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Application of Monte Carlo simulation for estimation of uncertainty of four-point roundness measurements of rolls

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\begin{abstract}
Large-scale rotors in the paper and steel industry are called rolls. Rolls are reground at regular intervals and roundness measurements are made throughout the machining process. Measurement systems for roundness and diameter variation of large rolls (diameter <2000 mm) are available on the market, and generally use two to four sensors and a roundness measurement algorithm. These methods are intended to separate roundness of the rotor from its movement. The hybrid four-point method has improved accuracy, even for harmonic component amplitudes. For reliable measurement results, every measurement should be traceable with an estimation of measurement uncertainty. In this paper, the Monte-Carlo method is used for uncertainty evaluation of the harmonic components of the measured roundness profile under typical industrial conditions. According to the evaluation, the standard uncertainties for the harmonic amplitudes with the hybrid method are below 0.5 \(\mu\)m for the even harmonics and from 1.5 \(\mu\)m to 2.5 \(\mu\)m for the odd harmonics, when the standard uncertainty for the four probes is 0.3 \(\mu\)m each. The standard uncertainty for roundness deviation is 3.3 \(\mu\)m.
\end{abstract}

\section{Introduction}

Roundness is defined by ISO 12181-1 \cite{1} and ISO 12181-2 \cite{2} as a geometrical property of a cross-section of a piece intended to be round. Roundness is an important feature of all rotating machines where smooth rotation of the rotors or even surface quality and even thickness of the end product are needed, such as paper machines, steel strip or sheet production, printing machines, engines and generators etc. In length metrology, diameter is often measured as a two-point measurement that is affected by out-of-roundness of the part. Measurements of roundness profiles are also useful when a specific harmonic component is critical or important, e.g. for vibration excitation. In laboratories, roundness measuring machines can measure deviation from roundness using a single sensor, as high-accuracy bearing assembly ensures that there is only a small rotational error in the radial direction \cite{3, 4, 5}.

In paper mills, roundness measurements are usually carried out with the roll placed on a lathe or grinding machine as shown in Fig. \ref{fig:1}. Heavy rolls rotate with their own bearings or are supported by sliding pads. With these measurement setups it is difficult to avoid a rotational error of the roll’s centreline; thus one- or two-point measurement methods cannot properly separate this rotational error from the geometry of the workpiece – hence the usage of multi-point measurement devices in the paper industry \cite{6}. Most of these devices are based on the Ozono method, where the roundness is calculated from weighted sensor signals in a given configuration around the rotor \cite{7}. In the steel industry the roundness tolerances of the rolls are not as tight as in the paper industry, thus a two-point measurement device is used, which is well suited for diameter variation profile measurement. Generally, in steel strip and paper production the diameter and the diameter variation profiles are more important than the roundness \cite{8, 9, 10, 11}.

The reliability of the measurement is naturally important for machined workpieces in production. Competitive production needs reliable information about the geometry of the workpiece or some specific dimension or feature of the workpiece, e.g. roundness profile. In modern machine tools for large scale rotors, i.e. in paper

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Heavy rolls rotate with their own bearings or are supported by sliding pads. With these measurement setups it is difficult to avoid a rotational error of the roll’s centreline; thus one- or two-point measurement methods cannot properly separate this rotational error from the geometry of the workpiece – hence the usage of multi-point measurement devices in the paper industry.}
\end{figure}

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\textsuperscript{*} Photo by RollResearch Int. Ltd.

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or steel mills, the reliability of the onsite measurement device is important also for the error compensation of the roll grinder or lathe. The control systems of the machine tools use the geometry information provided by the measurement device for error compensation; thus the measured geometry must be accurate for the compensation to be correct [8,10,11].

Uncertainty of a measurement can be evaluated using the “GUM” method, which uses a linear Taylor expansion of the measurement model with sensitivity coefficients [12]. If the measurement model is simple, this method is straightforward and used extensively. However, once the measurement model becomes complex, as with measurement of rolls, the sensitivity coefficients are difficult to evaluate.

In 2008, “Supplements to the GUM” were published describing the use of the Monte-Carlo method for uncertainty evaluation [13]. Using the Monte Carlo method the measurement is simulated using input quantities which are random, but follows probability density functions relevant to each uncertainty contribution to the measurement [14–16]. Its strength is that non-linearity in the measurement model is not a problem.

In this paper, the principle of the four-point method is described first. The application of the Monte-Carlo method for an uncertainty evaluation is presented next, and finally the simulation results are reported and discussed.

2. Material and methods

2.1. Roundness and Fourier series

The roundness profile is typically presented in polar coordinates, but for analytical purposes a more relevant presentation is the use of Fourier series terms. For roundness profile characterization only terms with \( n \geq 2 \) are significant, because the term \( n = 0 \) denotes the offset of the signal, i.e. the DC value, and the term \( n = 1 \) stands for the eccentricity of the roundness profile. Therefore, our results include only the terms \( n \geq 2 \).

One of the most common Fourier analyses is done with the fast Fourier transform (FFT) algorithm developed originally by Cooley & Tukey [17]. The inverse FFT algorithm can be used to compose the original measurement signal in the time domain from these complex numbers. Filtering of some unwanted frequencies or components is straightforward. The complex number representing the unwanted frequency or component is set to zero before the inverse FFT, an example of which is shown by Mosier-Boss et al. [18]. In the analysed measurement signals of our research, the FFT algorithm is used both for identifying certain harmonic components and for filtering purposes.

2.2. Four-point roll roundness measurement

The studied four-point roundness measurement method is a combination of the two-point method and the Ozono three-point method. Both are briefly discussed here.

2.2.1. Two-point method

The two-point method uses only two probes (Fig. 2A). In some applications, one of the probes can be replaced with a fixed point (Fig. 2B). Practical implementations of this type of device include modified roll callipers (see Fig. 3). These devices can also be used for absolute diameter measurements, if the distance between the probe and the follower or the distance between the two probes is known. Otherwise it can only measure the variation in the diameter when measuring a rotating object.

This method measures the diameter profile or diameter variation profile. In principle, the only difference between the two is that in variation profile, the average or minimum diameter value has been subtracted. The diameter variation measurement is commonly used for large roll grinding machines. There, the measured profile is inaccurately called the “roundness profile”, although a two-point measuring method cannot measure the true roundness profile because it suffers from harmonic filtration. Using this type of diameter-measuring device one cannot measure odd lobe shapes like triangular, 5-lobe, 7-lobe etc. geometries, because the method is unable to separate the form error of the cross-section from the error motion of the rotating axis [8,19,20].

Calculation of the measured diameter variation profile of a workpiece with the two-point method is straightforward, and includes only addition or subtraction depending on the orientation of the probes. If the values of the probes increase in the direction of the increasing diameter, the measured diameter variation \( \Delta d \) is:

\[
\Delta d (\theta) = s_1 (\theta) + s_4 (\theta),
\]

where \( s_1 \) and \( s_4 \) are measured sensor signals (see Fig. 3). Variation for radius \( \Delta r \) is:

\[
\Delta r (\theta) = \frac{s_1 (\theta) + s_4 (\theta)}{2},
\]

The harmonic amplitudes \( D_n \) are then calculated by Fourier transform of the roundness profile

\[
D_n = \mathcal{F} \left( \Delta r (\theta) \right),
\]

where \( n = 2, 4, N/2 \).

2.2.2. Three-point Ozono roundness measurement method

One of the first numerical methods in the literature for assessing roundness is that by Ozono [7]. The method is complex, thus only the basic principles are presented here. The roundness profile is determined by measuring run-out \( s_1 (\theta), s_2 (\theta) \) and \( s_3 (\theta) \) at three

![Fig. 2. Two-point measurement probes orientations with A) two sensors and B) one sensor.](Image 182.png)
different angles denoted by $\gamma_1$, $\gamma_2$ and $\gamma_3$. In practice, the first angle is set as $\gamma_1 = 0$ as shown in Fig. 4.

A roundness profile function $m(\theta_k)$ is introduced and denoted as

$$m (\theta_k) = s_1 (\theta_k) + w_2 s_2 (\theta_k) + w_3 s_3 (\theta_k),$$  \hspace{1cm} (4)

where $k = 0, 1, 2, \ldots, N - 1$, and where $N$ is the number of samples per revolution.

The idea is to eliminate centre point motion by using Eq. (4) with appropriate weighting factors $w_2$ and $w_3$. The weighting factors $w_2$ and $w_3$ are derived from the conditions

$$\sin \gamma_1 + w_2 \sin \gamma_2 + w_3 \sin \gamma_3 = 0$$  \hspace{1cm} (5)

$$\cos \gamma_1 + w_2 \cos \gamma_2 + w_3 \cos \gamma_3 = 0.$$  \hspace{1cm} (6)

Kato et al. [21] have developed a numerical method to optimize the measuring angles, resulting in

$\gamma_1 = 0^\circ$, $\gamma_2 = 38^\circ$, and $\gamma_3 = 67^\circ$.

As a function of observation angles the weighting factors $w_2$ and $w_3$ can be expressed as

$$w_2 = -\frac{\sin \gamma_3}{\sin (\gamma_3 - \gamma_2)} , \text{ and}$$  \hspace{1cm} (7)

$$w_3 = \frac{\sin \gamma_2}{\sin (\gamma_3 - \gamma_2)}.$$  \hspace{1cm} (8)

Previous studies show that the sensitivity of the algorithm is at its best with no major harmonic suppression when the number of lobes per revolution is below 35 [22]. The harmonic amplitudes $E_n$ are then calculated by Fourier transform of the roundness profile

$$E_n = \mathcal{F} \left( m \left( \theta_k \right) \right).$$  \hspace{1cm} (9)

### 2.2.3. Hybrid four-point roundness measurement

One of the multi-point methods commonly used in the roll geometry measurement devices of roll grinding machines is the hybrid four-point method. The method behind the hybrid four-point measurement device is based on the three-point Ozono method [7], but combined with the two-point measurement method. As mentioned above, the two-point measurement method (when only using sensors $S_1$ and $S_4$ in Fig. 5) suffers from harmonic filtration, making it unsuitable for the measurement of odd-numbered harmonic lobes of a roundness profile [9,19], but the even-numbered harmonic lobes are measured accurately. The hybrid four-point method presented originally by Väänänen [23] uses the Ozono method to measure the odd-numbered harmonic lobes and combines the result with the even-numbered lobes measured with the two-point method. Because of this, the hybrid four-point method should ensure an overall better accuracy compared with the Ozono or two-point method alone.

The harmonic amplitudes of the hybrid four-point method can be expressed as

$$G_n = |D_n|, \text{ if } n \text{ mod } 2 = 0$$

$$G_n = |E_n|, \text{ if } n \text{ mod } 2 = 1$$  \hspace{1cm} (10)

$n = 2 \ldots N$.

$G_n$ have the even amplitudes from the two-point method and the odd amplitudes from the Ozono method. Fig. 6 illustrates the
analytic principle of the method. The idea of combining harmonics from different sets of measurements has also been used in the calibration of roundness standards [24].

2.3. Measurement devices

There are several versions of the measurement device (see Figs. 1 and 7, Fig. 7). All of them have four probes attached either directly to a frame or to four radially adjustable rods on a frame. The rods are used to bring the probes into the measurement position, if rotors with different diameters are measured. Our simulations were based on the adjustable rod setup as shown in Fig. 7, which creates an additional source of error (rod alignment error, see Fig. 12).

2.3.1. Measurement frame

The frame of the measurement device is made of carbon fibre due to its low thermal expansion coefficient and lightness. There are two frame types, the first consisting of one piece with all of the rods and probes on the same frame (Fig. 7 right), the second comprising two parts (Fig. 7 left). The two-part frame may introduce an additional error source, but is outside the scope of this paper.

2.3.2. Probes

There are several alternatives for probes. Commonly used displacement probes are length gauges working internally with photoelectric scanning of a grating and a plunger with a ball touching the roll. For the chosen length gauge (Heidenhain MT 12) the measurement error was verified to be within ±0.2 μm when calibrated against a laser interferometer at the Finnish national metrology institute (VTT MIKES). Different versions with different measurement heads exist.

2.3.3. Calibration disc

For testing and calibration of roundness measurement devices, discs with different roundness properties are used. An example is shown in Fig. 8 with a disc diameter of around 500 mm.

In a previous work [9] calibration disc with a roundness profile containing 2–30 undulations per revolution (UPR) was designed and manufactured (Fig. 9). The roundness deviation of the profile was minimized by optimizing the phase angles of the individual harmonics. In the previous work [9] the calibration disc was measured with four laser probes and the measurement result averaged over 100 rotations. Same four-point method was used as in this paper.

The roundness profile of the calibration disc was manufactured by grinding. Due to the limitations of the grinding accuracy, the harmonic amplitudes of the ground profile differ from the nominal 10 μm by several micrometres. The development and use of the disc and two other similar discs with different profiles were presented elsewhere [25], as were the preliminary results [26], but two roundness measurements from the disc with the profile shown in Fig. 9 are discussed briefly here. One measurement was performed with a laboratory four-point test device, comprised mostly of the same parts as a commercial device but with a self-made frame. During this measurement the disc was rotated on roller bearings. The difference between the roundness plots in Fig. 10 is caused in part by differences in filtering. The RONt values were 109.0 μm measured by the reference device and 106.5 μm for the four-point device. For the harmonic amplitudes the differences were less than 1 μm (Fig. 11).
2.4. Uncertainty evaluation by simulation

2.4.1. Probability distributions

In general, uncertainty evaluation or uncertainty budgets have been used to identify predominant uncertainty sources. In a typical “Classic Gum” approach, all uncertainty components are collected into one table together with sensitivity coefficients. As noted elsewhere [27], there is no counterpart to equivalent sensitivity coefficients in the Monte Carlo method. However, it is possible to run the Monte Carlo simulation with one uncertainty source at a time while holding the other input quantities fixed at their best estimates [27]. A ‘non-linear’ sensitivity coefficient can be defined from the results [27].

The profile used in the simulation is the same as in Fig. 9. The algorithm doing the calculations for the four-point method (Eq. (10)) was acquired as an executable program, which takes measured data as an input file and calculates the harmonic amplitudes as a result of roundness. The principle for the Monte Carlo simulation is to generate synthetic data representing a roundness measurement, distorted with suitable distributions for error contributions. Next, uncertainty evaluation inputs are presented for a four-point measurement system in industrial use. The assumed uncertainty contributions are based on experience in typical industrial environments. The probability distribution functions (PDF) are assumed to follow a normal distribution where the standard deviation is a property of the variation of input values.

Expected error sources are the probes themselves and their angular orientation and positioning. Thermal expansion and vibration of the measurement frame (Figs. 1, 7 and 8) and movement of the centreline are other possible sources. The position of a length probe in the Ozono method should be at either $0^\circ$, $38^\circ$, $67^\circ$, or $180^\circ$ in polar co-ordinates (see Figs. 4 and 5). It is assumed that these positions differ from the nominal angular values with a standard uncertainty of $0.5^\circ$ (Fig. 12).

A standard uncertainty of $0.3\,\mu m$ for the scale error of length probes is assumed based on calibrations and experience.

The temperature of the instrument is assumed to be $20\,^\circ C$, with a standard uncertainty of $0.5\,^\circ C$. One measurement typically takes 10–30 s to complete. The effect of temperature is taken as a linear expansion of the whole measurement device during the measurements. Far more complex temperature effects may occur in different industrial environments, e.g. when measuring hot or...
warm workpieces. Modelling of these should be based on real temperature measurements and different scenarios and is outside the scope of this paper.

For length probes, an assumed alignment error with standard uncertainty of one degree is estimated. The resulting cosine error for an effective length of 1 mm is about half of the scale error of the probes and can be omitted, as preliminary analysis showed that also the scale error is of minor significance.

In all roundness measurements of rolls there is also some movement of the centreline of the roll. In the simulation it is assumed that the centre will move with an amplitude of 10 μm with a frequency twice the rotational frequency. The probability distribution function is arcsine, i.e. U-shaped representing the cyclic variation from -10 μm to 10 μm.

Significant error sources with their probability density functions are shown in Table 1. As there are four rods with probes, the number of separately simulated quantities is ten.

A script, written in Python and using the Scipy mathematical package, generates input data files representing simulated measurement data containing the desired PDFs and calls the executable analysis program for the four-point method. From a test run with no error contributions from the probes etc., the result shows that the algorithm works well. The Monte Carlo simulation with the PDFs of Table 1 is done with a large number of test runs, and from the results the mean and standard deviations of the outputted harmonic components is calculated. To evaluate the sensitivity for each uncertainty source, simulations are done with one error source at a time for alignment, probe error and temperature change. The standard deviation σ was set to 1.0° for alignment, 1.0 μm for scale error and 1.0 °C for temperature change. These results are relative to the selected σ value and serve to illustrate virtual sensitivity as discussed earlier.

3. Results and discussion

The output from the Monte Carlo simulations with 10 000 runs is shown in Figs. 13–17, where the different standard uncertainties are shown as error bars. Each simulation with 10 000 runs took half an hour on a Windows PC with Intel i7 processor. Fig. 13 shows the output from a simulation run with centre point movement as the only source of error. This simulation demonstrates the method in conditions where the σ values of PDFs of the measurement instrument are set to zero. The result shows that only harmonics are affected by the centreline movement used in the simulation. This is a limitation of the four-point method. The maximum deviation of the amplitude was less than 0.05 μm.

![Fig. 11. Comparison of the calculated harmonics of the four-point result and reference roundness result.](image)

![Fig. 12. Alignment/position error of a rod.](image)

### Table 1

Selected significant error sources. The notation of the PDFs follows specific guidelines [12].

<table>
<thead>
<tr>
<th>Quantity</th>
<th>PDF</th>
<th>Parameters</th>
<th>μ</th>
<th>σ</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alignment of the rods</td>
<td>N(μ, σ²)</td>
<td>0°</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scale error of the probe</td>
<td>N(μ, σ²)</td>
<td>0°</td>
<td>0.3 μm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temperature change</td>
<td>N(μ, σ²)</td>
<td>20 °C</td>
<td>0.5 °C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Movement of the centreline</td>
<td>U(a, b)</td>
<td></td>
<td>-10 μm</td>
<td>10 μm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The result shown in Fig. 14 is from a simulation run with all the error sources listed in Table 1. This represents the measurement uncertainty of the method under assumed typical measurement conditions in the industry. The uncertainties of the odd harmonic amplitudes are generally higher than for the even harmonics. This is a feature of the hybrid measurement method, where odd harmonics are calculated with the Ozono method and even harmonics with the two-point method. The deviation was also analysed from the simulated roundness. Fig. 14 also shows the average roundness curve, and from the results the standard uncertainty for roundness deviation was evaluated at 3.3 μm when filtered with a Gaussian filter with cut-off UPR 30.

The sensitivity results with the thermal error source show a similar increase in the uncertainties of the odd harmonic amplitudes (Fig. 15). The reason is also the same: different calculation methods for odd and even harmonics.

The sensitivity results with the four probe error sources show very small uncertainties for all of the probes (Fig. 16).

These simulations were run with one probe error value σ = 1 μm at a time. It seems that the hybrid four-point roundness measurement method is robust and not sensitive to probing error. However, with the Monte Carlo uncertainty evaluation there is a risk that the errors are averaged out too much. This happens if the error source in reality is not as random as assumed with normal distribution and zero expectation value μ. Some experiments are required before the low sensitivity obtained for the probing error can be finally concluded. These studies would include autocorrelation of points measured with one probe and cross-correlation for data between several probes. The probes are insensitive to small alignment errors (cosine error type), because their magnitude is negligible compared to other error sources.

The simulations for rod alignment error sensitivity produced clear uncertainties for odd harmonic amplitudes for the rods S₁.
Fig. 15. Output from Monte Carlo simulation, showing standard uncertainties as error bars. This simulation was run with a thermal error ($\sigma = 1 \, ^\circ C$).

Fig. 16. Outputs from four Monte Carlo simulations for probe error, with the standard uncertainties shown as error bars. These simulations were run with one probe error at a time ($\sigma = 1 \, \mu m$).
to S2, S1 and S4 have very small uncertainties for even harmonic amplitudes. The results are shown in Fig. 17.

The hybrid four-point roundness measurement method is similar to the Ozono method and sensitive to the errors in the S2 and S3 run-out signals, which is natural because the Ozono method forms part of the roundness calculation. Temperature variation affects the measurement result noticeably, but this is mitigated by the short measurement time, and there is normally no need to measure hot or warm workpieces since their geometry changes with temperature.

4. Conclusions

Knowledge of measurement uncertainty is a fundamental requirement arising from both practical problems, scientific issues and quality systems. Measurement of rolls in an industrial environment using a four-point measurement device is an example of a measurement with large economic impact where knowledge of measurement uncertainty has been weak or non-existent. To our knowledge this paper, which presents the evaluation of this measurement uncertainty, is the first of its kind. The influence of several uncertainty components is analysed and discussed. The results are characteristic of the hybrid four-point method, although unstable temperature conditions or the presence of vibrations may make additional uncertainty contributions in a very rough industrial environment.

With the present assumptions, the four-point hybrid algorithm works well. This in conformance with the good experience from industrial use. We also conclude that the predominant uncertainty contribution for a four-point measurement instrument is the positioning of rods of the probes S2 and S3. According to our evaluation, the standard uncertainties for harmonic amplitudes with the hybrid method are below 0.5 μm for even harmonics, and from 1.5 μm to 2.5 μm for odd harmonics. The uncertainties of the odd harmonic amplitudes are generally higher than for the even harmonics. The evaluated uncertainties are in line with measurements using a calibration disc. A further result is insensitivity to probe error. However, future research is needed to investigate the statistical properties of randomness in probe error, and to refine error modelling, before final conclusions can be drawn regarding robustness to probe error. Also, the uncertainty of the phase of harmonic components should be investigated further.

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