T :=$  an arborescence from  $\mathcal{T}$  sampled uniformly at random (u.a.r.)
  2. While  $d$  is not reached
    - a. Route along  $T$  (canonical mode)
    - b. If a failed edge is hit then
      - i. With probability  $q$ , replace  $T$  by an arborescence from  $\mathcal{T}$  sampled u.a.r.
      - ii. Otherwise, bounce the failed edge and update  $T$  correspondingly
- 

In the following sections, we first study the connection between arborescences of  $\mathcal{T}$  and failed links, and show how a part of their intricate interaction can be represented in a simple and an elegant way via, so-called, *meta-graph*. Afterwards, we show the RAND-BOUNCING-ALGO is  $(k - 1)$ -resilient and we analyze its efficiency.

## 5 Meta-graph, Good Arcs, and Good Arborescences

The goal of this section is to provide an understanding of the structural relation between the arborescences of  $\mathcal{T}$  when the underlying  $k$ -connected network has at most  $k - 1$  failed edges. The perspective that we are building here drives the construction of our randomized algorithm.

We start by introducing the notion of a *meta-graph*. To that end, we fix an arbitrary set of failed edges  $F$ . Throughout the section, we assume  $|F| < k$ , and define  $f := |F|$ . Then, we define a meta-graph  $H_F = (V_F, E_F)$  as follows:

- $V_F = \{1, \dots, k\}$ , where vertex  $i$  is a representative of arborescence  $T_i$ .
- For each failed edge  $e \in E$  belonging to at least one arborescences of  $\mathcal{T}$  we define the corresponding edge  $e_F$  in  $H_F$  in the following way:
  - $e_F := \{i, j\}$ , if  $e$  belongs to two different arborescences  $T_i$  and  $T_j$ ;
  - $e_F := \{i, i\}$ , i.e.  $e_F$  is a self-loop, if  $e$  belongs to a single arborescence  $T_i$  only.

Note that in our construction  $H_F$  might contain parallel edges. Intuitively, the meta-graph represents a relation between arborescences of  $\mathcal{T}$  for a fixed set of failed edges. We provide the following lemma as the first step towards understanding the structure of  $H_F$ .

► **Lemma 1.** *The set of connected components of  $H_F$  contains at least  $k - f$  trees.*

**Proof.** We give a proof by contradiction. To that end, assume that the set of connected components of  $H_F$ , denoted by  $\mathcal{C}$ , contains at most  $k - f - 1$  trees. Now, if  $C \in \mathcal{C}$  is a tree, we have  $|E(C)| = |V(C)| - 1$ , and  $|E(C)| \geq |V(C)|$  otherwise. We also have

$$\begin{aligned} \sum_{C \in \mathcal{C}} |E(C)| &= \sum_{C \in \mathcal{C} \text{ is not a tree}} |E(C)| + \sum_{C \in \mathcal{C} \text{ is a tree}} |E(C)| \\ &\geq \sum_{C \in \mathcal{C} \text{ is not a tree}} |V(C)| + \sum_{C \in \mathcal{C} \text{ is a tree}} (|V(C)| - 1). \end{aligned} \quad (1)$$

Next, following our assumption that  $\mathcal{C}$  contains at most  $k - f - 1$  trees, from (1) we obtain

$$\sum_{C \in \mathcal{C}} |E(C)| \geq \sum_{C \in \mathcal{C}} |V(C)| - (k - f - 1). \quad (2)$$

Furthermore, as by the construction we have  $\sum_{C \in \mathcal{C}} |V(C)| = |V_F| = k$ , (2) implies

$$\sum_{C \in \mathcal{C}} |E(C)| \geq |V_F| - (k - f - 1) = f + 1. \quad (3)$$

On the other hand, from the construction of  $H_F$  we have

$$\sum_{C \in \mathcal{C}} |E(C)| = f,$$

which leads to a contradiction with (3). ◀

Lemma 1 implies that the fewer failed edges there are, the larger fraction of connected components of the meta-graph  $H_F$  are trees. Note that an isolated vertex is a tree as well.

In the sequel, we show that each tree-component of  $H_F$  contains at least one vertex corresponding to an arborescence from which any bounce on a failed edge leads to the destination  $d$  without hitting any new failed edge. To that end, we introduce the notion of good arcs and good arborescences. We say that an arc  $(u, v)$  is a *good arc* of an arborescence  $T$  if on the (unique)  $v$ - $d$  path in  $T$  there is no failed edge. Let  $a = (i, j)$ , for  $i \neq j$ , be an arc of  $\vec{H}_F$ ,  $\{u, v\}$  be the edge that corresponds to  $a$ , and w.l.o.g. assume  $(u, v)$  is an arc of  $T_j$ . Then, we say  $a$  is a *well-bouncing arc* if  $(u, v)$  is a good arc of  $T_j$ . Intuitively, a well-bouncing arc  $(i, j)$  of  $\vec{H}_F$  means that by bouncing from  $T_i$  to  $T_j$  on the failed edge  $\{v, u\}$  the packet will reach  $d$  via routing along  $T_j$  without any further interruption. Finally, we say that an arborescence  $T_i$  is a *good arborescence* if every outgoing arc of vertex  $i \in V_F$  is well-bouncing.

► **Lemma 2.** *Let  $T$  be a tree-component of  $H_F$  s.t.  $|V(T)| > 1$ . Then,  $\vec{T}$  contains at least  $|V(T)|$  well-bouncing arcs.*

**Proof.** Let  $T_i$  be an arborescence of  $\mathcal{T}$  such that  $i \in V(T)$ . Then, by the construction of  $H_F$  we have that  $T_i$  contains a failed link. Next, a failed link closest to the root of  $T_i$  is a good arc of  $T_i$ . Therefore, for every  $i \in V(T)$ , we have that  $T_i$  contains an arc which is both good and failed. Furthermore, by the construction of  $H_F$  and the definition of well-bouncing arcs, we have that for every good, failed link of  $T_i$  there is the corresponding well-bouncing arc of  $\vec{T}$ . Also, observe that the construction of  $H_F$  implies that a well-bouncing arc corresponds to exactly one good-arc.

Now, putting all the observations together, we have that each  $T_i$ , for every  $i \in V(T)$ , has a good failed link which further corresponds to a well-bouncing arc of  $\vec{T}$ . As all the arborescences are arc-disjoint, and there are  $|V(T)|$  many of them represented by the vertices of  $T$ , we have that  $\vec{T}$  contains at least  $|V(T)|$  well-bouncing arcs. ◀

Now, building on Lemma 2, we prove the following.

► **Lemma 3.** *Let  $T$  be a tree-component of  $H_F$ . Then, there is an arborescence  $T_i$  such that  $i \in V(T)$  and  $T_i$  is good.*

**Proof.** Consider two cases:  $|V(T)| = 1$ , and  $|V(T)| > 1$ . In the case  $|V(T)| = 1$ ,  $T$  is an isolated vertex which implies that it has no outgoing arcs. Therefore,  $T$  represents a good arborescence.

If  $|V(T)| > 1$ , then from Lemma 2 we have that  $\vec{T}$  contains at most  $2(|V(T)| - 1) - |V(T)| < |V(T)|$  arcs which are not well-bouncing. This implies that there is at least one vertex in  $T$  from which every outgoing arc is well-bouncing. ◀

Let us understand what this implies. Consider an arborescence  $T_i$ , and a routing of a packet along it. In addition, assume that the routing hits a failed edge  $e$ , such that  $e$  is shared with some other arborescence  $T_j$ . Now, if  $e$  corresponds to a well-bouncing arc of  $\vec{H}_F$ , then by bouncing on  $e$  and routing solely along  $T_j$ , the packet will reach  $d$  without any further interruption. Lemma 3 claims that for each tree-component  $T$  of  $H_F$  there always exists an arborescence  $T_i$ , with  $i \in V(T)$ , which is good, i.e. every failed edge of  $T_i$  corresponds to a well-bouncing arc of  $\vec{H}_F$ .

We can now state the main lemma of this section.

► **Lemma 4.** *If  $G$  contains at most  $k - 1$  failed edges, then  $\mathcal{T}$  contains at least one good arborescence.*

**Proof.** We prove that there exists an arborescence  $T_i$  such that if a packet bounces on any failed edge of  $T_i$  it will reach  $d$  without any further interruption. Let  $F$  be the set of failed edges, at most  $k - 1$  of them. Then, by Lemma 1 we have that  $H_F$  contains at least  $k - f \geq 1$  tree-components. Let  $T$  be one such component.

By Lemma 3, we have that there exists at least an arborescence  $T_i$  such that every outgoing arc from  $i$  is well-bouncing. Therefore, bouncing on any failed arc of  $T_i$  the packet will reach  $d$  without any further interruption. ◀

## 6 Randomized Routing via Good Arborescences

In this section, we show that a set of routing functions for  $G$  obtained by RAND-BOUNCING-ALGO is  $(k - 1)$ -resilient. Note that our routing function (RND) maps an incoming edge and the set of active edges incident at  $v$  to a set of pairs  $(e, q)$ , where  $e$  is an outgoing edge and  $q$  is the probability of forwarding a packet through  $e$ . A packet is forwarded through a unique outgoing edge.

The section is structured as follows. As a prelude, we show a simple, yet inefficient, randomized routing algorithm, called RAND-ALGO, that although is  $(k - 1)$ -resilient, fails to achieve low expected number of hops in case of  $k - 1$  failed edges. We then apply our results from Section 5 to show that RAND-BOUNCING-ALGO is both  $(k - 1)$ -resilient and requires up to an order fewer number of hops, compared to RAND-ALGO, to reach the destination.

## 6.1 A Simple (Inefficient) Randomized Routing

Consider the following naive randomized algorithm RAND-ALGO for routing along arborescences. A packet is routed along the same arborescence until it either reaches its destination or hits a failed edge. In the latter case, it is rerouted along another arborescences chosen uniformly at random. We show that there exists a  $k$ -connected graph and a set of failed edges such that the expected number of tree switches that RAND-ALGO makes is  $\Omega(k^2)$ . This further implies that the expected number of hops is  $\Omega(Hk^2)$  in the worst case, where  $H$  is the length of a longest path in any arborescence and assuming that longest path in all the arborescences are up to a constant factor the same.

To prove the promised bound, we start by defining a  $2k$  edge connected graph  $G = (V, E)$  and its set of  $2k$  arc disjoint spanning trees  $T_0, \dots, T_{2k-1}$  as follows.

- Set  $V$  consists of a destination vertex  $d$  and  $4k$  additional vertices arranged into two equal-sized layers  $L_1 = \{v_0^1, \dots, v_{2k-1}^1\}$  and  $L_2 = \{v_0^2, \dots, v_{2k-1}^2\}$ .
- Set  $E$  is defined by the following four subgraphs: (1)  $L_2$  is a clique of size  $2k$ ; (2)  $(L_1, L_2)$  is a complete bipartite graph; (3) for each  $k = 0, \dots, k - 1$ , there is an edge  $(v_{2i}^1, v_{2i+1}^1)$  and (4) vertex  $d$  is connected to each vertex of  $L_1$ . There is no other edge included in  $G$ .

Next, we construct  $2k$  arc-disjoint spanning trees  $T_0, \dots, T_{2k-1}$  (see Fig. 2 for an example with  $k = 2$ ). We use  $[t]_0$  to denote set  $\{0, 1, 2, \dots, t - 1\}$ .

- For each  $i \in [k]_0$ , add the following arcs:
  - $(v_{2i+1}^2, v_{2i}^2)$ ,  $(v_{2i}^2, v_{2i}^1)$ ,  $(v_{2i}^1, v_{2i+1}^1)$ , and  $(v_{2i+1}^1, d)$  into  $T_{2i+1}$ ;
  - arcs  $(v_{2i}^2, v_{2i+1}^2)$ ,  $(v_{2i+1}^2, v_{2i+1}^1)$ ,  $(v_{2i+1}^1, v_{2i}^1)$ , and  $(v_{2i}^1, d)$  into  $T_{2i}$ .
- For each  $i \in [k]_0$ , and for each  $j \in [2k]_0 \setminus \{2i, 2i + 1\}$ , add the following arcs:
  - $(v_j^2, v_{2i}^2)$ , and  $(v_j^1, v_{2i}^1)$  into  $T_{2i}$ ;
  - $(v_j^2, v_{2i+1}^2)$ , and  $(v_j^1, v_{2i+1}^1)$  into  $T_{2i+1}$ .

Finally, consider a scenario in which edges  $(v_0^2, v_1^2)$ ,  $(v_2^2, v_3^2)$ ,  $\dots$ ,  $(v_{2k-4}^2, v_{2k-3}^2)$  and  $(v_0^1, v_1^1)$ ,  $(v_2^1, v_3^1)$ ,  $\dots$ ,  $(v_{2k-4}^1, v_{2k-3}^1)$ ,  $(v_{2k-2}^1, v_{2k-1}^1)$  failed.

We say that a packet is routed *downwards* (*upwards*) if it is routed from a vertex in  $L_2$  ( $L_1$ ) to a vertex in  $L_1$  ( $L_2$ ). Let  $E_d$  be the expected number, minimized over all the vertices, of tree switches of a packet that is routed downwards,  $E_u$  be the expected number of tree switches of a packet that is routed along  $T_i$  and is currently located at  $v_j^2$ , for some  $i \in [2k - 2]_0$ , and  $E_2$  be the expected number of tree switches of a packet that is originated by a vertex in  $L_2$ . Then, we can show.

► **Lemma 5.** *It holds  $E_u \geq \frac{3}{2k-1}E_d + \frac{2k-4}{2k-1}E_u + 1$ .*

**Proof.** Let  $p$  be routed along  $T_h$  and located at  $v_h^2$ , for some  $h \in [2k - 2]_0$ . W.l.o.g, let  $h = 0$ . By the construction of  $T_0$ , from  $v_0^2$  packet  $p$  should be forwarded to  $v_1^2$  but  $(v_0^2, v_1^2)$  has failed. So, from  $v_0^2$ ,  $p$  is forwarded downwards along  $T_1$ ,  $T_{2k-2}$  or  $T_{2k-1}$  with probability  $\frac{3}{2k-1}$  and routed along any other tree  $T_j$  to a vertex  $v_j^2$  in  $L_2$  with probability at least  $\frac{2k-4}{2k-1}$ . Hence, the lemma follows. ◀

► **Lemma 6.** *We have  $E_d \in \Omega(k^2)$ .*

**Proof.** By the construction, a packet routed downwards traverses arc  $(v_i^2, v_i^1)$  of  $T_i$ . W.l.o.g, let  $p$  be routed along  $(v_{2i}^2, v_{2i}^1)$  of  $T_{2i}$ . As  $(v_{2i}^1, v_{2i+1}^1)$ , which belongs to  $T_{2i}$ , has failed,  $p$  is rerouted along  $T_j$  for some  $j \in [2k]_0 \setminus \{2i\}$ . Among them, only  $T_{2i+1}$  has a path from  $v_{2i}^1$  to  $d$  that does not contain any failed link.  $T_{2i+1}$  is chosen with probability  $\frac{1}{2k-1}$ .

If any other tree  $T_j$  is chosen except  $T_{2k-2}$  and  $T_{2k-1}$ , which happens with probability  $\frac{2k-4}{2k-1}$ , then  $p$  is rerouted through  $T_j$  from  $v_{2i}^1$  to a vertex  $v_j^2$  in  $L_2$ , and hence

$$E_d \geq \frac{2k-4}{2k-1} E_u + 1. \quad (4)$$

Putting together with (4) and Lemma 5 we obtain  $E_d \in \Omega(k^2)$ .  $\blacktriangleleft$

We finally observe that any packet originated at a vertex of  $L_2$  is routed downwards at least once before reaching the destination vertex, i.e.,  $E_2 \geq E_d = \Omega(k^2)$ , which proves the following theorem.

► **Theorem 7.** *For any  $k > 0$ , there exists a  $2k$  edge-connected graph, a set of  $2k$  arc-disjoint spanning trees, and a set of  $2k-1$  failed edges, such that the expected number of tree switches with RAND-ALGO is  $\Omega(k^2)$ .<sup>1</sup>*

## 6.2 Correctness of Randomized-Bouncing Routing

In this section we prove that RAND-BOUNCING-ALGO eventually delivers a packet to  $d$ , i.e. it avoids loops, and in the next section we analyze its efficiency.

Assume that we, magically, know whether the arborescence we are routing along is a good one or not. Then, on a failed edge we could bounce if the arborescence is good, or switch to the next arborescence otherwise. And, we would not even need any randomness. However, we do not really know whether an arborescence is good or not since we do not know which edges will fail. To alleviate this lack of information we use a random guess. So, each time we hit a failed edge we take a guess that the arborescence is good, where the parameter  $q$  estimates this likelihood. Notice that RAND-BOUNCING-ALGO implements exactly this approach. As an example, consider Fig. 1. If a packet originated at  $a$  is first routed through **Red** and the corresponding outgoing edge  $\{a, c\}$  is failed, then the packet is forwarded with probability  $q$  to **Blue** or **Green** chosen u.a.r., and with probability  $1-q$  it is bounced to **Green**, which shares the outgoing failed edge  $\{a, c\}$  with **Red**. By the following lemma we show that this approach leads to  $(k-1)$ -resilient routing.

► **Lemma 8.** *RAND-BOUNCING-ALGO produces a set of  $(k-1)$ -resilient routing functions.*

**Proof.** By Lemma 4 we have that there exists at least one arborescence  $T_i$  of  $\mathcal{T}$  such that bouncing on any failed edge of  $T_i$  the packet will reach  $d$  without any further interruption. Now, as on a failed edge algorithm RAND-BOUNCING-ALGO will switch to  $T_i$  with positive probability, and on a failed edge of  $T_i$  the algorithm will bounce with positive probability, we have that the algorithm will eventually reach  $d$ .  $\blacktriangleleft$

<sup>1</sup> In the extended version of this paper [9], we show a more involved example for which RAND-ALGO makes  $\Omega(|V|k^2)$  hops, in expectation, to deliver a packet to  $d$ .

### 6.3 Number of Switches of Rand-Bouncing-Algo

In this subsection we analyze the expected number of times  $I$  the packet is rerouted from one arborescence to another one in RAND-BOUNCING-ALGO. As we are interested in providing an upper bound on  $I$ , we make the following assumptions. First, we assume that bouncing from an arborescence which is not good the routing always bounces to an arborescence which is not good as well. Second, we assume that only by bouncing from a good arborescence the routing will reach  $d$  without switching to any other arborescence. Third, we assume that there are exactly  $k - f$  good arborescences, which is the lower bound provided by Lemma 1 and Lemma 3. Clearly, these assumptions can only lead to an increased number of iterations compared to the real case. Finally, for the sake of brevity we define  $t := \frac{f}{k}$ .

Now, we are ready to start with the analysis. As the first step we define a random variable, where in the definitions  $T$  is the arborescence variable from algorithm RAND-BOUNCING-ALGO,

$X :=$  number of times a failed edge is hit before reaching  $d$  if routing on  $T$ .

Let  $T_{init}$  be the first arborescence that we consider in RAND-BOUNCING-ALGO. Then,  $\mathbb{E}[I]$  is upper-bounded by

$$\mathbb{E}[I] \leq \Pr[T_{init} \text{ is not good}] \mathbb{E}[X|T_{init} \text{ is not good}] + \Pr[T_{init} \text{ is good}] \mathbb{E}[X|T_{init} \text{ is good}], \quad (5)$$

where from our assumptions we have

$$\Pr[T_{init} \text{ is not good}] = t, \text{ and } \Pr[T_{init} \text{ is good}] = 1 - t.$$

To simplify calculations, let  $X_P$  and  $Y_P$  be *pessimistic* upper bound on conditional expected values. That is, let  $X_P$  be the same as  $\mathbb{E}[X|T_{init} \text{ is not good}]$  and  $Y_P$  as  $\mathbb{E}[X|T_{init} \text{ is good}]$  under assumption that: the packet always hits a failed edge unless it bounces on a good arborescence; and, whenever packet bounces on a non-good arborescence it switches to a non-good one.

Now, let us express  $X_P$  and  $Y_P$  as functions in  $X_P$ ,  $Y_P$ ,  $q$ , and  $t$ , while following our assumptions. If  $T$  is not a good arborescence, then a routing along  $T$  will hit a failed edge. If it hits a failed edge, with probability  $1 - q$  the routing will bounce and switch to a non good arborescence. With probability  $qt$  the routing scheme will set  $T$  to be a non good arborescence, and with probability  $q(1 - t)$  it will set  $T$  to be a good arborescence. Formally, we have

$$X_P = 1 + qtX_P + q(1 - t)Y_P + (1 - q)X_P. \quad (6)$$

Applying an analogous reasoning about  $Y_P$ , we obtain

$$Y_P = 1 + qtX_P + q(1 - t)Y_P. \quad (7)$$

Observe that the equations describing  $X_P$  and  $Y_P$  differ only in the term  $(1 - q)X_P$ . This comes from the fact that bouncing on a good arborescences the packet will reach  $d$  without hitting any other failed edge.

By some simple calculations (see [9]), we obtain

$$\mathbb{E}[I] \leq U(q) := \frac{t}{(1 - q)q(1 - t)} + \frac{1}{1 - q}. \quad (8)$$

Now we can prove the following lemma.

► **Lemma 9.** *We have that*

$$\mathbb{E}[I] \leq 2 + 4 \frac{t}{1-t} = 2 + 4 \frac{f}{k-f}.$$

**Proof.** From (8) we have  $\mathbb{E}[I] \leq U(q)$ . Setting  $q = 1/2$  we obtain

$$U(1/2) \leq 2 + 4 \frac{t}{1-t},$$

and by plugging  $t = f/k$  the lemma follows. ◀

Note that if we know  $f$  in advance, or have some guarantee in terms of an upper bound on  $f$ , we can derive parameter  $q$  that improves the running time of RAND-BOUNCING-ALGO, as provided by the following lemma.

► **Lemma 10.**  *$U(q)$  is minimized for  $q = q^* := 1 - (1 + \sqrt{t})^{-1}$ , and equal to*

$$U(q^*) = \frac{1 + \sqrt{t}}{1 - \sqrt{t}}. \quad (9)$$

**Proof.** Consider  $U(q)'$ , which is

$$U(q)' = \frac{t(1-q)^2 - q^2}{(1-q)^2 q^2 (t-1)}.$$

In order to find the value of  $q$  that minimizes  $U(q)$ , denote it by  $q^*$ , we find the roots of  $U(q)' = 0$  with respect to  $q$ . There is only one positive solution of equation  $U(q)' = 0$ , which is also the minimizer  $q^*$ , and is equal to  $q^* = 1 - \frac{1}{1+\sqrt{t}}$ , as desired.

Finally, substituting  $q^*$  into (8) and simplifying the expression we obtain (9). ◀

Observe that

$$U(q^*) \leq \frac{4}{1 - \frac{f}{k}}.$$

Therefore, if  $f = \alpha k$ , i.e., only a fraction of the edges fail, we obtain  $U(q^*) \leq \frac{4}{1-\alpha}$ . This means that the expected number of arborescence switches does not depend on the number of failed edges but on the ratio between this number and the connectivity of the graph. Otherwise, if  $f = k - 1$ , we have that the expected number of arborescence switches is bounded by  $4k$ , which is linear w.r.t. to the connectivity of the graph. Combining these conclusions with Lemma 8, we obtain the following.

► **Theorem 11.** *Given a  $k$ -connected graph  $G$ , destination  $d$  and a decomposition of  $G$  into  $k$  arc-disjoint arborescences  $\mathcal{T}$  rooted at  $d$ , there exists a  $(k-1)$ -resilient algorithm that delivers a packet to  $d$  after  $O\left(\frac{k}{k-f}H\right)$  hops in expectation, where  $H$  is the length of a longest path of any arborescence of  $\mathcal{T}$  and  $f$  the number of failed edges. The algorithm uses randomization only when encounters a failed edge. In particular, if  $f = 0$ , the algorithm is deterministic.*

## 6.4 An Extension: Rerouting in a Non-uniform Manner

In this section we briefly study non-uniform choice of arborescence used for rerouting in algorithm RAND-BOUNCING-ALGO. To motivate that discussion, consider a scenario in which a packet hits a failed edge  $u, v$  while routed along arborescence  $T$ . Wlog, assume  $T = T_k$ . Furthermore, assume that path  $v-d$  along every other arborescence does not contain any

failed link. Therefore, switching from  $T_k$  to any other arborescence the packet will reach  $d$  without any further interruption. If the packet is rerouted at step 2.2.(a) of algorithm RAND-BOUNCING-ALGO but not bounced, then the rerouting tree is chosen uniformly at random. It further means that the expected number of edges the packet will traverse before reaching  $d$  from  $v$  is

$$E_U = \sum_{i=1}^{k-1} \frac{dist_{T_i}(v)}{k-1} = \frac{\sum_{i=1}^{k-1} dist_{T_i}(v)}{k-1},$$

where  $dist_{T_i}(a)$  is the number of the edges on the unique path from  $a$  to  $d$  along arborescence  $T_i$ .<sup>2</sup> However, the distances from  $v$  to  $d$  along different arborescences might significantly differ. This naturally suggests us to consider a non-uniform distribution of arborescences chosen at step 2.2.(a) of RAND-BOUNCING-ALGO, as we do in the rest of this section.

For each vertex  $v \neq d$  and each arborescence  $T_i$  define probability  $p_v^i$  as

$$p_v^i := \frac{\frac{1}{dist_{T_i}(v)}}{\sum_{j=1}^{k-1} \frac{1}{dist_{T_j}(v)}}.$$

The expected number of the edges the packet will traverse if each arborescence is chosen with respect to the distribution given by  $p_v$  is

$$E_{NU} = \sum_{i=1}^{k-1} p_v^i dist_{T_i}(v) = \sum_{i=1}^{k-1} \frac{1}{\sum_{j=1}^{k-1} \frac{1}{dist_{T_j}(v)}} = \frac{k-1}{\sum_{i=1}^{k-1} \frac{1}{dist_{T_i}(v)}}.$$

Now we would like to show that indeed  $E_U \stackrel{?}{\geq} E_{NU}$ . But, it is the same as showing that

$$(k-1)^2 \stackrel{?}{\leq} \sum_{i=1}^{k-1} dist_{T_i}(v) \sum_{i=1}^{k-1} \frac{1}{dist_{T_i}(v)}.$$

However, the latter follows from Cauchy-Schwarz inequality as

$$(k-1)^2 = \left( \sum_{i=1}^{k-1} \sqrt{dist_{T_i}(v)} \sqrt{\frac{1}{dist_{T_i}(v)}} \right)^2 \leq \sum_{i=1}^{k-1} dist_{T_i}(v) \sum_{i=1}^{k-1} \frac{1}{dist_{T_i}(v)}.$$

Hence,  $E_U \geq E_{NU}$ , as advertised.

We note that this example is a potential scenario that might occur. However, and unfortunately, in case of failures we are unable to detect whether the described situation has occurred or not. Nevertheless, we believe that in practical applications the non-uniform choice of arborescences used for rerouting, as described above, would result in a more efficient routing than its uniform counterpart.

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<sup>2</sup> As a remark, the best option in this scenario would be to reroute the packet along the arborescence  $T_i$  such that  $i = \arg \min_i dist_{T_i}(v)$ . Unfortunately, our model does not provide the information whether there is any failed edge on path  $v-d$  along  $T_i$  or not.

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